Algorithmic approaches to distributed adaptive transmit beamforming

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***Abstract*—We present three approaches for algorithmic improvements on distributed adaptive transmit beamforming in wireless sensor networks. These algorithms reduce the time to synchronise carrier signals among nodes compared to the global random search approaches commonly applied to this problem. For a local random search heuristic we provide an asymptotic bounds on the optimisation time. All approaches are also studied in mathematical simulations on greater networks sizes.**

I. Introduction

By phase coherent superimpositioning of RF transmit signal components from wireless sensor nodes, the characteristics of the received RF sum signal can be improved [1], [2]. In particular, the transmission distance of tiny sized, low power sensor nodes can be increased by such collaborative transmission schemes. The general idea behind these transmission schemes is similar to multiple input, single output (MISO) schemes [3]. In order to improve the transmission quality, spatial diversity is deployed. However, as single nodes are typically of very restricted computing capabilities and, more importantly, are designed to have minimum dimensions, the integration of several antennas together with the respective computing logic is not feasible.

Alternatively, antennas of distinct transmit nodes are combined to build the antenna array. Since communication among nodes in a sensor network is wireless and unreliable and as oscillators and clocks in distinct nodes are independent, the main issues of these approaches lie in the accurate synchronisation of clocks and carrier phases that is required in order to concentrate a transmission beam on the location of a remote receiver. Furthermore, as wireless sensor nodes might be brought out in a random manner, for example, when nodes are dropped from a plane or when they are placed on non-solid ground like water, nodes have to provide these transmission schemes through self organisation.

One approach to achieve this synchronisation among nodes is virtual MIMO in wireless sensor networks [4]. In virtual MIMO, single antenna nodes are cooperating to establish a multiple antenna wireless sensor network. Virtual MIMO has capabilities to adjust to different frequencies and is highly energy efficient [5]. However, the implementation of MIMO capabilities in WSNs requires accurate time synchronisation, complex transceiver circuity and signal processing that might surcharge the power consumption and processing capabilities of simple sensor nodes.

When the receiver node is also involved in the synchronisation of nodes by providing a feedback on the channel quality this is typically referred to as feedback-based approaches. For the synchronisation of nodes we generally distinguish between closed-loop and open-loop feedback based approaches, depending on whether the feedback is computed in an open-loop or closed-loop communication. Closed-loop feedback based approaches include full-feedback techniques, in which carrier synchronisation is achieved in a master-slave manner. The phase-offset between the destination and a source node is corrected by the receiver node. Diversity between transmit signals is achieved over CDMA channels [6]. This approach is applicable only to small network sizes and requires sophisticated processing capabilities at the source nodes.

A simpler approach is the one-bit feedback based closed-loop synchronisation considered by Mudumbai, Hespanha, Madhow and Barriac in [7]. The authors describe an iterative process in which the source nodes randomly adapt their carrier phases. This random process is guided by a one-bit feedback on the synchronisation quality that is computed by the destination node. For a network size of n nodes and k possible phase offsets for transmit signal components the synchronisation time of the approach was bounded by $\mathcal{O}(n \cdot \log(n) \cdot k)$ [8].

For the calculation of this bound the process to alter the phase offsets of carrier signal components was considered to follow a uniform distribution. Other authors assume a normal distribution for this process [9], [10], [11]. In [12] it was shown that the optimisation time is improved by factor two when a node as response to a negative feedback from the receiver applies a complementary phase offset instead of simply reversing its modification.

In both cases, general purpose global search mechanisms are applied. We present in the following sections algorithmic approaches that better exploit the properties of the problem scenario and show in mathematical simulations that the performance can be improved by the proposed algorithms.

