Simple Rectilinear Polygons are Perfect under Rectangular Vision

Esther Arkin
Joseph S. B. Mitchell
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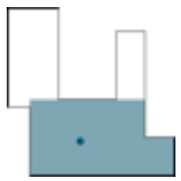
The Art Gallery Problem, Guards and Witnesses



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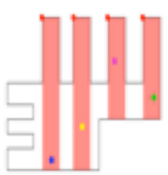
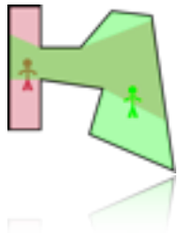


Perfectness

Outlook



The Art Gallery Problem, Guards and Witnesses

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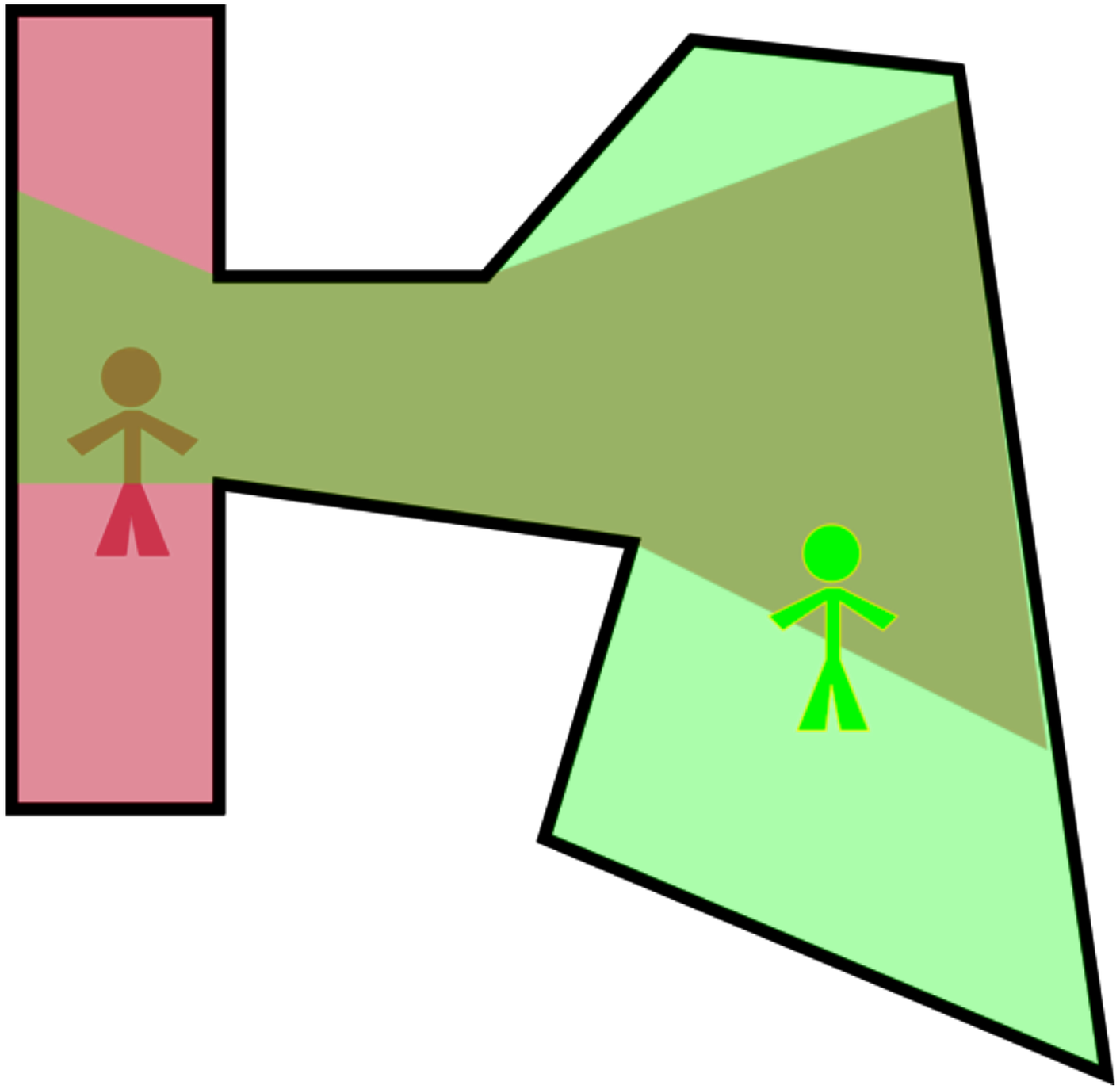

