AN ASYMPTOTICALLY OPTIMAL APPROACH TO THE DISTRIBUTED ADAPTIVE TRANSMIT BEAMFORMING IN WIRELESS SENSOR NETWORKS

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ABSTRACT

We present an asymptotically optimal solution for feedback based distributed adaptive transmit beamforming in wireless sensor networks. This solution utilizes feedback provided by a remote receiver in order to estimate optimum phase offsets of individual carrier signals. In a mathematical simulation we show that the global random search approach, which was applied in prior studies of this scenario, is outperformed by the proposed algorithm. Furthermore, we study the performance and feasibility of distributed adaptive transmit beamforming for two mobility models and derive the maximum possible velocities of nodes for both approaches.

1. INTRODUCTION

In the scenario of distributed adaptive transmit beamforming for wireless sensor networks, a set of wireless nodes of a sensor network combine their carrier signals to reach a distant receiver as a distributed beamformer. When carrier symbols of n transmit nodes are tightly synchronized, the received signal strength RSS_{sum} of the sum signal at a remote receiver can be greatly improved compared to the received signal strength RSS_i of individual signal components $i \in [1, \ldots, n]$.

A solution to synchronize carrier signals of distributed wireless nodes is virtual/cooperative MIMO for wireless sensor networks [1, 2, 3]. In virtual MIMO for wireless sensor networks, single antenna nodes cooperate to establish a multiple antenna wireless sensor network. The nodes broadcast their data in the network using a TDMA-scheme. After propagating the data, all nodes transmit simultaneously identical signals acting as a multiple antenna system.

Virtual MIMO has the capability of adjusting to different frequencies and is highly energy efficient [4, 5]. However, the implementation of MIMO capabilities in WSNs requires accurate time synchronization, complex transceiver circuits and signal processing that might exceed the power consumption and processing capabilities of simple sensor nodes.

Alternatives to virtual MIMO are closed-loop feedback based approaches to distributed adaptive beamforming in wireless sensor networks. For these methods, a receiver controls the synchronization of transmit nodes by correcting the phase offset among carrier signals of transmitters. This approach, however, is restricted to networks of small size and requires considerable processing capabilities at the source nodes [6].

In the case of wireless sensor nodes, which are typically limited in their processing power and energy consumption, less computationally demanding closed-loop synchronization approaches are better suited to synchronize carrier signals of transmit nodes. In [7], a computatinally less demanding one-bit feedback based approach for closed-loop synchronization is detailed.

In this iterative process, n source nodes $i \in [1, \ldots, n]$ randomly adapt the phases γ_i of their carrier signal

$$\Re\left(m(t)e^{j(2\pi(f_c+f_i)t+\gamma_i)}\right). \quad (1)$$

In this equation, f_i describes the frequency offset of the carrier signal component from node i to a common carrier frequency f_c.

A possible scenario for distributed adaptive transmit beamforming in wireless sensor networks is depicted in figure 1. In this example, the remote receiver is located on a helicopter. Wireless sensor nodes are distributed in an agricultural setting to collect relevant data about the plants on a field. As the transmission power of each single node is too weak to reach the distant receiver, a set of nodes may transmit identical data simultaneously as a distributed beamformer. We assume that this data was exchanged among nodes beforehand and that initially, phase offsets γ_i of carrier signals are independently and identically distributed (i.i.d.). The tight synchronization among carrier phases of transmitting nodes is achieved in an iterative manner, as depicted in the figure.

The synchronization process is initialized by the remote receiver. Afterwards, the following four steps are iterated until sufficient synchronization is achieved.

Step 1: Each source node i adjusts its carrier phase offset γ_i and frequency offset f_i randomly.

Step 2: The source nodes transmit to the destination simultaneously as a distributed beamformer.

Step 3: The receiver estimates the level of phase synchronization of the received sum signal; for instance by the SNR.

Step 4: This value is broadcast as a feedback to the network. Nodes interpret this feedback and adapt the phase of their carrier signal accordingly.

After the achievement of synchronization, data can be transmitted by the nodes as a distributed beamformer.