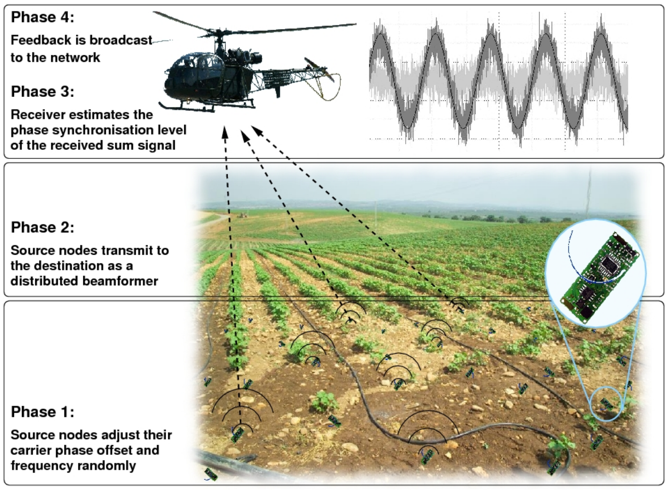

Fig. 1. A schematic overview on feedback based closed-loop distributed adaptive transmit beamforming in wireless sensor networks

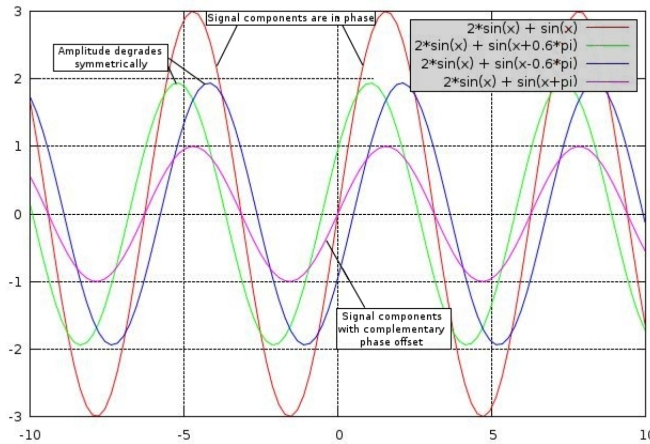

Fig. 2. Example of sinusoid sum signals. The amplitude of the sum signal degrades symmetrically when the phase offset between the two signal components increases

II. Distributed adaptive beamforming in wireless sensor networks

Distributed adaptive transmit beamforming can be implemented as an iterative process to synchronise signal phases of RF transmit signal components at a remote receiver without inter-node communication [9]. We assume that, for a network of size n, initially the phase offsets γ_i of carrier signals $\Re\left(e^{j(2\pi(f+f_i)t+\gamma_i)}\right);\ i \in \{1..n\}$ are arbitrarily distributed. When a receiver requests a transmission from the network, carrier phases are synchronised in an iterative manner as depicted in figure 1.

The four phases detailed in the figure are iterated repeatedly until a stop criteria is met (e.g. maximum iteration count or sufficient synchronisation). The computational complexity for each one of the transmit nodes is low in this approach as only the phase and frequency of the RF transmit signal is adapted according to a random process.

III. Analytic consideration

Recent approaches to distributed adaptive transmit beamforming in wireless sensor networks utilise a random search approach that could in each iteration reach any point in the search space with positive probability. We define a search point as one combination of phase offsets for all RF signal components and a neighbourhood relation over the difference in phase offsets in these configurations. As the search space is multimodal, no local optima exist so that in an arbitrary neighbourhood around a given search point the fitness value is either optimal or search points with an improved fitness value exist.

We can also show that, when the distance to the optimum and to the worst point is greater than the neighbourhood radius, the algorithm has an equal chance to improve or worsen the fitness score. We see this as follows. Assume that the fitness of the sum signal is proportional to the amplitude of the signal. When $n-1$ signal components are received simultaneously at a given frequency f the resulting sum signal is of the same frequency as the individual RF signal components. When the phase offset of the n-th signal component is identical or complementary to the phase offset of the sum signal, the amplitude is at its maximum or minimum, respectively. Inbetween these two values it degrades symmetrically. Consequently, when the minimum or maximum is not inside the neighbourhood, an equal number of points incorporate better or worse fitness values (cf. figure 2).

