Minimum Covering with Travel Cost*

Sándor P. Fekete[1], Joseph S.B. Mitchell[2,**], and Christiane Schmidt[1,***]

[1] Algorithms Group, Braunschweig Institute of Technology, Germany
{s.fekete,c.schmidt}@tu-bs.de
[2] Department of Applied Mathematics and Statistics, Stony Brook University
jsbm@ams.stonybrook.edu

Abstract. Given a polygon and a visibility range, the Myopic Watchman Problem with Discrete Vision (MWPDV) asks for a closed path P and a set of scan points S, such that (i) every point of the polygon is within visibility range of a scan point; and (ii) path length plus weighted sum of scan number along the tour is minimized. Alternatively, the bicriteria problem (ii') aims at minimizing both scan number and tour length. We consider both lawn mowing (in which tour and scan points may leave P) and milling (in which tour, scan points and visibility must stay within P) variants for the MWPDV; even for simple special cases, these problems are NP-hard.

We sketch a 2.5-approximation for rectilinear MWPDV milling in grid polygons with unit scan range; this holds for the bicriteria version, thus for any linear combination of travel cost and scan cost. For grid polygons and circular unit scan range, we describe a bicriteria 4-approximation. These results serve as stepping stones for the general case of circular scans with scan radius r and arbitrary polygons of feature size a, for which we extend the underlying ideas to a $\pi(\frac{r}{a}+\frac{r+1}{2})$ bicriteria approximation algorithm. Finally, we describe approximation schemes for MWPDV lawn mowing and milling of grid polygons, for fixed ratio between scan cost and travel cost.

1 Introduction

Covering a given polygonal region by a small set of disks or squares is a problem with many applications. Another classical problem is finding a short tour that visits a number of objects. Both of these aspects have been studied separately, with generalizations motivated by natural constraints.

In this paper, we study the combination of these problems, originally motivated by challenges from robotics, where accurate scanning requires a certain

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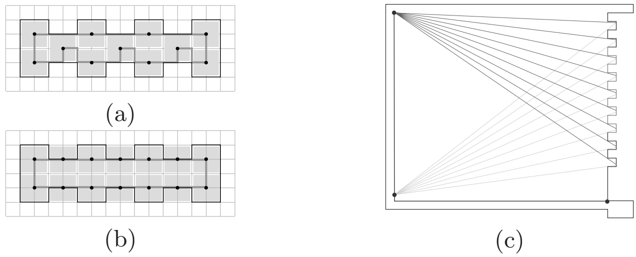

Fig. 1. (a) An MWPDV solution with a minimum number of scans; (b) an MWPDV solution with a minimum tour length. (c) A minimum guard cover may involve scan points that are not from an obvious set of candidate points.

amount of time for each scan; obviously, this is also the case for other surveillance tasks that combine changes of venue with stationary scanning. The crucial constraints are (a) a limited visibility range, and (b) the requirement to stop when scanning the environment, i.e., with vision only at discrete points. These constraints give rise to the *Myopic Watchman Problem with Discrete Vision* (MWPDV), the subject of this paper.

For a scan range that is not much bigger than the feature size of the polygon, the MWPDV combines two geometric problems that allow approximation schemes (minimum cover and TSP). This makes it tempting to assume that combining two approximation schemes will yield a polynomial-time approximation scheme (PTAS), e.g., by using a PTAS for minimum cover (Hochbaum and Maass [11]), then a PTAS for computing a tour on this solution. As can be seen from Figure 1 (a) and (b), this is not the case; moreover, an optimal solution depends on the relative weights of tour length and scan cost. This turns the task into a bicriteria problem; the example shows that there is no simultaneous PTAS for both aspects. As we will see in Sections 3 and 4, a different approach allows a simultaneous constant-factor approximation for both scan number and tour length, and thus of the combined cost. We show in Section 7, a more involved integrated guillotine approach allows a PTAS for combined cost in the case of a fixed ratio between scan cost and travel cost.

A different kind of difficulty is highlighted in Figure 1 (c): For a visibility range r that is large compared to the feature size a, it may be quite hard to determine a guard cover of small size. In fact, there is no known constant-factor approximation for minimum guard cover in general polygons; currently, the best result is an $O(\log OPT)$-approximation by Efrat and Har-Peled [7]. In addition, the optimal solution may change significantly with the relative weights between tour length: If tour length dominates the number of scans, an optimal tour can be forced to follow the row of niches on the right. We will show in Section 6 how to obtain a constant-factor approximation for bounded value $\frac{r}{a}$.