The strength of feedback based closed-loop distributed adaptive beamforming in wireless sensor networks is its simplicity and low processing requirements, which make it feasible for application in networks of energy and processing power restricted sensor nodes. Inter-node communication is not required for the synchronization process. It is even possible to synchronize a set of nodes that are out of reach of each other (although in this case a coordinated transmission of identical data subsequent to the synchronization is not possible).

For this process a global random search method was studied by various authors [8, 9, 10, 11, 12, 13].

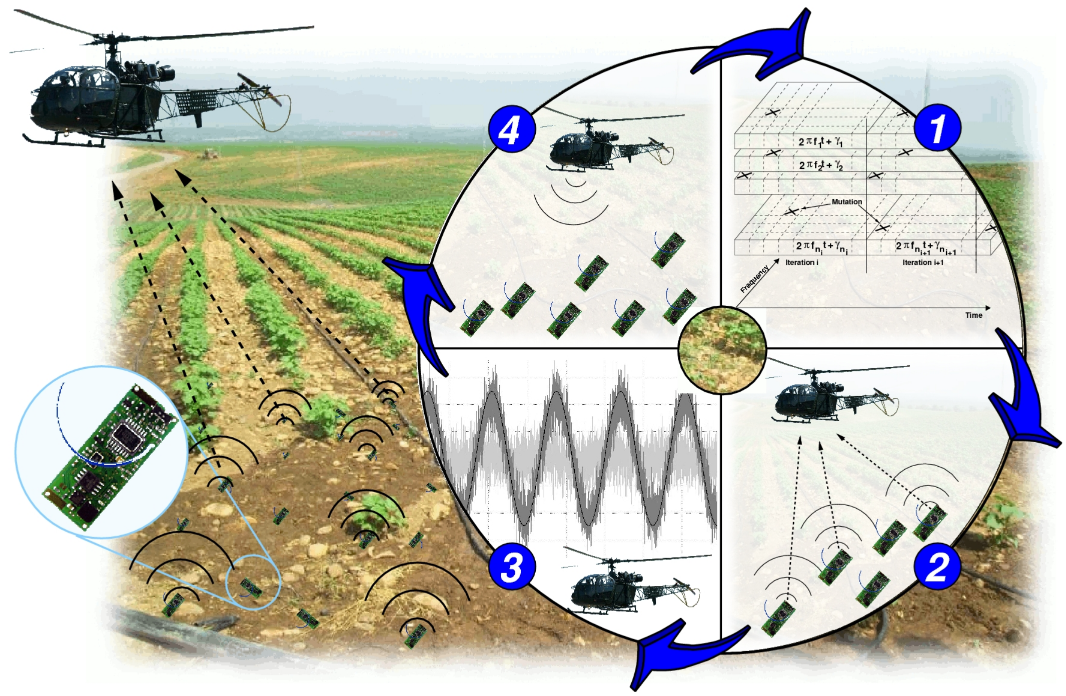


Figure 1: Schematic illustration of feedback based distributed adaptive beamforming in wireless sensor networks

The considered search approaches differ in the actual implementation of the random process (for instance, normal or uniformly distributed) that are utilized to alter the phase offset of carrier signals from source nodes [14, 15, 16]. However, for all studies, the phase offset γ_i applied is chosen from the complete phase space $\gamma_i \in [-\pi, \pi]$.

This synchronization scheme, however, does not provide the optimum performance. The synchronization speed diminishes as carrier signals approach the optimum synchronization since the probability to reach a better synchronisation decreases with increasing synchronisation quality. In section 3, we present a synchronization approach that achieves the optimum asymptotic synchronization time for distributed adaptive beamforming in wireless sensor networks. In our approach, each node estimates its optimum phase offset by solving a multi variable equation that describes the feedback function.

Furthermore, we consider mobility in section 4 in the scenario of distributed adaptive transmit beamforming in wireless sensor networks and compare the synchronization performance of our proposed algorithm to the classic random search approach.

2. CALCULATION OF OPTIMUM PHASE OFFSETS FOR CARRIER SIGNALS

In the global random search approach for synchronizing carrier phases described above, the phase offset of each node is chosen uniformly at random from the whole phase space $\gamma_i \in [-\pi, \pi]$ in distinct iterations. Since deterioration of fitness values is not allowed, an optimum phase offset for transmit nodes is gradually achieved during these iterations.

