

Guarding Offices with Maximum Dispersion*

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Abstract

We investigate the DISPERSIVE ART GALLERY PROBLEM with vertex guards for a class of orthogonal polygons that reflect the properties of real-world floor plans: These *office-like* polygons consist of rectangular rooms and corridors. In the dispersive variant of the ART GALLERY PROBLEM, the objective is not to minimize the number of guards but to maximize the minimum distance between any pair of guards, called the *dispersion distance*.

We prove that determining whether a guard set can achieve a dispersion distance of 4 in office-like polygons, where vertices are restricted to integer coordinates, is NP-complete. Complementarily, we present a simple worst-case optimal algorithm that guarantees a dispersion distance of 3. For the more restricted class of hole-free independent office-like polygons, we propose a dynamic programming approach that computes optimal solutions in polynomial time.

1 Introduction

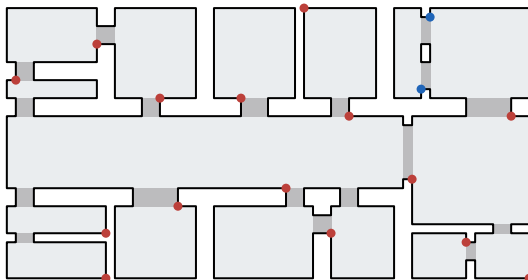
The ART GALLERY PROBLEM is a widely studied problem in computational geometry: Given a polygonal region, the objective is to select a minimum number of points (*guards*) such that every point within the region is visible from at least one guard. Since its introduction by Victor Klee in 1973, numerous variants of the problem have been studied [9, 11, 12].

An important application involves deploying sensors across a designated area, where a key challenge is balancing sensor spacing with coverage requirements. While increasing the distance between sensors reduces mutual interference and helps preserve data integrity, the network must remain sufficiently dense to ensure complete coverage and continuous, reliable signal transmission throughout the region. This trade-off is captured by the DISPERSIVE ART GALLERY PROBLEM, which is defined as follows: Given a polygon \mathcal{P} and a rational number ℓ , determine whether there exists a guard set \mathcal{G} for \mathcal{P} such that the geodesic distance between any pair of guards in \mathcal{G} is at least ℓ . Notably, this formulation focuses solely on pairwise distances between guards rather than minimizing their total number.

* An extended abstract of this paper appeared in the proceedings of the 50th International Symposium on Mathematical Foundations of Computer Science (MFCS 2025) DOI: 10.4230/LIPIcs.MFCS.2025.46. This work was partially supported by the German Research Foundation (DFG), grants 444569951 and 522790373. Moreover, it was supported by the National Science Foundation (CCF-2007275), and by the Austrian Science Fund (FWF) 10.55776/DOC183.

42nd European Workshop on Computational Geometry, Hagen, Germany, March 25–27, 2026.
This is an extended abstract of a presentation given at EuroCG'26. It has been made public for the benefit of the community and should be considered a preprint rather than a formally reviewed paper. Thus, this work is expected to appear eventually in more final form at a conference with formal proceedings and/or in a journal.

In this paper, we study the tractability of the problem in the setting of vertex guards for a class of orthogonal polygons that capture essential structural characteristics of real-world floor plans and rectangular galleries [4]. These polygons, referred to as *office-like polygons*¹, consist of orthogonal rooms and corridors; see Figure 1 for an illustrative example.



■ **Figure 1** An example of an office-like polygon with three holes, together with a guard set of maximum dispersion. The blue guards have minimum distance among all pairs of guards.

1.1 Our Contributions

We summarize our main results for the DISPERSIVE AGP with vertex guards in office-like polygons under r -visibility and L_1 -geodesics:

- We give a worst-case optimal method showing that office-like polygons with integer vertices always admit a guard set with dispersion distance 3, which is sometimes necessary.
- We show that deciding the existence of a guard set with a dispersion distance of 4 is NP-complete, even for integer-coordinate polygons, as well as polyominoes.
- We present a dynamic programming algorithm for independent office-like polygons without holes to efficiently compute guard sets with maximum dispersion in polynomial time.

Due to space restrictions, proofs and technical details can be found in the full version [5].

1.2 Previous work

Prior work studied the DISPERSIVE AGP for vertex guards in polyominoes under L_1 -geodesic distance and r -visibility [10]. The problem of deciding whether a thin polyomino admits dispersion distance 5 is NP-complete, whereas simple polyominoes admit worst-case optimal guard sets with dispersion distance 3 computable in linear time. These results were later extended to polygons [6]: deciding dispersion distance 2 is NP-complete for polygons with holes, and even polygons without holes may require dispersion distance 2, which can be achieved using a constructive algorithm.