Related Work. Closely related to practical problems of searching with an autonomous robot is the classical theoretical problem of finding a *shortest watchman tour*; e.g., see [4,5]. Planning an optimal set of scan points (with unlimited visibility) is the *art gallery problem* [14]. Finally, visiting all grid points of a given set is a special case of the classical *Traveling Salesman Problem* (TSP); see [12]. Two generalizations of the TSP are the so-called *lawn mowing* and *milling problems*: Given a cutter of a certain shape, e.g., an axis-aligned square, the *milling problem* asks for a shortest tour along which the (center of the) cutter moves, such that the entire region is covered and the cutter stays inside the region at all times. Clearly, this takes care of the constraint of limited visibility, but it fails to account for discrete visibility. At this point, the best known approximation method for milling is a 2.5-approximation [2]. Related results for the TSP with neighborhoods (TSPN) include [6,13]; further variations arise from considering online scenarios, either with limited vision [3] or with discrete vision [10,8], but not both. Finally, [1] consider covering a set of points by a number of scans, and touring all scan points, with the objective function being a linear combination of scan cost and travel cost; however, the set to be scanned is discrete, and scan cost is a function of the scan radius, which may be small or large.

For an online watchman problem with unrestricted but discrete vision, Fekete and Schmidt [10] present a comprehensive study of the milling problem, including a strategy with constant competitive ratio for polygons of bounded feature size and with the assumption that each edge of the polygon is fully visibly from some scan point. For limited visibility range, Wagner et al. [15] discuss an online strategy that chooses an arbitrarily uncovered point on the boundary of the visibility circle and backtracks if no such point exists. For the cost they only consider the length of the path used between the scan points, scanning causes no cost. Then, they can give an upper bound on the cost as a ratio of total area to cover and squared radius.

Our Results. On the positive side, we give a 2.5-approximation for the case of grid polygons and a rectangular range of unit-range visibility, generalizing the 2.5-approximation by Arkin, Fekete, and Mitchell [2] for continuous milling. The underlying ideas form the basis for more general results: For circular scans of radius $r = 1$ and grid polygons we give a 4-approximation. Moreover, for circular scans of radius r and arbitrary polygons of feature size a, we extend the underlying ideas to a $\pi(\frac{r}{a} + \frac{r+1}{2})$-approximation algorithm. All these results also hold for the bicriteria versions, for which both scan cost and travel cost have to approximated simultaneously. Finally, we present a PTAS for MWPDV lawn mowing, and sketch a PTAS for MWPDV milling, both for the case of fixed ratio between scan cost and travel cost.

2 Notation and Preliminaries

We are given a polygon P. In general, P may be a polygon with holes; in Sections 3, 4 and 5, P is an axis-parallel polygon with integer coordinates.

Our robot, R, has discrete vision, i.e., it can perceive its environment when it stops at a point and performs a scan, which takes c time units. From a scan point p, only a ball of radius r is visible to R, either in L_∞- or L_2-metric. A set $\mathcal{S}$ of scan points *covers* the polygon P, if and only if for each point $q \in P$ there exists a scan point $p \in \mathcal{S}$ such that q sees p (i.e., $qp \subset P$) and $|qp| \leq r$.

We then define the *Myopic Watchman Problem with Discrete Vision* (MWPDV) as follows: Our goal is to find a tour T and a set of scan points $\mathcal{S}(T)$ that covers P, such that the total travel and scan time is optimal, i.e., we minimize $t(T) = c \cdot |\mathcal{S}(T)| + L(T)$, where $L(T)$ is the length of tour T. Alternatively, we may consider the bicriteria problem, and aim for a simultaneous approximation of both scan number and tour length.

3 NP-Hardness

Even the simplest and extreme variants of MWPDV lawn mowing are still generalizations of NP-hard problems; proofs are omitted.

Theorem 1. *(1) The MWPDV is NP-hard, even for polyominoes and small or no scan cost, i.e.,* $c \ll 1$ *or* $c = 0$.