Instead of requesting randomly chosen points from the search space continuously, a more ambiguous approach is to estimate the feedback function to be able to calculate the optimum configuration of carrier phases.

A possible description of the feedback calculated by the remote receiver is, for example, the SNR. This value increases when carrier phases are well synchronized and decreases with worse synchronization. Basically, the more the carrier synchronization deviates from an optimum synchronization, the smaller is the SNR. We quantify this offset with the root of the mean square error (RMSE) of the received sum signal from all carrier signals

$$\zeta_{\text{sum}} = \Re\left(m(t)e^{j2\pi f_c t}\sum_{i=1}^{n}\text{RSS}_i e^{j(\gamma_i+\phi_i+\psi_i)}\right) \quad (2)$$

and an optimum superimposed carrier signal

$$\zeta_{\text{opt}} = \Re\left(m(t)\text{RSS}_{\text{opt}}e^{j(2\pi f_c t+\gamma_{\text{opt}}+\phi_{\text{opt}}+\psi_{\text{opt}})}\right). \quad (3)$$

In these formulae, the values $\gamma_i + \phi_i + \psi_i$ and $\gamma_{\text{opt}} + \phi_{\text{opt}} + \psi_{\text{opt}}$, which constitute the overall phase offset at the receiver node, denote the carrier phase offset γ_i (γ_{opt}) for the transmitted signal, the phase offset ϕ_i (ϕ_{opt}) due to the delay in signal propagation and the phase offset ψ_i (ψ_{opt}) caused by the local oscillators at the nodes not being synchronized.

When the deviation between the current phase offset of node i and the optimum phase offset of this node increases, the RMSE-value increases as well.

For a given configuration of carrier phase offsets, the function describing the fitness curve of the feedback function when one node alters its phase offset while all other carrier phases remain fixed, can be derived experimentally as follows. Observe that the fitness function can be described as a function

$$\mathscr{F}(\gamma_i) = A \times sin(\gamma_i + \phi) + c \quad (4)$$

where A denotes the amplitude and ϕ the phase offset of the fitness function. The value γ_j denotes the phase offset of the i-th carrier signal and c is a suitable constant.

The reason that this is a periodic sinusoid function can be seen as follows. When all but one carrier are fixed, the RMSE-value is determined by the phase-offset between the optimum sum signal ζ_{opt} and the non-fixed carrier. When the carrier is modified in γ_i, the resulting SNR follows a sinusoid function.

A, ϕ, and c are three unknowns that can be calculated when three distinct function values for this function are known. These three function values for three distinct phase offsets $\gamma_1, \gamma_2, \gamma_3$ of a carrier signal can be calculated when the one node with a non-fixed carrier signal acquires the corresponding feedback values from the remote receiver. In figure 2(a), we depict the accuracy at which the RMSE fitness function can be estimated by this procedure. The dashed line in the figure depicts the fitness function estimated from three distinct feedback requests, while the solid line is created from 100 feedback calculations in a Matlab-based simulation environment. From the calculated expected fitness function, we can determine the optimum phase offset for this carrier signal.

We have experienced a maximum deviation

$$\frac{\sum_{i=1}^{n_{\text{samples}}} |\mathscr{F}'(\gamma_i) - \mathscr{F}^*(\gamma_i)|}{\sum_{i=1}^{n_{\text{samples}}} \mathscr{F}^*(\gamma_i)} \tag{5}$$

between the approximated and the measured RMSE values of less than 0.01 when all but one node kept their phase offset constant. In equation (5) $\mathscr{F}'(\gamma_i)$ denotes the estimated fitness value for a phase offset of γ_i while $\mathscr{F}^*(\gamma_i)$ denotes the correct value for this phase offset. Since inter-node communication is not assumed in the scenario of distributed adaptive transmit beamforming in wireless sensor networks, more than one node might alter its phase offset at once. In this case, we can show that the calculated fitness curve deviates more significantly from the actual fitness function. Figure 2(b) represents the approximation of the fitness function when three nodes change their phase offset simultaneously. The deviation between the approximated and the measured values is greater than that in the previous case and reaches values of more than 0.03.

This observation leads to two important conclusions. The first conclusion is that a precise calculation of the optimum phase offset for a single node is possible, when only one sender changes the phase offset of its carrier signal during a single iteration. The second fact is that verification of the correctness of the results is possible, by measuring the significance of the deviation between the calculated and the actual fitness function.

