Institut für Mathematische Optimierung
TU Braunschweig

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SS 2006

Coding Theory / Discrete Mathematics II
Assignment 4 (May 18, 2006)

(This assignment is due on May 26, 2006, 1.00 p.m., by dropping it into the wooden box **in front of** F 310)

Exercise 1 (Ring of polynomials):

Let K be a field and $K[x]$ the ring of polynomials over K.

a) Let $h \in K[x] \setminus \{0\}$ and $h(\alpha) = 0$ for some $\alpha \in K$. Show that $h = g \cdot (x - \alpha)$ for $q \in K[x]$.

b) Let $f \in K[x] \setminus \{0\}$ with degree 2 or 3. Show that f is irreducible if and only if $f(\alpha) \neq 0$ for every $\alpha \in K$.

(Hint: Use Theorem 3.5 about the division of polynomials to prove part a).)

(15+15 Points)

Exercise 2 (Irreducible polynomials):

a) Determine all irreducible polynomials of degree 2 in $\mathbb{Z}_3[x]$.

b) Write $x^5 - x^4 - x^3 + x + 1$ as a product of irreducible polynomials over $\mathbb{Z}_3$.

c) Write $x^4 + 8x^2 + 15$ as a product of irreducible polynomials over $\mathbb{R}$.

(15+8+7 Points)