(2) The MWPDV is NP-hard, even for polyominoes and small or no travel cost, i.e., $c \gg 1$ *or* $t(T) = |\mathcal{S}|$.

4 Approximating Rectilinear MWPDV Milling for Rectangular Visibility Range

As a first step (and a warmup for more general cases), we sketch an approximation algorithm for rectilinear visibility range in rectilinear grid polygons.

Our approximation proceeds in two steps; details can be found in [9].

(I) Construct a set of scan points that is not larger than 2.5 times a minimum cardinality scan set.
(II) Construct a tour that contains all constructed scan points and that does not exceed 2.5 times the cost of an optimum milling tour.

First we describe how to construct a covering set of scan points:

1. Let S_{4e} be the "even quadruple" centers of all 2x2-squares that are fully contained in P, and which have two even coordinates.
2. Remove all 2x2-squares corresponding to S_{4e} from P; in the remaining polyomino P_{4e}, greedily pick a maximum disjoint set S_{4o} of "odd quadruple" 2x2-squares.
3. Remove all 2x2-squares corresponding to S_{4o} from P_{4e}; greedily pick a maximum disjoint set S_3 of "triple" 2x2-squares that cover 3 pixels each in the remaining polyomino $P_{4e,4o}$,
4. Remove all 2x2-squares corresponding to S_3 from $P_{4e,4o}$; in the remaining set $P_{4e,4o,3}$ of pixels, no three can be covered by the same scan. Considering edges between pixels that can be covered by the same scan, pick a minimum set of ("double" S_2 and "single" S_1) scans by computing a maximum matching.

Claim 1. The total number of scans is at most 2.5 times the size of a minimum cardinality scan set.

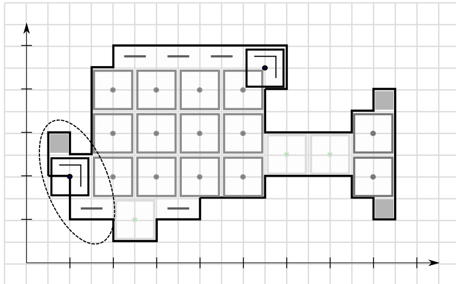

Fig. 2. An example for our approximation method: The set of "even quadruple"scans is shown in gray; the "odd quadruple" scans are light gray. A possible (greedy!) set of "triple" scan is shown in black, leaving the maximum matching (and the corresponding "double scans") shown in dark gray. The leftover single pixels are filled squares. The ellipse indicates a part that is covered by three scans instead of two: the triple scan with adjacent single and double scans could be covered by two triple scans.

Claim 2. All scan points lie on a 2.5-approximative milling tour.

The tour consists of (a) a "boundary" part following the contour of the polygon; (b) a "strip" part that covers the interior; (c) a "matching" part that allows an Eulerian tour. The cost for (a) and (b) is $L(T^*)$, while (c) can be bounded by $L(T^*)/2$. Proofs are omitted for lack of space.

Theorem 2. *A polyomino P allows a MWPDV with rectangular vision solution that contains at most 2.5 times the minimum number of scans necessary to scan the polygon, and has tour length at most 2.5 times the length of an optimum milling tour.*

5 Approximating Rectilinear MWPDV Milling for Circular Visibility Range

When considering a circular scan range, one additional difficulty are boundary effects of discrete scan points: While continuous vision allows simply sweeping a corridor of width $2r$, additional cleanup is required for the gaps left by discrete vision; this requires additional mathematical arguments.

We overlay the polyomino with a point grid as in Figure 3, left, i.e., a diagonal point grid with L_2-distance of $\sqrt{2}$. These are used as scan points; it is relatively straightforward to prove that this number is within a factor of 4 of the optimum number of scans.

For the movement between interior scan points and the boundary we use horizontal strips located on grid lines (and distance 1 to the boundary). As before, these are combined with a boundary tour. As strip ends do not fully extend to the boundary of the polygon, we link pairs of strips and connect them to the left boundary, other scan points are visited by paths of length 2 from the boundary. This can be achieved at the cost of one additional tour; see Figure 3, right, for an example.