3. A NUMERIC ALGORITHM

This section describes the steps of the synchronization procedure for our numeric algorithm. Every four iterations are logically grouped. In these four iterations a node may either participate by calculating its optimum phase offset γ_i^* (active node), or it may transmit its carrier signal unmodified (passive node).

The synchronization begins as the receiver starts sending the feedback messages. A message consists of the following fields: the measured RMSE-value, the iteration number, and a flag which indicates whether the synchronization has been completed or not. When receiving the feedback message, a transmit node sets its iteration counter to the iteration value in the feedback, so that all nodes have the same iteration number.

At the beginning of a cycle (i.e. the iteration number is divisible by 4), a node i becomes an active participant with probability p_i and stays passive otherwise. A reasonable choice is $p_i = \frac{1}{n}$ (for a network of n nodes) so that one out of n nodes is active on average in each iteration.

An active node :

1. Transmit its carrier signal with three distinct phase offsets $\gamma_1 \neq \gamma_2 \neq \gamma_3$ and measures the feedback generated by the remote receiver. Feedback value and corresponding phase offset are stored by the node.
2. From these three feedback values and phase offsets, it estimates the feedback function (cf. section 2) and calculates the optimum phase offset γ_i^*.
3. Transmit a fourth time with $\gamma_4 = \gamma_i^*$.
4. If the deviation is less than 0.01 according to equation (5), it stores γ_i^* as the optimal phase offset and, otherwise discards it.

A passive node :

1. Transmits the carrier signal four times with identical phase offset γ_i.

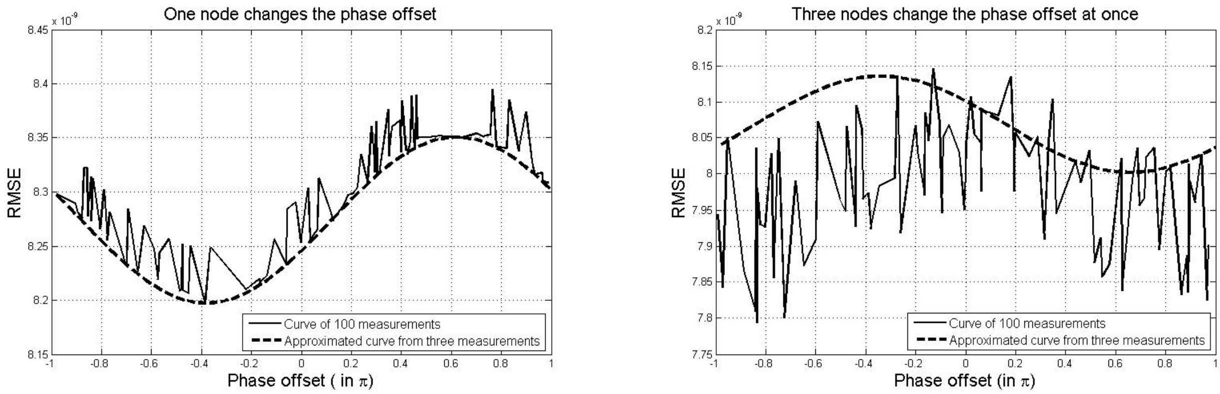
In order to decrease the number of time slots, in which either more than one node or no node actively participate, the nodes may adjust the value of p_i. After receiving the fourth feedback message, an active node i that has calculated γ_i^* successfully, becomes a passive node for a certain number of iterations. The node sets $p_i = 0$ to reduce the interference for other active nodes. All passive nodes, which register an improvement of the feedback value after the fourth transmission, assume that a node has calculated its γ_i^* successfully and alter their p_i-value to $p_i = \frac{1}{n - \text{successful phase alterations}}$. The probability that a node successfully calculates γ_i^* is

$$\binom{n}{1} \cdot \frac{1}{n} \cdot \left(1 - \frac{1}{n}\right)^{n-1}$$
$$\left(1 - \frac{1}{n}\right)^{n-1} \tag{6}$$

Figure 3(a) shows the relative deviation of the phase offsets γ_i among 100 nodes. The results have been obtained in a Matlab-based simulation environment.