This means that an algorithm with restricted neighbourhood size has a probability of 0.5 to increase the fitness value for a long time during the optimisation until the optimum point is within the neighbourhood. The price for this high probability to improve the fitness value in each iteration is that the chance to reach the optimum in one step (as possible with an unrestricted neighbourhood size) is lost. Since this event is very unlikely, we are easily prepared to pay this price. We provide an upper bound on the optimisation time of a local random search approach for distributed adaptive beamforming in wireless sensor networks.

To ease the analysis we model the optimisation problem in a binary representation. We assume that each one of the sensor nodes is able to apply k distinct phase offsets of transmit RF signals. For each of the n nodes, we binary encode the distinct phase offsets by $\log(k)$ bits. Overall, a binary string of length $n \cdot \log(k)$ describes the contribution of phase offsets in one iteration. The neighbourhood of a given configuration is defined by all configurations with hamming distance up to κ for suitable κ. We assume that configurations are encoded such that the hamming distance between two configurations increases with increasing difference in phase offsets.

A. An upper bound

We analyse the count of bit mutations of this representative bitstring until a bitstring is found that encodes the phase configurations of a global optimum. We choose the mutation probability as $\frac{1}{n}$ for a network of n nodes. In this case, the

$\log(k)$ bits that represent the phase offset of the corresponding node are altered uniformly at random inside the neighbourhood boundaries. With Chernoff bounds we can show that w.h.p. the hamming distance to an optimum is not much smaller than $\frac{n \cdot \log(k)}{2}$. As long as the optimum is far away (i.e. outside the neighbourhood boundaries), the probability to reach a new search point with better fitness value is at least $\frac{1}{2}$ as detailed above. During the synchronisation, when i signal components are already in phase, the fitness value is thus improved, when one of the $n-i$ non-optimal signal components is improved and the i already optimal ones are left unchanged. This happens with probability at least

$$\begin{aligned} & (n-i) \cdot \tfrac{1}{n} \cdot \tfrac{1}{2} \cdot \left(1-\tfrac{1}{n}\right)^{n-i} \\ = & \tfrac{n-i}{2n} \cdot \left(1-\tfrac{1}{n}\right)^{n-i}. \end{aligned} \tag{1}$$

Because of $\left(1-\frac{1}{n}\right)^{n} < e < \left(1-\frac{1}{n}\right)^{n-1}$ we obtain the probability s_i that a higher fitness value is reached as

$$s_i \geq \frac{n-i}{2en}. \tag{2}$$

The expected number of mutations to increase the fitness value is bounded from above by s_i^{-1}. But how many distinct fitness values are possible? In [13] we have shown that the configurations can be roughly divided into k fitness based partitions. These are mostly given by the number of configurations in which a fixed number of RF signal components is in phase. Consequently, we obtain an upper bound on the expected optimisation time as

$$\begin{aligned} E[T_{\mathcal{P}}] \leq & \textstyle\sum_{i=0}^{k} \frac{2en}{n-i} & = 2en \cdot \sum_{i=1}^{k+1} i^{-1} \\ < & \ 2en \cdot \ln(k+1) & = O\left(n \cdot \log(k)\right) \end{aligned} \tag{3}$$

This means that after $\mathcal{O}(n \cdot \log(k))$ iterations we expect that the fitness value has reached a region near the optimum so that the distance to the optimum is smaller than the neighbourhood size. Consequently, the optimisation from there on is more complicated since the count of points that decrease the fitness value increases with decreasing distance to the optimum. In the worst case, the probability to increase the fitness values is $\frac{1}{N}$ with N denoting the neighbourhood size. Similarly to our consideration above, we estimate the expected optimisation time for this phase as $\mathcal{O}(N \cdot n \cdot \log(k))$. A weak estimation of $N = \mathcal{O}(k)$ will lead to an upper bound on the overall optimisation time of

$$\mathcal{O}(k \cdot n \cdot \log(k)). \tag{4}$$

IV. Algorithmic improvements and simulation studies

In the following sections we detail simulation studies conducted on the topic of distributed adaptive transmit beamforming in wireless sensor networks. In particular, we study algorithms that improve the synchronisation performance. These implement a local random search approach, the utilisation of nodes with pre-synchronised phase offsets and an approach to re-consider nodes that have positive impact on the fitness value.