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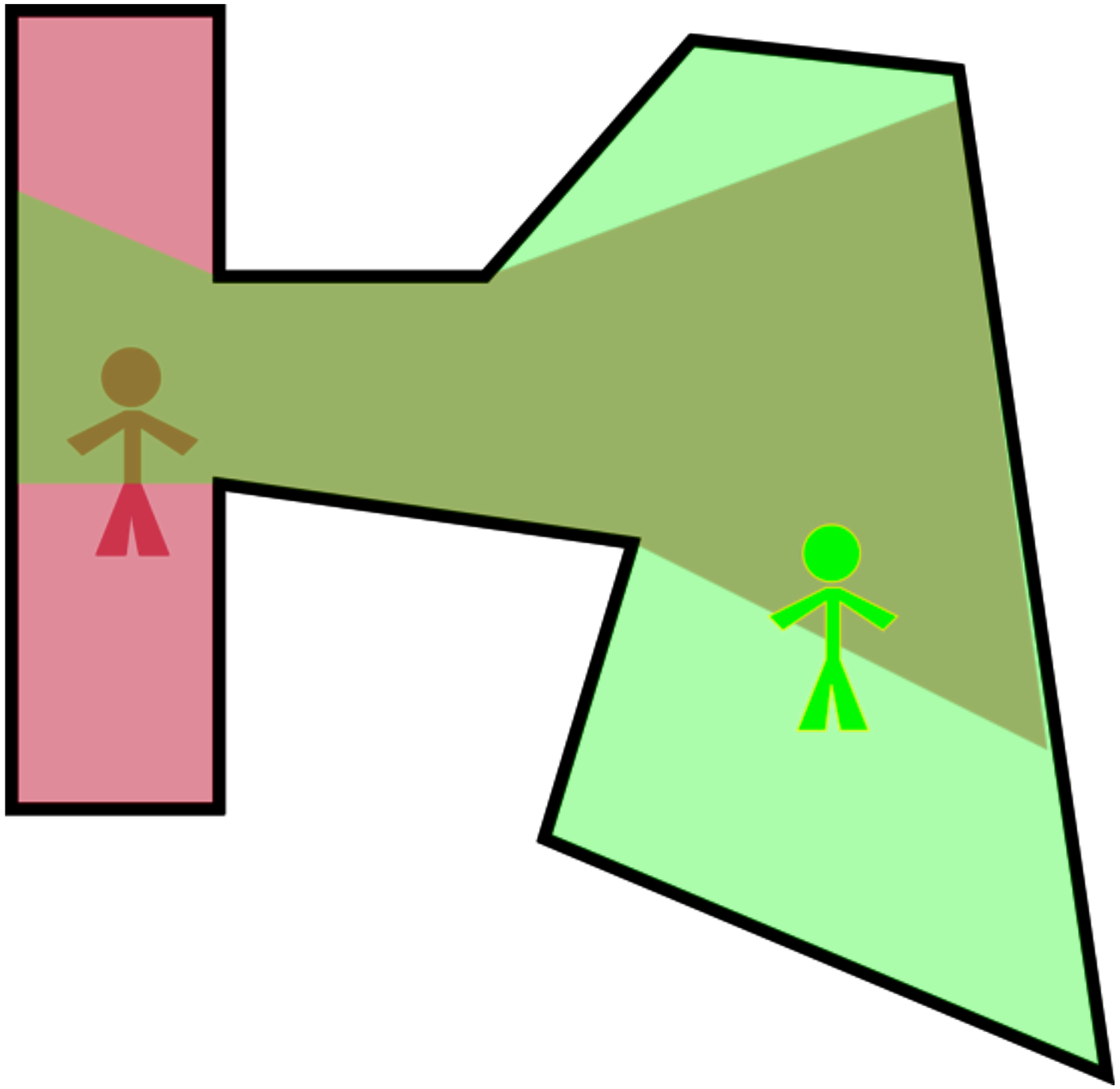
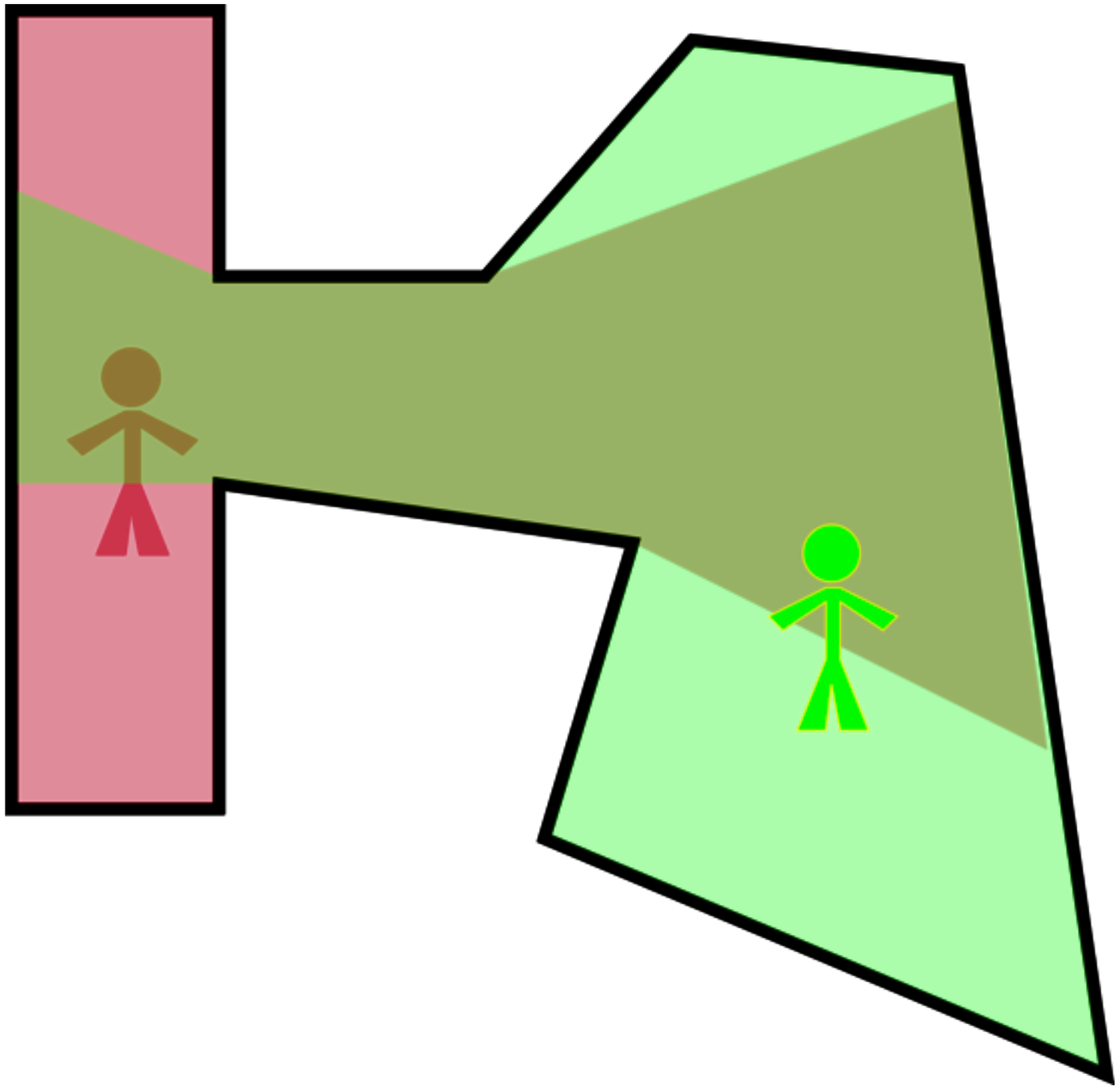