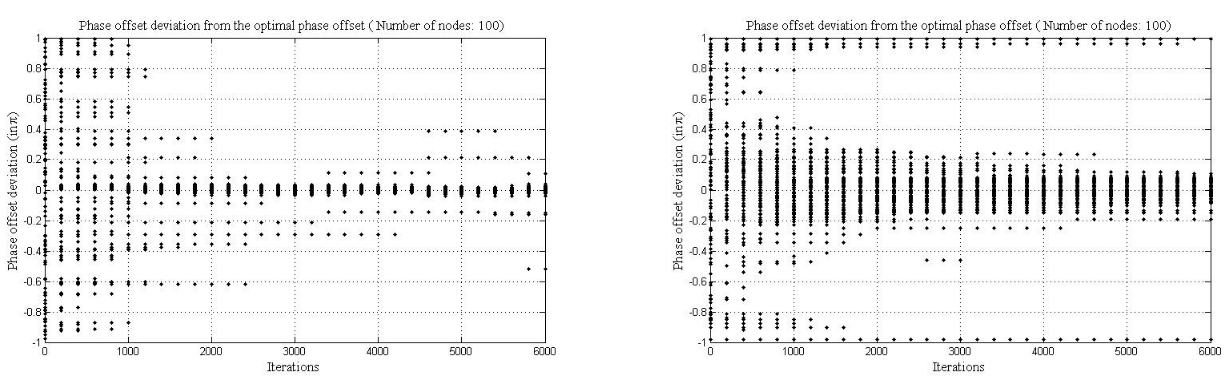
All nodes were configured to transmit at a frequency of 2.4 GHz. The sender nodes utilize a transmission power of 0.1 mW. Ambient White Gaussian Noise (AWGN) with a noise power of $-103dBm$ is applied as proposed in [17]. 100 nodes are placed in a $30m \times 30m \times 30m$ field. The transmit nodes are distributed randomly at the bottom of the field and the receiver is placed initially at the center of the field's top, so that the minimum possible distance between a sender and receiver is 30 meters.

In the simulation, the Doppler effect due to node mobility was taken into consideration. Individual signal components are summed up at the receiver node to generate the superimposed sum signal ζ_{sum}. Path loss was calculated by the Friis



(a) RMSE-γ-relationship when only one sender node changes the phase offset (b) RMSE-γ-relationship when three sender nodes change the phase offset at once

Figure 2: Approximation of the RMSE- phase offset- relationship



(a) Results using the numeric algorithm

(b) Results using a global random search approach

Figure 3: Deviation of the phase offsets from the optimal phase offsets using the numerical and the random method

free space equation [18]

$$P_{rx} = P_{tx} \left(\frac{\lambda}{2\pi d} \right)^2 G_{tx} G_{rx} \quad (7)$$

with $G_{tx} = G_{rx} = 1$. Shadowing and signal reflection were disregarded so that only the direct signal component is utilized.

After about 1500 iterations most of the nodes (about 90 %) have near optimum phase offsets. The global random search approach, however, that is typically utilized for distributed adaptive beamforming in wireless sensor networks has a greatly degraded performance (cf. figure 3(b)).

In our current implementation, we require about $12n$ iterations for all nodes to finally find and set the optimum phase offset of their carrier signal. This is asymptotically optimal, since the optimum phase offset of the carrier signal has to be calculated for each single node. As the network is of size n, a synchronization time of $O(n)$ is asymptotically optimal.

4. CONSIDERATION OF MOBILITY

In current studies on distributed adaptive beamforming in wireless sensor networks, all nodes are considered static. An interesting case to be studied is that of node mobility. We present results from a Matlab-based simulation environment, where mobility is applied to transmitter or receiver nodes. All other parameters of this simulation are identical to the simulation scenario detailed above.

We implemented a global random search approach and our numerical algorithm for this scenario, as well as two mobility models. The first mobility model is a random-walk model, whereas the second one is a linear model.

Nodes in the random-walk model travel in non-specified directions. After every iteration the movement direction is altered uniformly at random. The distance traveled between two consecutive iterations is constant and depends on the speed of motion specified. In the linear model, the senders or the receiver move in a constant direction and with a constant speed.