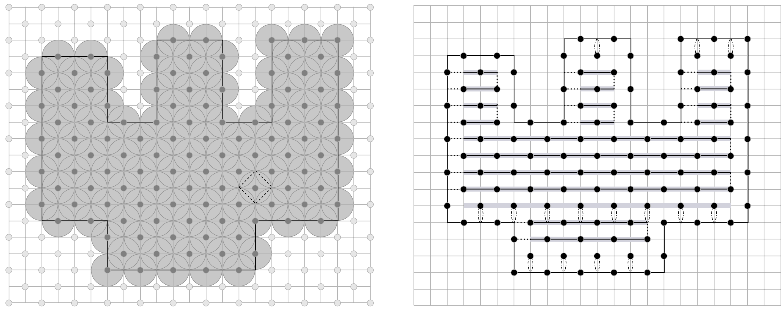

Fig. 3. Left: Point grid (light gray) with grid points within a polyomino (black) in dark gray. Circular visibility ranges of the grid points covering the plane, one square of side length $\sqrt{2}$ is indicated by a dashed line. Right: A polyomino P with the tour given by our strategy. Scan points are displayed in black. The horizontal strips of total length L_{strips} are indicated in bold light gray, the tour is in black for the links to the boundary and between strips (dotted) as well as for connections of points (dash-dotted) and parts located on the strips are indicated as a continuous line. (The rest of the tour runs on the boundary.)

Theorem 3. *A polyomino P allows a MWPDV solution for a circular visibility range with $r = 1$ that is 4-competitive.*

6 Approximating General MWPDV Milling for a Circular Visibility Range

In this section we discuss MWPDV milling for a circular visibility range r in general polygons. As discussed in Section 1, even the problem of minimum guard coverage has no known constant-factor approximation; therefore, we consider a bounded ratio r/a between visibility range and feature size.

Just as in the rectilinear case for a rectilinear scan range, our approximation proceeds in the two steps (I) and (II), see Section 4.

We start with a description of the second step, which will form the basis for the placement of scan points. Just as in the rectilinear case, we consider three parts.

(1) A "boundary" part: We use two "boundary tours" within distance of (at most) $\frac{1}{2}r$ and (at most) $\frac{3}{2}r$ to the boundary, $TR1$ and $TR2$ of length L_{TR1} and L_{TR2}, respectively. With $L_{\delta B}$ denoting the length of the boundary δB of B ($B \subset P$ is the inward offset region of all points within P that are feasible placements for the center of a milling cutter), we get:

$$L_{TR1} + L_{TR2} = 2 \cdot L_{\delta B} \leq 2 \cdot L(T^*) \quad (1)$$

(The length of the three tours differs at the vertices: drawing a line perpendicular there from $TR2$ to $TR1$ the Intercept Theorem shows that the distance to the diagonal through the vertices of all tours on $TR1$ is twice as much as on the boundary tour with distance r to the boundary.)

The two "boundary" tours allow us to cover a corridor of width $2r$ with a bounded number of scans, while (1) enables us to bound the tour length in terms of the optimal length.

(2) A "strip" part: For the interior we use strips again: $P_{int} := P \backslash P_{\delta B}$—if nonempty—can be covered by a set of k_1 horizontal strips Σ_i^1. The y-coordinates of two strips differ by multiples of $2r$. We can consider another set of strips, Σ_i^2, shifted by r. Then, let $L_{str}^j = \sum_{i=1}^{k_j} L_{\Sigma_i^j}$. Similar to the argument for L_∞, we have $L_{str}^1 + L_{str}^2 \leq 2 \cdot L(T^*)$.

(3) A "matching" part: In order to combine the two "boundary parts" and the two sets of strips for a tour we add two more set of sections.
- The center lines of the strips have a distance of r to the boundary, thus they do not yet touch $TR1$. Consequently, we add $1/2r$ to each center line (on each end). For that purpose, we consider the matchings as defined above. (Consider the endpoints of strips on δB_i: every δB_i contains an even number of such endpoints. Hence, every δB_i is partitioned into two disjoint portions, $M_1(\delta B_i)$ and $M_2(\delta B_i)$. Using the shorter of these two $(M_*(\delta B_i))$ for every δB_i we obtain for the combined length, L_M: $L_M \leq L_{str}/2 \leq L(T^*)/2$.) Because two strips are at least a distance of r apart, the connection to $TR1$ costs less than $1/2 \cdot L_M \leq 1/2 \cdot L_{str}/2 \leq L(T^*)/4$.
- Moreover, we consider the above matchings defined on $TR1$ and insert the shorter sections of the disjoint parts, $(M_*^1(\delta B_i))$, for every δB_i. The Intercept Theorem in combination with the analogously defined sections on $TR2$ enables us to give an upper bound of $L_{M^1} \leq L_{str} \leq L(T^*)$.