The classic AGP in office-like polygons was studied in [4]. For hole-free office-like polygons, a greedy algorithm computes guard sets of minimum cardinality, polynomial-time algorithms based on matching and flow techniques exist for r -independent instances, and the general problem under r -visibility is NP-hard. More broadly, the AGP is $\exists\mathbb{R}$ -complete under classic line visibility [1], remains NP-hard for r -visibility [8], and becomes polynomial-time solvable for hole-free orthogonal polygons under r -visibility [13].

¹ This class of polygons was originally introduced by Cruz and Tomás [4] using the Portuguese expression SCOT for *salas e corredores ortogonais*, i.e., orthogonal rooms and corridors.

1.3 Preliminaries

Given any polygon \mathcal{P} , a subset $\mathcal{G} \subset \mathcal{P}$ of points is called a *guard set* for \mathcal{P} if every point $p \in \mathcal{P}$ is *visible* to at least one point $g \in \mathcal{G}$. We restrict ourselves to vertex guards, i.e., guards are only placed on vertices of \mathcal{P} . Moreover, we adopt the *r-visibility* model, where two points p and q are said to *see each other* if the axis-aligned rectangle defined by p and q is entirely contained within \mathcal{P} . We measure the *distance* between two points in \mathcal{P} as *L_1 -geodesics*. With this, the *dispersion distance* ℓ of a guard set \mathcal{G} is then defined as the minimum distance between any pair of guards.

An *office-like polygon* $\mathcal{P} = (\mathcal{R}, \mathcal{C})$ is an orthogonal polygon made up from a collection of rectangular rooms \mathcal{R} that are linked by a collection of rectangular corridors \mathcal{C} such that all rectangles in $\mathcal{R} \cup \mathcal{C}$ have pairwise disjoint interiors. A corridor connects two rooms if two opposite edges of the corridor are *strictly contained* in edges of the two rooms, one in each room; in particular, the corridor is *narrower* than the rooms, and no vertex of a room coincides with a vertex of a corridor. An example of an office-like polygon with several rooms and corridors, as well as a set of guards is depicted in Figure 1. Two corridors C_1 and C_2 are *independent* if no guard placed on a vertex of C_1 also guards C_2 . If every pair of corridors is independent, then the polygon is *independent*.

2 Worst-case optimality

In this section, we present a worst-case optimal algorithm for the DISPERSIVE AGP in office-like polygons with vertices at integer coordinates.

► **Theorem 2.1.** *For office-like polygons with vertices at integer coordinates, there always exists a guard set with dispersion distance at least 3, which is sometimes optimal.*

As an initial observation, one can easily construct instances in which every feasible guard set has a dispersion distance of at most 3; such an instance is illustrated in Figure 2(a). In this polygon, at least one guard is required in each of the three unit-square corridors, which necessarily forces at least one pair of guards to be at distance at most 3 from one another. Importantly, this polygon can be used as a building block, as it can be extended arbitrarily in the directions indicated by the arrows.

We now present the high-level idea of an algorithm that constructs guard sets with a dispersion distance of 3, which is therefore optimal in the worst case. The algorithm works in three phases. During each phase, a set of guards is placed while maintaining the invariant that every newly placed guard is at distance at least 3 from all guards placed earlier.

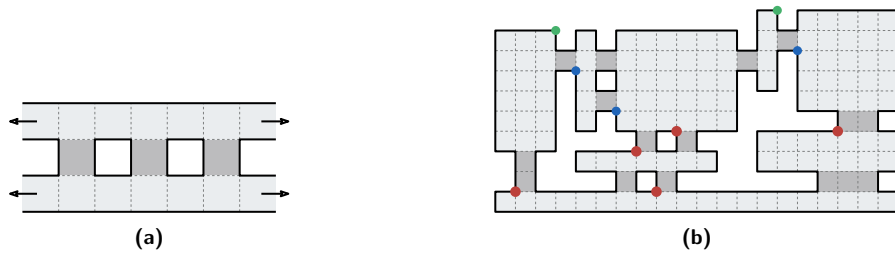
Phase (1): Place guards at corridor vertices for all corridors that connect two rooms vertically, ensuring full coverage of these corridors.

Phase (2): Apply the same procedure to corridors that connect two rooms horizontally.