In order to quantify the maximum speed at which a synchronization is possible, we define a synchronization as successful if the signal strength achieved is at least 75% of the signal strength possible with perfect synchronisation. All the obtained simulation results are depicted in figure 4 and figure 5 and are discussed in the following sections.

4.1 Performance of the global random search approach

In our first scenario, the receiver node moves in a random walk mode, where all transmit nodes remain static. We observed that a successful synchronization in this scenario is possible with a movement speed of 5m/sec at most. Figure 4(a) shows the relative phase offset of individual carrier signals. The standard deviation σ of the relative phase offset of all nodes is about 0.1π for about 95% of all nodes after 6000 iterations. Figure 4(b) shows that the signal strength exceeds the 75% threshold we defined, so that the movement speed is considered as feasible.

However, when transmitters follow the random-walk movement algorithm while the receiver is not moving, the maximum speed is about 2 m/sec (cf. figures 4(c) and 4(d)).

For the linear movement, the maximum relative speed between transmit and receive nodes with the global random search implementation is 30 m/sec regardless of whether the receive or the transmit nodes are moving (cf. figure 4(e) and figure 4(f)).

4.2 Performance of the numeric algorithm

For the proposed numerical algorithm, we have applied the same settings as in section 4.1. When the receiver moves in a random-walk model while the transmitters are static, the movement speed of 5 m/sec is easily supported (cf. figure 5(a) and figure 5(b)).

In the figures, the standard deviation σ of the relative phase offset among all nodes is about 0.03 π. Figure 5(c) depicts the phase deviations while the receiver is static and the transmit nodes are moving in random directions at even 5 m/sec. In this case, a standard deviation of $\sigma = 0.22\pi$ is achieved after 6000 iterations and the signal strength is strong enough for successful transmission (cf. figure 5(d)).

Finally we conclude that the numeric method enables higher movement speeds as well as an improved synchronization performance. The maximum relative movement speed for the linear movement model is about 60 m/sec. Figure 5(e) depicts deviations of phase offsets, where the standard deviation of the relative phase offset among all nodes in this case is $\sigma = 0.18\pi$ for about 95 % of all nodes and the signal strength (figure 5(f)) is above the required threshold.

5. CONCLUSION

We have introduced a numeric approach to distributed adaptive beamforming in wireless sensor networks. The algorithm achieves an asymptotic simulation time of $O(n)$, which means that it is an asymptotically optimal solution. In mathematical simulations, we could show that the standard global random search approach is in fact outperformed. Moreover, we have studied the impact of mobility on the synchronization performance of both approaches. For a random walk model and a linear movement model, both approaches have been compared in mathematical simulations.

The numeric synchronization method allows movement speeds of more than 200 km/h at a distance of about 30 meters.

For obtaining experimental results in near realistic settings, we are currently working on the implementation of both approaches with USRP software radios.

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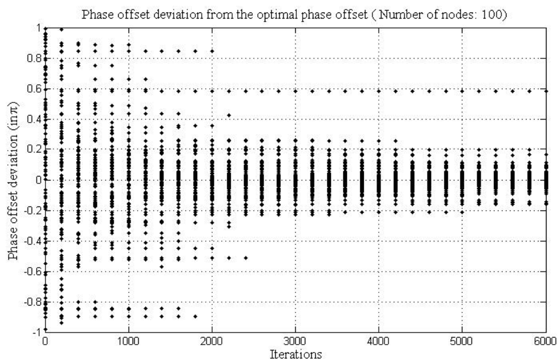
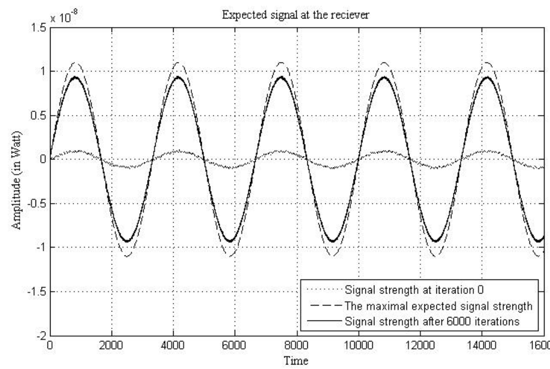
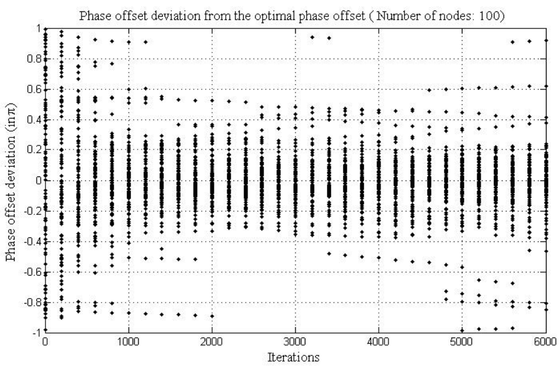


