**Institut für Mathematische Optimierung**
**TU Braunschweig**

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**SS 2006**

# Coding Theory / Discrete Mathematics II
# Assignment 9 (July 06, 2006)

(This assignment is due on July 13, 2006, 1.00 p.m., by dropping it into the wooden box **in front of** F 310)

**Exercise 1 (Bounds for codes):**

The following theorem is a result of the discussion from the end of the lecture.

**Theorem 5.6:** *There exists a linear code of length n, dimension k, and distance d if*

$$\binom{n-1}{0} + \binom{n-1}{1} + \ldots + \binom{n-1}{d-2} < 2^{n-k}$$

(a) Does there exist a linear code of length $n = 9$, dimension $k = 2$, and distance $d = 5$?

(b) What is a lower and an upper bound on the dimension, k, of a code with $n = 9$ and $d = 5$? (Use Theorem 5.2 and Theorem 5.6.)

(c) What does Theorem 5.6 tell you about the existence of a code of length $n = 15$, dimension $k = 7$ and distance $d = 5$?

(8+14+8 Points)

**Exercise 2 (Perfect codes):**

A code $C$ of length $n$ and odd distance $d = 2t + 1$ is called a perfect code if $C$ attains the Hamming bound; that is

$$|C| = \frac{2^n}{\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{t}}$$

(a) Show that any perfect code of length and distance $2t + 1$ has exactly 2 codewords.

(b) Can there exist a perfect code of length $n = 7$ and distance $d = 3$?

(c) Show that $\binom{n}{0} + \binom{n}{1} = 2^r$, for $n = 2^r - 1$.

(14+8+8 Points)