Phase (3): Place guards in any rooms that remain uncovered after the previous phases.

During the initial two phases, guards are placed along the left boundaries of vertical corridors and along the bottom boundaries of horizontal corridors. As a result, any room that remains uncovered after these two phases must contain at least one vertex whose distance to every guard placed in an incident corridor is at least three. Figure 2(b) illustrates the guard placements computed by our algorithm. Guards placed during the first phase are shown in red, those placed in the second phase in blue, and those placed in the final phase in green.

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■ **Figure 2** A dispersion distance of 3 is (a) sometimes best possible and (b) always realizable.

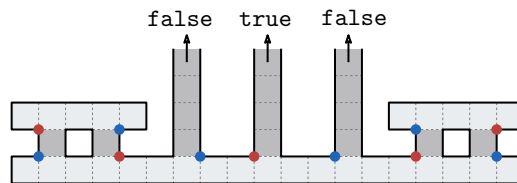
3 Computational complexity

We investigate the computational complexity of the respective decision problem, and show:

► **Theorem 3.1.** *Deciding whether there exists a guard set with a dispersion distance of 4 in independent office-like polygons with vertices at integer coordinates is NP-complete.*

Membership in NP is straightforward; thus, we focus on the NP-hardness reduction. We reduce from PLANAR 3SAT WITH EXACTLY THREE OCCURRENCES PER VARIABLE [2, 3, 7], where each variable appears twice negated and once unnegated. Given such a Boolean formula φ , we compute a rectilinear embedding of its clause-variable incidence graph and straightforwardly replace variables and clauses by the corresponding gadgets.

Variable gadget. The variable gadget, shown in Figure 3, admits exactly two feasible guard sets with a dispersion distance of 4, corresponding to the variable being assigned **true** (red vertices) or **false** (blue vertices). Crucially, no single guard set can simultaneously observe both the corridor labeled **true** and the corridor labeled **false**.



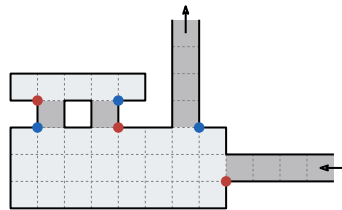
■ **Figure 3** Variable gadget for office-like polygons with vertices at integer coordinates.

Clause gadget. Different clause gadgets accommodate both two- and three-literal clauses, as illustrated in Figure 4. In each gadget, guard sets with dispersion distance 4 cover all but one incoming corridor, and no such set can cover all corridors simultaneously; vertices with the same color indicate membership in the same guard set. Consequently, at least one corridor must be guarded from outside the gadget, ensuring that the corresponding clause gadget is satisfied under the associated truth assignment.

Bending gadget. The bending gadget, depicted in Figure 5, implements a 90° edge-bend. It effectively propagates an assignment across the turn: if the incoming corridor is guarded from outside the gadget, the outgoing corridor can be guarded from inside (blue vertices), and vice versa (red vertices). Importantly, no guard set with dispersion distance 4 can simultaneously cover both corridors from within it.



■ **Figure 4** Clause gadgets for office-like polygons with vertices at integer coordinates.



■ **Figure 5** The bending gadget enables 90° turns within office-like polygons.

These gadgets enable us to prove that a Boolean formula is satisfiable if and only if the constructed polygonal region admits a guard set with dispersion distance 4, thereby establishing NP-hardness. Moreover, as illustrated in the figures, each constructed polygonal region is in fact a polyomino. Consequently, the reduction applies directly to polyominoes as well, and the result holds even under this restriction.

4 DP for hole-free independent office-like polygons

We now present a polynomial-time dynamic programming algorithm that computes the optimal solution for independent office-like polygons without holes. The main component of this approach is to solve geometric independent set problems.

► **Theorem 4.1.** *For independent office-like polygons without holes, there exists a polynomial-time algorithm that computes guard sets with maximum dispersion.*

The high-level idea of our approach is as follows: Consider a hole-free independent office-like polygon \mathcal{P} . We solve the decision problem of whether there exists a guard set with dispersion distance at least ℓ , and then perform a binary search over the $\mathcal{O}(n^2)$ candidate values implied by all pairwise vertex distances in \mathcal{P} to solve the maximization problem.