In the simulations, only thermal noise (AWGN) at $-103dBm$ was considered and no interference from other sources than the nodes in the network. However, with additional interference we expect similar results but a decreased transmission range. Reflections and multi-path-propagation have not been considered since, in a typical scenario we assume that nodes are placed on a ground level with a receiver above them with direct line of sight.

In the simulations, 100 nodes have been placed uniformly at random on an area of $30m \times 30m$ with one receiver in the distance of $30m$ above the centre of this area. Each node has a transmission power of $P_{TX} = 1mW$ and transmits at 2.4 GHz. The transmission power at the receiver is calculated according to the Friis free-space equation

$$P_{RX} = P_{TX} \left(\frac{\lambda}{2\pi d}\right)^2 G_{RX} G_{TX} \tag{5}$$

with $G_{TX} = G_{RX} = 0$ [14].

We measure the progress of an algorithm by the RMSE of the received RF sum signal to the RF sum signal that could be expected when signal phases are perfectly aligned:

$$RMSE = \sqrt{\sum_{t=0}^{\tau} \frac{\left(\sum_{i=1}^{n} s_i + s_{noise}(i) - s^*\right)^2}{n}}. \tag{6}$$

All figures depicted in the following show the median RMSE values from 10 individual simulations with identical parameter settings that are reached after various iterations of the algorithms together with the standard deviation.

A. Impact of the choice of nodes for synchronisation

In distributed adaptive beamforming, several nodes in a network collaboratively reach a distant receiver. The synchronisation time for these nodes is dependent on the number of nodes that participate in the synchronisation (cf. [8], [10]). Consequently, when not all nodes are required to reach a given distance, it is beneficial to utilise only a subset of nodes. In particular, we would like to pick those nodes for the synchronisation process that are well pre-synchronised so that the initial fitness value is already high. We conducted several simulation runs in which a set of pre-synchronised nodes is identified in random experiments prior to the synchronisation. In these random experiments a set of 100 randomly chosen nodes transmit simultaneously. The fitness value reached in this simultaneous transmission is stored for each experiment. After all random experiments are completed the synchronisation is conducted with the nodeset that scored the best fitness value during the random experiments. Figure 3 details the simulation results. For a network size of 1000 and 120 nodes, 100 nodes are picked in 500 and 100 random experiments, respectively. These simulations are compared to simulation results in which a network if 100 nodes is synchronised with uniformly distributed phase alterations and a mutation probability of 0.01 but without pre-synchronisation of nodes.

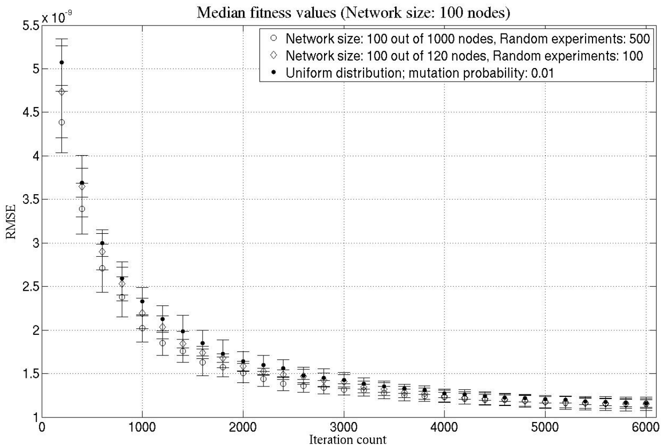

Fig. 3. Performance of distributed adaptive beamforming in WSNs when participating nodes are chosen based on random experiments

