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## Discrete Mathematics I Assignment 4 (23.11.2005)

(This assignment is due on 30.11.2005, 1.00 p.m., by dropping it into the wooden box in front of F 310.)

### Exercise 1 (Fibonacci numbers):

The Fibonacci numbers  $f_n$ ,  $n \geq 1$  are defined as follows:

$$f_1 = 1, f_2 = 1 \text{ and } f_n = f_{n-1} + f_{n-2} \text{ for all } n \geq 3.$$

Prove by mathematical induction that,

- a)  $f_{n+3} - f_n = 2f_{n+1}$  for all  $n \geq 1$ .
- b)  $\sum_{i=1}^n f_{2i} = f_{2n+1} - 1$  for all  $n \geq 1$ .

(15+15 Points)

### Exercise 2 (Factors and multiples):

- a) Prove that  $5|(n^5 - n)$  for all  $n \geq 1$ .
- b) Prove that  $6|n(n+1)(n+2)$  for all  $n \geq 1$  (without using mathematical induction).

(15+15 Points)