Given: Polygon P

How many guards do we need to monitor P?

The Art Gallery Problem, Guards and Witnesses



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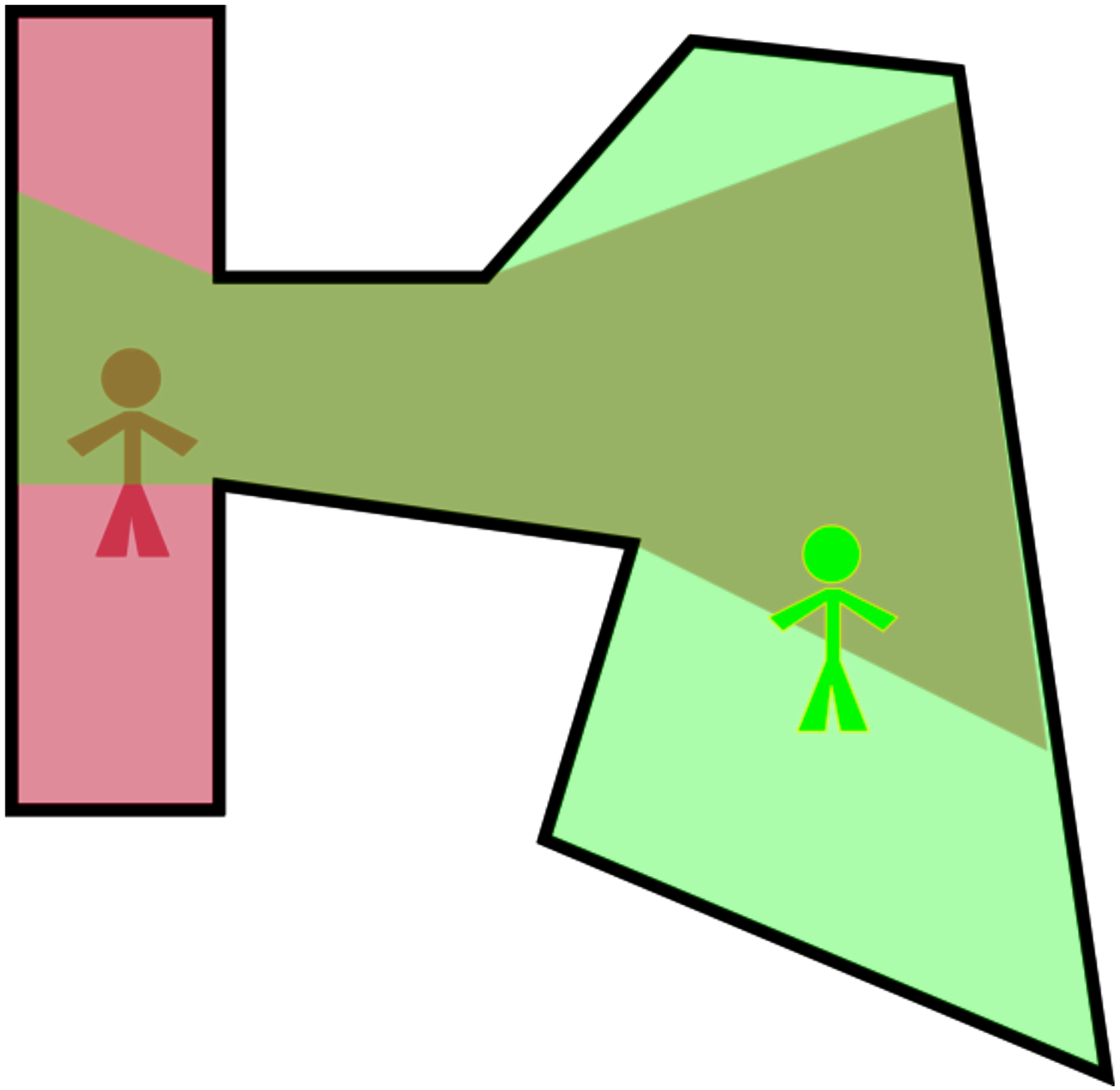

