

Prof. Dr. Sándor Fekete
Nils Schweer

Discrete Mathematics I Assignment 8 (December 21, 2005)

(This assignment is due on January 11, 2006, 1.00 p.m., by dropping it into the wooden box in front of F 310)

Exercise 1 (Chinese Remainder Theorem):

Use the technique from the proof of the Chinese Remainder Theorem to determine all solutions of the following system of congruences:

$$x \equiv 1 \pmod{2}, x \equiv 2 \pmod{3}, x \equiv 3 \pmod{5}, x \equiv 4 \pmod{11}$$

(30 Points)

Exercise 2 (Euler's φ -function):

Euler's φ -function

$$\varphi(n) = \sum_{i=1}^n \left\lfloor \frac{1}{\gcd(i, n)} \right\rfloor$$

counts the number of positive integers less than or equal to n that are relatively prime to n .

- a) Determine $\varphi(4)$, $\varphi(10)$, $\varphi(13)$.
- b) Prove that if p is prime then $\varphi(p) = p - 1$.

(30 Points)