Point Guards and Point Clouds: Solving General Art Gallery Problems

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1. INTRODUCTION

In this video, we illustrate how one of the classical areas of computational geometry has gained in practical relevance, which in turn gives rise to new, fascinating geometric problems. In particular, we demonstrate how the robot platform IRMA3D can produce high-resolution, virtual 3D environments, based on a limited number of laser scans. Computing an optimal set of scans amounts to solving an instance of the Art Gallery Problem (AGP): Place a minimum number of stationary guards in a polygonal region P, such that all points in P are guarded.

As first proven by Chvátal [2] and shown by Fisk [6] in a beautiful and concise proof, $\lfloor \frac{n}{3} \rfloor$ guards are sometimes necessary and always sufficient for a simple polygon P with n vertices. See O'Rourke [9] for an early overview.

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Algorithmically, the AGP is NP-hard, even for a simply connected polygonal region P [8]. Eidenbenz et al. [4] showed that for a region with holes, finding an optimal set of vertex guards is at least as hard as Set Cover, so there is little hope of achieving a better approximation guarantee than $\Omega(\log n)$. It seems unlikely that this gets any easier when allowing general point guards, as there is no known simple characterization of a discrete candidate set of guard locations. All this show the difficulty of the AGP, but it does not rule out methods that combine structural insights with powerful mathematical tools in order to achieve provably optimal solutions for instances of interesting size.

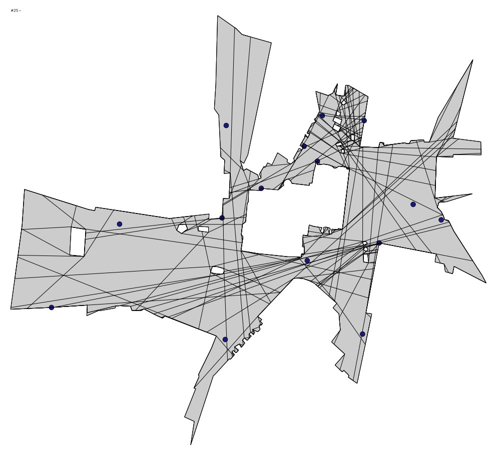

Figure 1: A real-life AGP instance with 15 holes and 332 vertices: the city center of Bremen.

Computing optimal solutions for general AGP instances is not only relevant from a theoretical point of view, but has also gained in practical importance in the context of modeling, mapping and surveying complex environments, such as in the fields of architecture, robotics and medicine.

2. OUR WORK

Irma3D (**I**ntelligent **R**obot for **M**apping **A**pplications in 3D) is an autonomous robot; see Fig. 2. Its main sensor is a Riegl VZ-400 laser scanner. A typical 3D laser scan needs 3 minutes, producing up to 20 million highly precise 3D measurements of the surrounding. A globally consistent scan matching is used to merge the 3D scans to a single scene [1]. Irma3D is built of a Volksbot RT-3 chassis; it uses the Xsens MTi IMU and odometry to sense its own position.

Figure 2: IRMA3D in front of the town hall scanning the city square of Bremen.

Lately, the groups in Campinas and Braunschweig have independently started to combine methods from integer (IP) and linear programming (LP) with non-discrete geometry in order to obtain optimal solutions; first for the discrete case of vertex guards [3], but now also for general point guards.

The algorithm in [10] computes lower and upper bounds for the AGP, based on computing finite set-cover instances with the help of a state-of-the-art IP solver. To generate a lower bound, a finite set of witness candidates is chosen and a restricted AGP is solved, in which only the witnesses have to be covered. For this, it suffices to extract a finite set of potential guard positions from the visibility arrangement of the witness set in order to ensure optimality. Similarly, finite sets of potential witness positions for a given finite guard set can be extracted from the visibility arrangment of the guards. This allows it to compute upper and lower bounds for the optimal AGP value by solving discrete set cover instances. The algorithm of [10] iterates between generating tighter lower and upper bounds by refining the witness and guard candidate sets along the iterations. It stops when lower and upper bounds coincide. Although no theoretical convergence has been established, in tests, the approach is able to yield optimal solutions for a large variety of instance classes, even for polygons with up to a thousand vertices.

An approach presented in [7] considers a similar primal-dual scheme, but focuses on the linear relaxation of the primal guard cover, whose dual is the witness packing problem.

$$\min \quad \sum_{g \in G} x_g \tag{1}$$

$$\text{s.t.} \quad \sum_{g \in G \cap \mathcal{V}(w)} x_g \geq 1 \quad \forall w \in W \tag{2}$$

$$0 \leq x_g \leq 1 \qquad \forall g \in G \tag{3}$$

Allowing fractional guard values leads to identical optimal primal and dual optimal solutions. In order to eliminate fractional solutions, we can apply appropriate cutting planes derived from the set cover polytope; see [5] for details.

$$\sum_{g \in J_2 \cap G} 2x_g + \sum_{g \in J_1 \cap G} x_g \geq 2 \tag{4}$$

As it turns out [5], only a small subset of these inequalities matter in the context of AGP instances. Together with a similar primal-dual iteration scheme such as the one in [10], we can find optimal integral solutions for a large range of benchmark instances, including the one shown in Figure 1.

3. THE VIDEO

The video opens with a city scene, to be scanned by the robot. This gives rise to the AGP, introduced in the next sequence. Then an IP approach for general point guards is presented, based on an analysis of possible guard and witness positions in the arrangement of visibility polygons. This is followed by the description of an LP approach, which is combined with ideas for eliminating fractional vertices by means of cutting planes; the method is then applied to the city instance from the introduction. Finally, the resulting scans are combined into a virtual flight through the city environment, both in visible light and in infrared.

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