Institut für Mathematische Optimierung
TU Braunschweig

Prof. Dr. Sándor Fekete
Nils Schweer

SS 2006

Coding Theory / Discrete Mathematics II
Assignment 7 (June 22, 2006)

(This assignment is due on June 29, 2006, 1.00 p.m., by dropping it into the wooden box **in front of** F 310)

Exercise 1 (Cyclic codes):
Which of the following codes are cyclic codes?

(a) $C_1 = \{000, 100, 011, 111\}$

(b) $C_2 = \{000, 110, 101, 011\}$

(10 Points)

Exercise 2 (Properties of cyclic codes):

(a) Let v be a word of length n. The cyclic shift $\pi(v)$ of v is the word of length n obtained from v by taking the last digit of v and moving it to the beginning and all other digits moving one position to the right. Show that $\pi(v+w) = \pi(v) + \pi(w)$ and $\pi(\alpha \cdot v) = \alpha \cdot \pi(v)$ for $\alpha \in \{0, 1\}$.

(b) Let v be a word of length n and $S = \{v, \pi(v), \pi^2(v), \ldots, \pi^{n-1}(v)\}$. We know that $C := < S >$ is a linear code. Show that it is also cyclic. (We use the notation $\pi^2(v) := \pi(\pi(v))$, $\pi^3(v) := \pi(\pi(\pi(v)))$, etc.)

(30 Points)

Exercise 3 (Cyclic shifts):
We use the notation from Exercise 2.

(a) Find all words of length n such that $\pi(v) = v$.

(b) Find all words of length 6 such that $\pi^2(v) = v$.

(c) Let k and n be two numbers from $\mathbb{N}$ such that k divides n. Determine the number of words of length n which satisfy $\pi^k(v) = v$.

(20 Points)