Starting on some point on $TR1$, tracing the strips, and the inner "boundary" $TR2$ at once when passing it yields a tour; the above inequalities show that $L(T) \leq 21/4 \cdot L(T^*)$.

Now we only have to take care of (I), i.e., construct an appropriate set of scan points. For the "boundary" part we place scans with the center points located on $TR1$ and $TR2$ in distance $\sqrt{3} \cdot r$ if possible, but at corners we need to place scans, so the minimum width we are able to cover with the two scans (on both tours) is a. For the "strip" part the distance of scans is also $\sqrt{3} \cdot r$ on both strip sets, exactly the distance enabling us to cover a width of r.

It remains to consider the costs for the scans. We start with the inner part. Taking scans within a distance of $\sqrt{3} \cdot r$, we may need the length divided by this value, plus one scan. We only charge the first part to the strips, the (possible) additional scans are charged to the "boundary" part, as we have no minimum length of the strips. The optimum cannot cover more than πr^2 with one scan. Let $L_{str} = max(L_{str}^1, L_{str}^2)$:

$$|\mathcal{S}(T^*)| \geq \frac{L_{str}}{\pi r/2}, \quad |\mathcal{S}(T)| \leq \frac{2L_{str}}{\sqrt{3} \cdot r} \;\Rightarrow\; \frac{|\mathcal{S}(T)|}{|\mathcal{S}(T^*)|} \leq \frac{2L_{str}}{\sqrt{3} \cdot r} \cdot \frac{\pi r/2}{L_{str}} = \frac{\pi}{\sqrt{3}} \qquad (2)$$

Finally, we consider the "boundary". We assume $L_{\delta B} \geq 1$. So $|\mathcal{S}(T^*)| \geq \frac{L_{\delta B}}{\pi r/2}$. We may need to scan within a distance of a—on two strips—, need additional scans and have to charge the scans from the "strip" part, hence, this yields: $|\mathcal{S}(T)| \leq \frac{L_{\delta B}}{a/2} + 1 + \frac{L_{\delta B}}{r}$. Consequently, for $r \geq a$: $\frac{|\mathcal{S}(T)|}{|\mathcal{S}(T^*)|} \leq \frac{\pi r}{a} + \frac{\pi r}{2} + \frac{\pi}{2}$.

Theorem 4. *A polygon P allows a MWPDV solution that contains at most a cost of $max(\frac{21}{4}, \frac{\pi r}{a} + \frac{\pi r}{2} + \frac{\pi}{2})$ times the cost of an optimum MWPDV solution (for $r \geq a$).*

Note that Theorem 4 covers the case from Section 5; however, instead of the custom-built factor of 4 it yields a factor of $2 \cdot \pi$.

7 A PTAS for MWPDV Lawn Mowing

We describe here the following special case, which we generalize in the full paper. Consider a polyomino P (the "grass") that is to be "mowed" by a $k \times k$ square, M. At certain discrete set $\mathcal{S}(T)$ of positions of M along a tour T, the mower is activated (a "scan" is taken), causing all of the grass of P that lies below M at such a position to be mowed. For complete coverage, we require that P be contained in the union of $k \times k$ squares centered at points $\mathcal{S}(T)$. Between scan positions, the mower moves along the tour T.

In this "lawn mower" variant of the problem, the mower is not required to be fully inside P; the mower may extend outside P and move through the exterior of P, e.g., in order to reach different connected components of P. Since P may consist of singleton pixels, substantially separated, the problem is NP-hard even for $k = 1$, from TSP.

Here we describe a PTAS for the problem. We apply the m-guillotine method, with special care to handle the fact that we must have full coverage of P. Since the problem is closely related to the TSPN [6,13], we must address some of the similar difficulties in applying PTAS methods for the TSP: in particular, a mower centered on one side of a cut may be responsible to cover portions of P on the opposite side of the cut.

