Institut für Mathematische Optimierung
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WS 2005/2006

Discrete Mathematics I
Assignment 3 (16.10.2005)

(This assignment is due on 23.11.2005, 1.00 p.m., by dropping it into the wooden box **in front of** F 310)

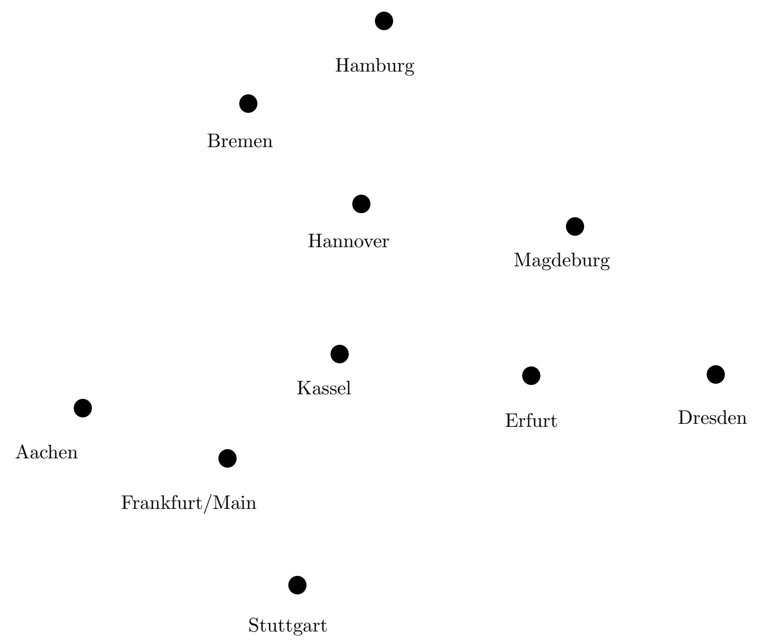

Figure 1: 10 cities in Germany

There is a direct connection between any two cities. The following table shows the length of every connection.

		2	3	4	5	6	7	8	9	10
1	Aachen	37	65	45	24	48	35	31	50	41
2	Bremen		48	35	45	11	12	28	25	65
3	Dresden			22	49	49	39	40	23	53
4	Erfurt				27	38	29	14	21	44
5	Frankfurt/Main					51	36	19	45	20
6	Hamburg						15	31	27	66
7	Hannover							24	14	53
8	Kassel								25	36
9	Magdeburg									57
10	Stuttgart									

Exercise: (Traveling Salesman Problem)
In the following we consider the above instance of the Traveling Salesman Problem.

In the first step you have to find an optimal solution to this instance. (One way of obtaining a good solution may be guessing.) Let L denote the value of your solution.

In the second step you are asked to **prove** a lower bound for the optimal solution L^*. A lower bound is a number that is not larger than *a*ll feasible solutions to the problem. (E.g. you may say that every tour contains at least 1 connection. So 11 is a lower bound since it is the smallest value from the table.) Let S denote the size of your lower bound.
Note, enumerating all possible tours will not be accepted as a proof for a lower bound.

Best possible would be to show that $S = L$ but that may be very complicated. If you obtain the same lower bound that we worked out you will get 60 points. Otherwise you will get

$$\mathbf{60 \cdot S/L \ points}.$$

If you do not prove a lower bound S will be set to zero. If you do not give a feasible solution L will be set to ∞. In both cases you do not get any points.

Recall:

Traveling Salesman Problem (TSP)
Given a collection of cities and the cost of travel between each pair of them, the traveling salesman problem, or TSP for short, is to find the cheapest way of visiting all of the cities and returning to your starting point.