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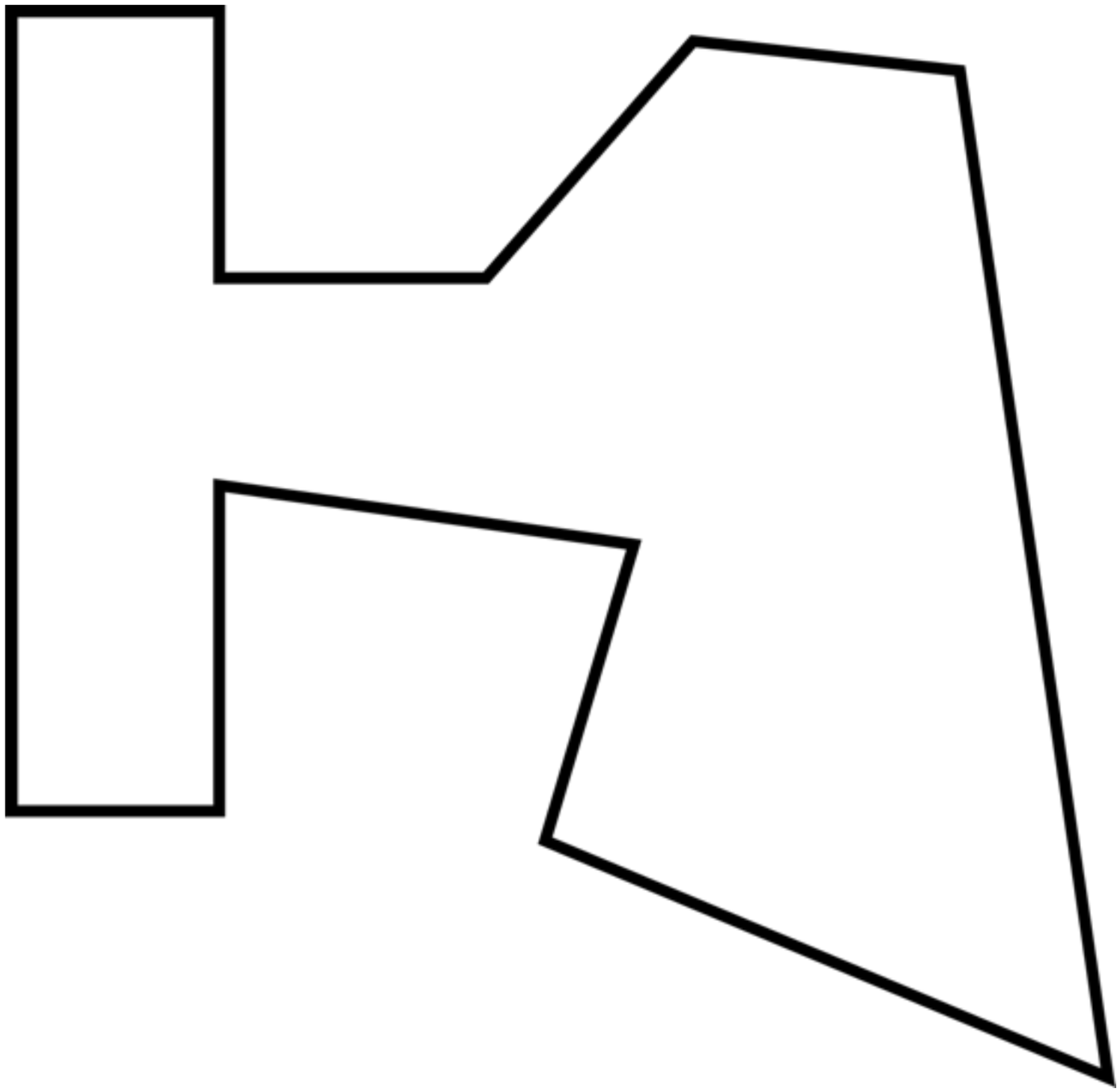
In general:

NP-hard

- point guards with holes [O'Rourke & Supowit 1983]
- vertex guards without holes [Lee & Lin 1986]
- point guards without holes [Aggarwal 1986]