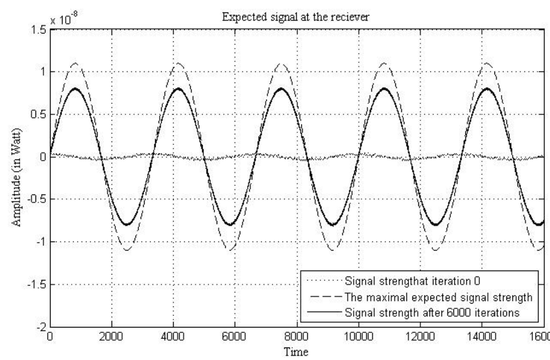
(a) Deviation of the phase offsets from their optimal values when the receiver moves at 5 m/sec following a random-walk model



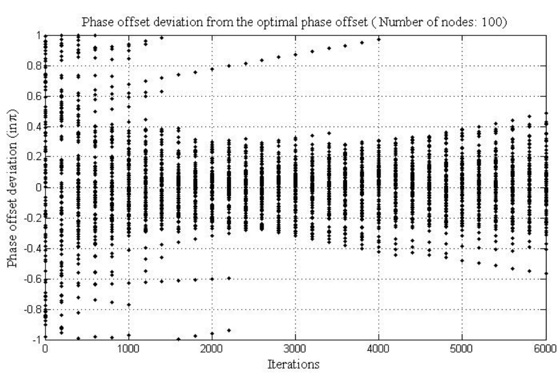
(b) Signal strength at the receiver when moving at 5 m/sec following a random-walk model



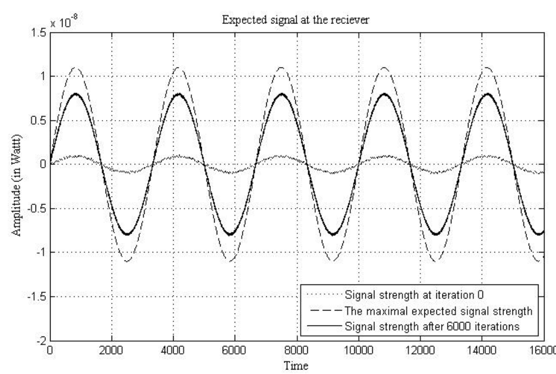
(c) Deviation of the phase offsets from their optimal values when the transmit nodes move at 2 m/sec following a random-walk model



(d) Signal strength at the receiver when the transmit nodes move at 2 m/sec following in a random-walk model

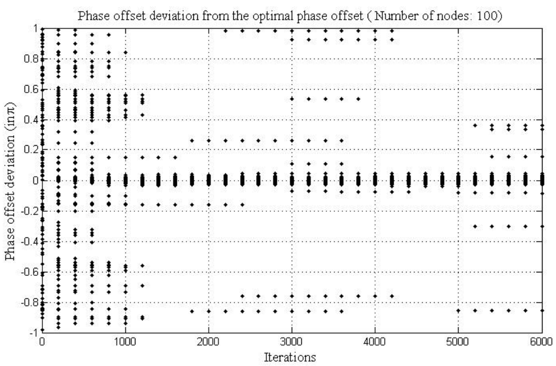


(e) Deviation of the phase offsets from their optimal values when nodes move at 30 m/s in a linear mode