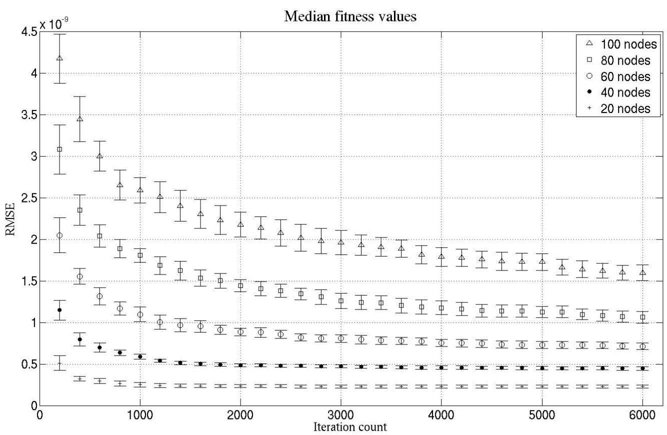

Fig. 4. The synchronisation performance for various network sizes in a uniformly distributed process. Nodes adapt their carrier phase in each iteration with probability 0.05.

We see that the pre-synchronised node-sets reach better fitness values earlier in the simulation. in particular, an improvement is already visible for 100 nodes chosen in 100 random experiments out of 120 nodes. This shows that we can benefit from the node choice already with few random experiments and with network sizes that deviate from the required network size only slightly. However, observe also that the lead in fitness value is shrinking with increasing iteration count.

B. Impact of the network size and hierarchical clustering

When the count of nodes that participate in the synchronisation process is altered, this also impacts the performance of the synchronisation process. We conducted several simulations with network sizes ranging from 20 to 100 nodes. Figure 4 depicts the performance of the synchronisation processes.

In these simulations, we set the probability for single nodes to adapt the phase offset of their respective carrier signals as 0.05 and utilised a uniformly distributed phase alteration process. We see that the maximum fitness value achieved is lower for smaller network sizes. This is due to the RMSE measure that compares the achieved sum signal to an expected optimum superimposed signal. As the count of participating nodes diminishes, also the amplitude of the optimum signal decreases. As expected, the optimum value is reached earlier for smaller network sizes.

From section III we learned that the upper bound on the synchronisation time is more than linear in n. As, however, the received signal strength of the superimposed sum signal (RSS_{sum}) changes linear with the network size n, the overall energy consumption and synchronisation time might benefit from a reduced number of nodes transmitting at increased power level.

We propose the following hierarchical clustering scheme that synchronises all transmit nodes iteratively in clusters of reduced size.

1) Determine clusters (e.g. by a random process initialised by the receiver node)
2) Guided by the receiver node, synchronise clusters successively as described above with possibly increased transmit power for nodes. When cluster ι is sufficiently synchronised, all nodes in this cluster sustain their carrier signal and stop transmitting until all other clusters are synchronised.
3) At this stage, carrier signals in each one of the clusters are in phase but carrier phases of distinct clusters might differ. To achieve overall synchronisation, we build an overlay-cluster of representative nodes from all clusters. These nodes are then synchronised.
4) Nodes in all clusters alter the phase of their carrier signal by the phase offset experienced by the corresponding representative node. Let $\zeta_i = \Re\left(\text{RSS}_i e^{j2\pi f_c t(\gamma_i)}\right)$ and $\zeta_i' = \Re\left(\text{RSS}_i e^{j2\pi f_c t(\gamma_i')}\right)$ be the carrier signals of representative node i from cluster ι before and after synchronisation between representative nodes was achieved. A node h from the same cluster ι will then alter its carrier signal $\zeta_h = \Re\left(\text{RSS}_h e^{j2\pi f_c t(\gamma_h)}\right)$ to $\zeta_h' = \Re\left(\text{RSS}_h e^{j2\pi f_c t(\gamma_h+\gamma_i-\gamma_i')}\right)$. Under ideal conditions, all nodes should then be in phase although an overall synchronisation was not applied.
5) To account for synchronisation errors a final synchronisation phase in which all nodes participate concludes the overall synchronisation process.

Observe that all coordination is initiated by the receiver node so that no inter-node communication is required for coordination. Clusters are formed by a random process and synchronisation between and within clusters is achieved by the distributed adaptive beamforming approach described above.