At the core of the method is a structure theorem, which shows that we can transform an arbitrary tour T, together with a set $\mathcal{S}(T)$ of scan points, into a tour and scan-point set, $(T_G, \mathcal{S}(T_G))$, that are m-guillotine in the following sense: the bounding box of the set of $k \times k$ squares centered at $\mathcal{S}(T)$ can be recursively partitioned into a rectangular subdivision by "m-perfect cuts". An axis-parallel cut line ℓ is m-perfect if its intersection with the tour has $O(m)$ connected components and its intersection with the union of $k \times k$ disks centered at scan points consists of $O(m)$ disks or "chains of disks" (meaning a set of disks whose centers lie equally spaced, at distance k, along a vertical/horizontal line).

The structure theorem is proved by showing the following lemma; the proof is omitted due to lack of space and can be found in the full version of the paper.

Lemma 1. *For any fixed $m = \lceil 1/\epsilon \rceil$ and any choice of $(T, \mathcal{S}(T))$, one can add a set of* doubled bridge segments*, of total length $O(|T|/m)$, to T and a set of $O(|\mathcal{S}(T)|/m)$* bridging scans *to $\mathcal{S}(T)$ such that the resulting set, $(T_G, \mathcal{S}(T_G))$, is m-guillotine, with points $\mathcal{S}(T_G)$ on tour T_G and with T_G containing an Eulerian tour of $\mathcal{S}(T_G)$.*

The algorithm is based on dynamic programming to compute an optimal m-guillotine network. A subproblem is specified by a rectangle, R, with integer coordinates. The subproblem includes specification of *boundary information* for each of the four sides of R. The boundary information includes: (i) $O(m)$ integral points ("portals") where the tour is to cross the boundary, (ii) at most one (doubled) bridge and one disk-bridge (chain) per side of R, with each bridge having a parity (even or odd) specifying the parity of the number of connections to the bridge from within R, (iii) $O(m)$ scan positions (from $\mathcal{S}(T)$) such that a $k \times k$ square centered at each position intersects the corresponding side of R, (iv) a connection pattern, specifying which subsets of the portals/bridges are required to be connected within R. We summarize:

Theorem 5. *There is a PTAS for MWPDV lawn mowing of a (not necessarily connected) set of pixels by a $k \times k$ square.*

The Milling Variant. Our method does apply also to the "milling" variant of the MWPDV, in which the scans all must stay within the region P, provided that P is a simple rectilinear polygon. The details are rather involved and not included here. The main idea is this: Subproblems are defined, as before, by axis-aligned rectangles R. The difficulty now is that the restriction of R to P means that there may be many ($\Omega(n)$) vertical/horizontal chords of P along one side of R. We can ignore the boundary of P and construct an m-bridge (which we can "afford" to construct and charge off, by the same arguments as above) for T, but only the portions of such a bridge that lie inside P (and form chords of P) are traversable by our watchman. For each such chord, the subproblem must "know" if the chord is crossed by some edge of the tour, so that connections made inside R to a chord are not just made to a "dangling" component. We cannot afford to specify one bit per chord, as this would be $2^{\Omega(n)}$ information. However, in the case of a simple polygon P, no extra information must be specified to the subproblem – a chord is crossed by T if and only if the mower (scan) fits entirely inside the simple subpolygon on each side of the chord. Exploiting this fact, we are able to modify our PTAS to apply to MWPDV problem within a simple rectilinear polygon.

Theorem 6. *There is a PTAS for MWPDV milling of a simple rectilinear polygon by a $k \times k$ square.*

8 Conclusion

A number of open problems remain. Is it possible to remove the dependence on the ratio (r/a) of the approximation factor in our algorithm for general MWPDV milling? This would require a breakthrough for approximating minimum guard cover; a first step may be to achieve an approximation factor that depends on $\log(r/a)$ instead of (r/a).

For combined cost, we gave a PTAS for a lawn mowing variant, based on guillotine subdivisions. The PTAS extends to the milling case for simple rectilinear polygons. It is likely that the PTAS extends to other cases too (circular scan disks, Euclidean tour lengths), but the generalization to arbitrary domains with (many) holes seems particularly challenging. Our method makes use of a fixed ratio between scan cost and travel cost; as discussed in Figure 1, there is no PTAS for the bicriteria version.

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