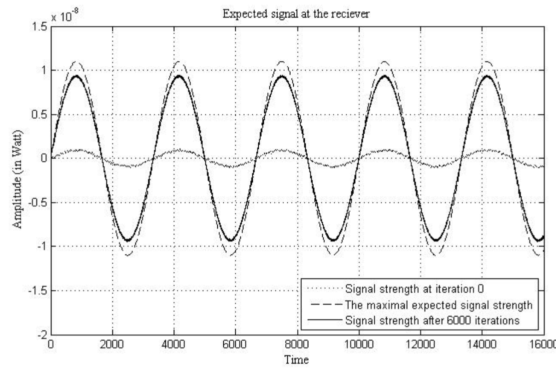


(f) Signal strength at the receiver when nodes move at 30 m/s in a linear mode

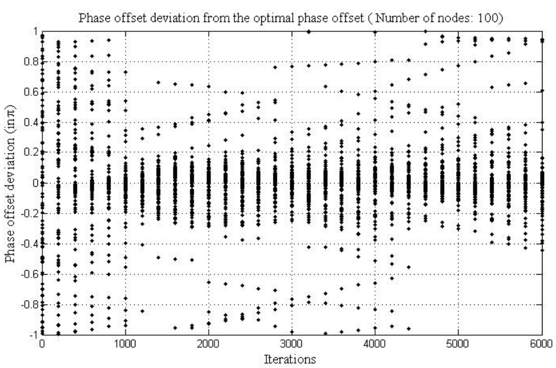
Figure 4: Performance of the evolutionary approach to distributed adaptive transmit beamforming for wireless sensor networks in a Matlab-based simulation environment



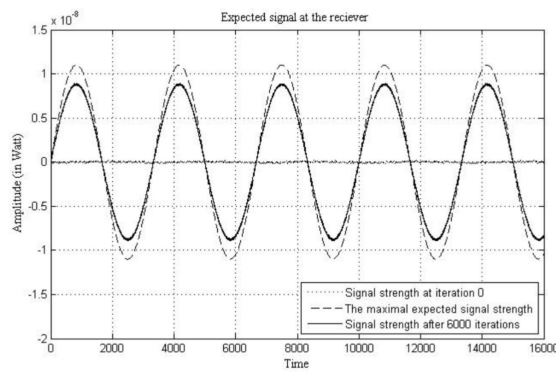
(a) Deviation of the phase offsets from their optimal values when the receiver moves at 5 m/sec following a random-walk model



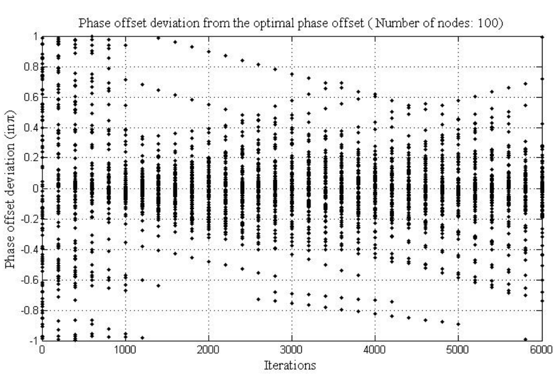
(b) Signal strength at the receiver when moving at 5 m/sec following a random-walk model



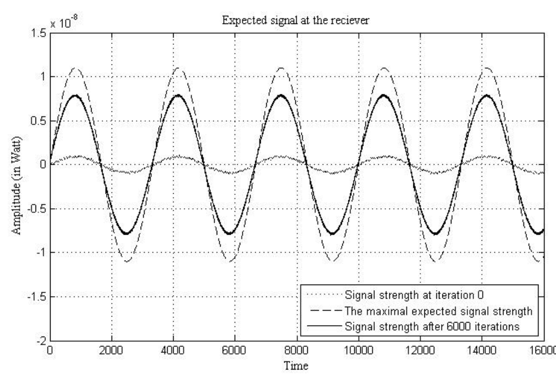
(c) Deviation of the phase offsets from their optimal values when the transmit nodes move at 5 m/sec following in a random-walk model



(d) Signal strength at the receiver when transmit nodes move at 5 m/sec following a random-walk model



(e) [Deviation of the phase offsets from their optimal values when nodes move at 60 m/s in a linear mode



(f) Signal strength at the receiver when nodes move at 60 m/s in a linear mode

Figure 5: Performance of the numeric approach to distributed adaptive transmit beamforming for wireless sensor networks in a Matlab-based simulation environment

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