Depending on the size of the network, more than one hierarchy stage might be optimal for the optimisation time and the energy consumption. In order to estimate the optimal hierarchy depth and the optimum cluster size, the size of the network must be computed. We assume that the nodes themselves do not know of the network size. This means that the remote receiver derives the network size, calculates optimal cluster sizes and hierarchy depths and transmits this information to

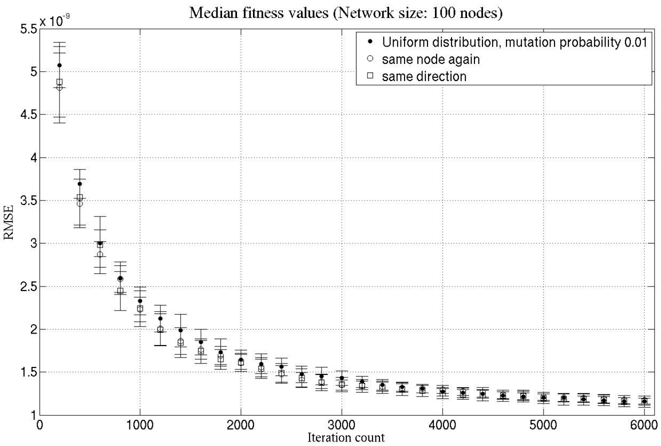

Fig. 5. Performance of distributed adaptive beamforming in WSNs when successful nodes are re-considered for mutation

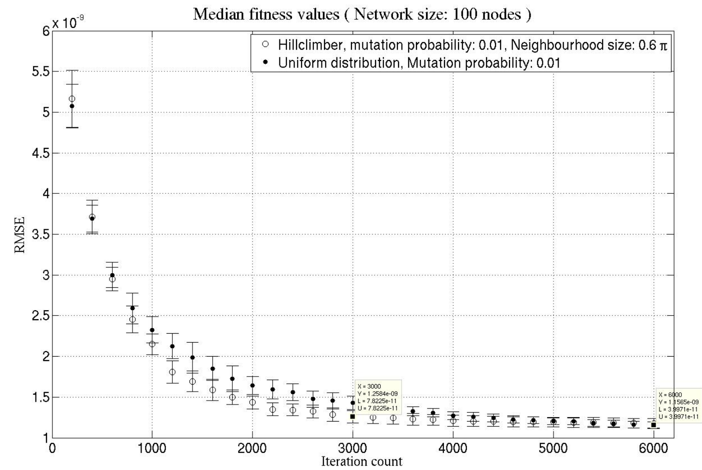

Fig. 6. Performance of the local random search implementation for distributed adaptive beamforming in wireless sensor networks

the nodes in the network. In [15] it was demonstrated that the superimposed sum signal from arbitrarily synchronised nodes over some time interval is sufficient to compute an estimation on the number of transmitting nodes.

C. Impact of reelecting successful nodes

For distributed adaptive beamforming, an optimum is reached when all received transmit signal components are in phase. In the random synchronisation process, this situation is approached iteratively, which means that several steps to an optimum are required. In particular, as long as the optimum phase offset for one transmit signal component is not reached, the fitness value can be improved by sufficiently altering the phase offset for this transmit signal component again. To exploit this property we modify the implementation of the original approach in such a way that transmit signal components that were altered in an iteration in which the fitness value was increased are altered again in the next iteration. The intuition behind this approach is that for a transmit signal component which was successfully altered we expect that the optimum phase offset is not yet reached so that further benefit is possible. We implemented two distinct approaches. The first approach is to alter the phase offset of the same transmit signal component again uniformly at random. For the second implementation also the direction in which the phase was altered is sustained. Figure 5 depicts simulation results for these two approaches when compared to the standard uniformly distributed phase alteration approach. We observe that both approaches have similar performance and improve the fitness early in the synchronisation process. With increasing iteration count, however, the standard approach catches up.

