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Lineare Optimierung
Übung 11 vom 25.01.05

(Abgabe bis zum 02.02.2005, 9:45 durch Einwurf in den Übungskasten im vierten Stock des Forumsgebäudes)

Aufgabe 1 (New York Times und schwierige Probleme):

Betrachte folgenden Artikel aus der *New York Times* vom 27. November 1979.

An Approach to Difficult Problems

Mathematicians disagree as to the ultimate practical value of Leonid Khachiyan's new technique, but concur that in any case it is an important theoretical accomplishment.

Mr. Khachiyan's method is believed to offer an approach for the linear programming of computers to solve so-called "traveling salesman" problems. Such problems are among the most intractable in mathematics. They involve, for instance, finding the shortest route by which a salesman could visit a number of cities without his path touching the same city twice.

Each time a new city is added to the route, the problem becomes very much more complex. Very large numbers of variables must be calculated from large numbers of equations using a system of linear programming. At a certain point, the compexity becomes so great that a computer would require billions of years to find a solution.

In the past, "traveling salesmen" problems, including the efficient scheduling of airline crews or hospital nursing staffs, have been solved on computers using the "simplex method" invented by George B. Dantzig of Stanford University.

As a rule, the simplex method works well, but it offers no guarantee that after a certain number of computer steps it will always find an answer. Mr. Khachiyan's approach offers a way of telling right from the start whether or not a problem will be soluble in a given number of steps.

Two mathematicians conducting research at Stanford already have applied the Khachiyan method to develop a program for a pocket calculator, which has solved problems that would not have been possible with a pocket calculator using the simplex method.

Mathematically, the Khachiyan approach uses equations to create imaginary ellipsoids that encapsulate the answer, unlike the simplex method, in which the answer is represented by the intersections of the sides of polyhedrons. As the ellipsoids are made smaller and smaller, the answer is known with greater precision. MALCOLM W. BROWNE

Entscheide für jede Aussage: ist sie (a) richtig, (b) falsch, (c) irreführend oder/und (d) äquivalent zu einer bekannten Vermutung, deren Lösung vermutlich dem Herrn Browne nicht bekannt war ?

(20 Punkte)

Aufgabe 2 (Ellipsoidmethode):

In der j-ten Iteration der Ellipsoidmethode seien $a_j = (0,0)^T$, $A_j = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}$ gegeben, und sei $x + y \leq -1$ eine der Ungleichungen. Stelle diese Situation graphisch dar. Bestimme a_{j+1} und A_{j+1} und stelle das zugehörige Ellipsoid graphisch dar.

(15 Punkte)

Aufgabe 3 (Größe der Ecken von Polyedern):

Seien $P = \{x \in \mathbb{R}^4 \mid Ax \leq b, x \geq 0\}$ und $Q = \{x \in \mathbb{R}^3 \mid Bx \leq d, x \geq 0\}$, wobei

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & -1 & 1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad d = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}.$$

(a) Sei $v = (v_1, v_2, v_3, v_4)$ eine beliebige Ecke von P und sei v_i, $1 \leq i \leq 4$, eine beliebige Koordinate von v. Gib obere Schranken für den Absolutbetrag des Zählers von v_i, für den Absolutbetrag des Nenners von v_i und für $|v_i|$ an. Verwende dazu den entsprechenden Satz aus der großen Übung. Löse dieselbe Aufgabe für eine beliebige Ecke $q = (q_1, q_2, q_3)$ von Q.

(b) Verbessere diese Schranken deutlich unter Verwendung der Cramerschen Regel mit den konkreten Matrizen.

(12+13 Punkte)