For the decision problem, we associate with \mathcal{P} a tree $G(\mathcal{P}) = (V, E)$ defined as follows: There is a node $v \in V$ for each room R_v of \mathcal{P} , and an edge $e = \{v, w\} \in E$ exactly when the corresponding rooms R_v and R_w are connected by a corridor C_e . We fix an arbitrary in-arborescence G' of $G(\mathcal{P})$ with all edges directed toward a root, see Figure 6(a).

For each node v , we define a corresponding subproblem, which is solved only after all predecessor nodes have been marked as processed, thereby enabling a bottom-up construction of the solution. To process the subproblem at node v , we remove the room R_v and, if present, the corridor C_a corresponding to the outgoing arc a , thereby partitioning \mathcal{P} into several subpolygons, see Figure 6(b). We slightly abuse notation by letting C_a refer to the corridor associated with the underlying edge of a .

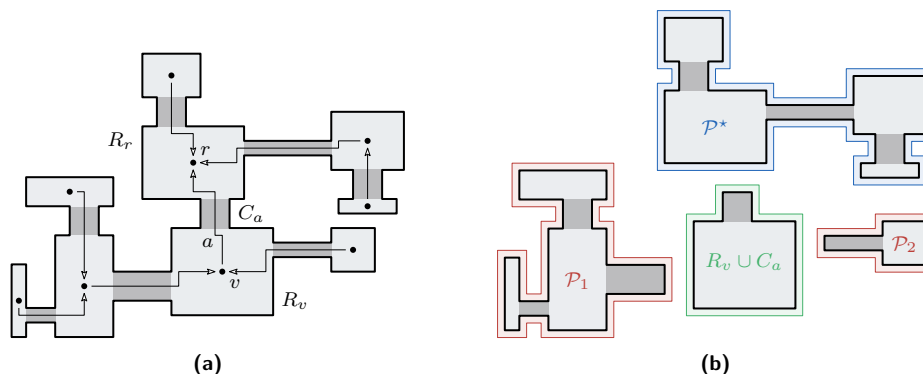


Figure 6 A hole-free independent office-like polygon \mathcal{P} with (a) an in-arborescence of $G(\mathcal{P})$ and (b) the subpolygons considered for the subproblem at node v . Feasible guard sets for \mathcal{P}_1 and \mathcal{P}_2 are already available, guard placements in $R_v \cup C_a$ are enumerated, and \mathcal{P}^* is ignored.

Let $\mathcal{P}^* \subseteq \mathcal{P}$ denote the subpolygon that contains the room R_r of the root r of G' . By convention, if $v = r$, we set $\mathcal{P}^* = \emptyset$. Our objective is to compute a constant number of *candidate* guard sets for $\mathcal{P} \setminus \mathcal{P}^*$ with the following property: if there exists a guard set for \mathcal{P} with dispersion distance ℓ , then such a guard set can be obtained by placing guards at vertices of \mathcal{P}^* and choosing one of these candidate sets for $\mathcal{P} \setminus \mathcal{P}^*$. The fact that a constant number of candidates suffices without knowing \mathcal{P}^* is argued using the independence of \mathcal{P} .

At this stage, for each subpolygon created by the removal of R_v and C_a , except for \mathcal{P}^* , we have already recursively computed a constant number of candidate guard sets. Hence, it remains to select exactly one guard set from each of these subpolygons and to combine them with one of the constantly many possible guard placements in $R_v \cup C_a$, obtained by explicitly testing all feasible options, so as to construct a valid guard set for $\mathcal{P} \setminus \mathcal{P}^*$.

This selection task can be formulated as a geometric independent set problem: from each subpolygon, select exactly one guard set such that the pairwise distance between guards from different subpolygons is at least ℓ . We address this through a sequence of greedy selection procedures. Embedding this decision routine as a subroutine within the above-mentioned binary search framework yields a guard set with maximum dispersion.

5 Conclusions

We conducted a comprehensive study of the DISPERSIVE AGP in office-like polygons and obtained several results. Most notably, we proved that deciding whether a dispersion distance of 4 can be achieved in office-like polygons with holes and vertices at integer coordinates is NP-complete. This hardness result straightforwardly extends to polyominoes.

Complementing this, we established a tight bound by showing that a guard set with dispersion distance 3 always exists and can be computed by a simple algorithm. Moreover, there exist office-like polygons in which a dispersion distance of 3 is necessary, making the algorithm worst-case optimal.

For independent office-like polygons without holes, we developed a dynamic programming algorithm that computes an optimal solution in polynomial time. Whether this approach can be extended to non-independent polygons remains an open problem.

Finally, the variant involving point guards remains widely open.

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