D. A local random search algorithm

For our implementation of the local random search algorithm we modify the phase alteration mechanism applied by individual nodes. While for the original random search approach the phase offset is chosen uniformly at random from all possible phase offsets in $[-\pi, \pi]$, we restrict the set of possible future phase offsets to a subset of this range: $\gamma \in [\sigma_1, \sigma_2]$ with $-\pi \leq \sigma_1 < \sigma_2 \leq \pi$. Figure 6 shows the simulation results of the algorithm. We observe that the hill-climbing approach reaches lower fitness values faster than the original approach. In particular, the near optimal fitness value reached by the original algorithm after 6000 iterations is achieved by the hill climber already after about 3000 iterations.

V. Conclusion

We studied aspects of distributed adaptive beamforming in wireless sensor networks. In particular, we analysed iterative random feedback based closed-loop synchronisation. We derived an upper bound on the synchronisation time of a local random search implementation. Furthermore, we proposed three modifications of the global search approaches that improve the synchronisation performance of distributed adaptive beamforming in wireless sensor networks. For these approaches we achieved quantitative results in mathematical simulations for a network of 100 nodes.

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References

[1] J. Laneman and G. Wornell, “Energy-efficient antenna sharing and relaying for wireless networks,” in *Proceedings of the IEEE Wireless Communication Networking Conference*, 2000, p. 294.

[2] J. Lanemann, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behaviour,” *Transactions on inforamtion Theory*, vol. 50, no. 12, December 2004.

[3] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. Cambridge University Press, 2005. [Online]. Available: http://www.eecs.berkeley.edu/ dtse/book.html

[4] M. Youssef, A. Yousif, N. El-Sheimy, and A. Noureldin, “A novel earthquake warning system based on virtual mimo wireless sensor netwroks,” in *Canadian conference on electrical and computer engineering*, April 2007, pp. 932–935.

[5] S. K. Jayaweera, “Energy efficient virtual mimo based cooperative communications for wireless sensor networks,” *IEEE Transactions on Wireless communications*, vol. 5, no. 5, pp. 984–989, May 2006.

[6] Y. Tu and G. Pottie, “Coherent cooperative transmission from multiple adjacent antennas to a distant stationary antenna through awgn channels,” in *Proceedings of the IEEE Vehicular Technology Conference*, 2002, pp. 130–134.

[7] R. Mudumbai, J. Hespanha, U. Madhow, and G. Barriac, “Distributed transmit beamforming using feedback control,” *IEEE Transactions on Information Theory*, (In review).

[8] S. Sigg and M. Beigl, “Randomised collaborative transmission of smart objects,” in *2nd International Workshop on Design and Integration principles for smart objects (DIPSO2008) in conjunction with Ubicomp 2008*, September 2008.

[9] M. Seo, M. Rodwell, and U. Madhow, “A feedback-based distributed phased array technique and its application to 60-ghz wireless sensor network,” in *IEEE MTT-S International Microwave Symposium Digest*, 2008, pp. 683–686.

[10] R. Mudumbai, D. R. Brown, U. Madhow, and H. V. Poor, “Distributed transmit beamforming: Challenges and recent progress,” *IEEE Communications Magazine*, pp. 102–110, February 2009.

[11] R. Mudumbai, B. Wild, U. Madhow, and K. Ramchandran, “Distributed beamforming using 1 bit feedback: from concept to realization,” in *Proceedings of the 44th Allerton conference on communication, control and computation*, 2006, pp. 1020–1027.

[12] J. A. Bucklew and W. A. Sethares, “Convergence of a class of decentralised beamforming algorithms,” *IEEE Transactions on Signal Processing*, vol. 56, no. 6, pp. 2280–2288, June 2008.

[13] S. Sigg and M. Beigl, “Collaborative transmission in wsns by a (1+1)-ea,” in *Proceedings of the 8th International Workshop on Applications and Services in Wireless Networks (ASWN’08)*, 2008, (to appear).

[14] T. Rappaport, *Wireless Communications: Principles and Practice*. Prentice Hall, 2002.

[15] A. Krohn, “Superimposed radio signals for wireless sensor networks,” Ph.D. dissertation, Technical University of Braunschweig, January 2007.