APX-hard [Eidenbenz, Stamm & Widmayer 2001]

The Art Gallery Problem, Guards and Witnesses



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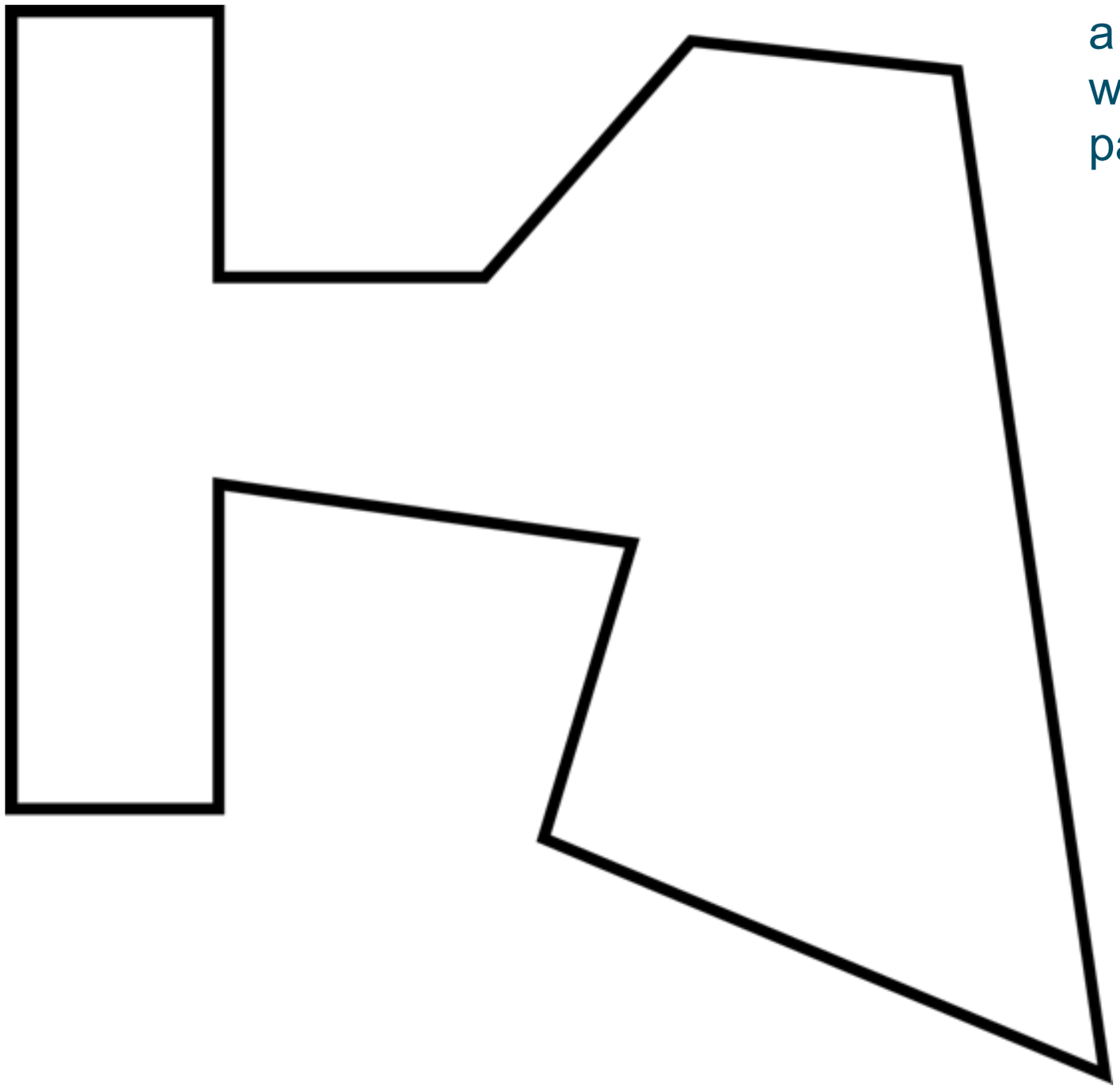
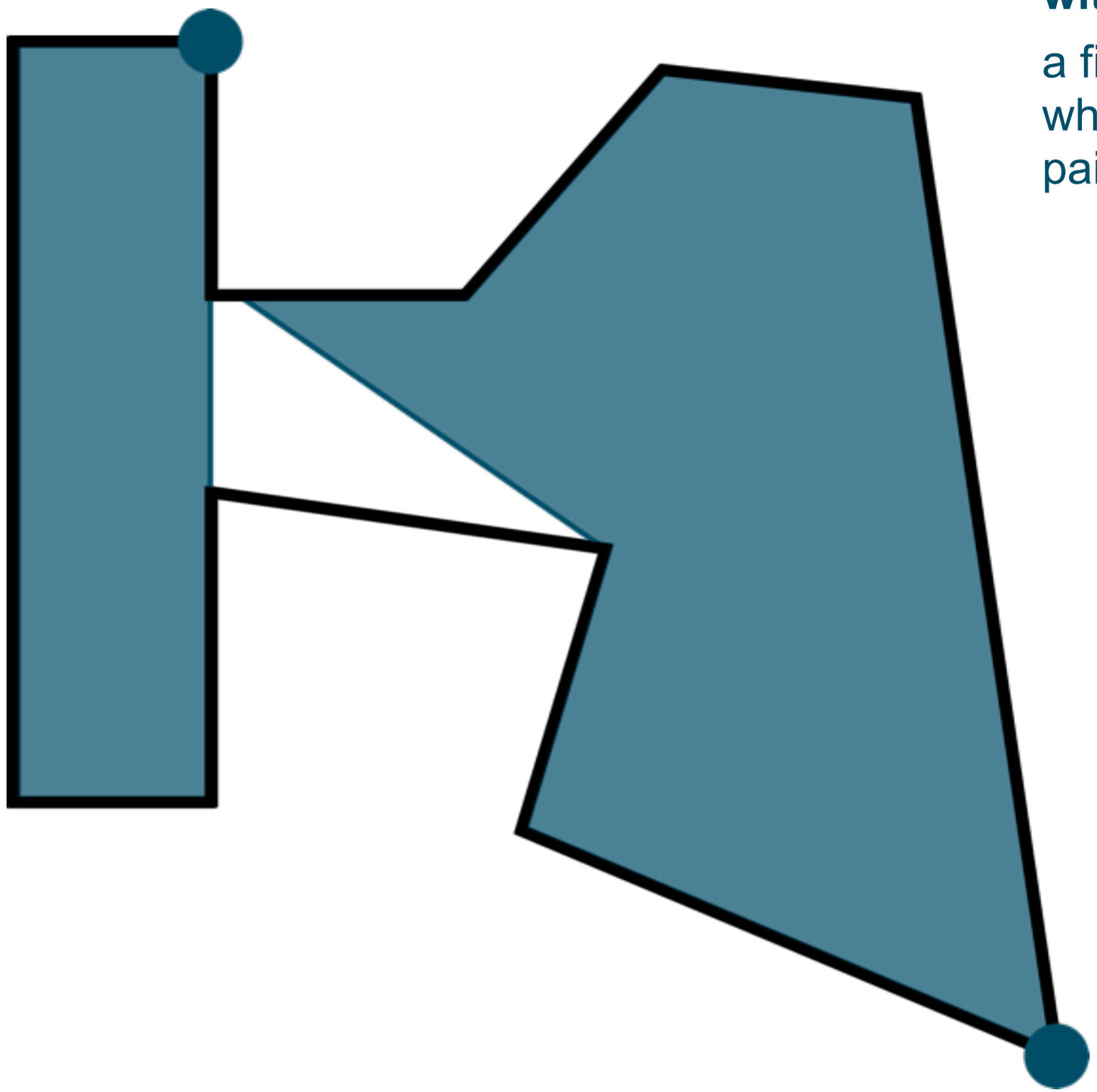


visibility-independent set of **witness points**:

a finite set of points $W \subset P$, whose visibility polygons are pairwise disjoint.

The Art Gallery Problem, Guards and Witnesses



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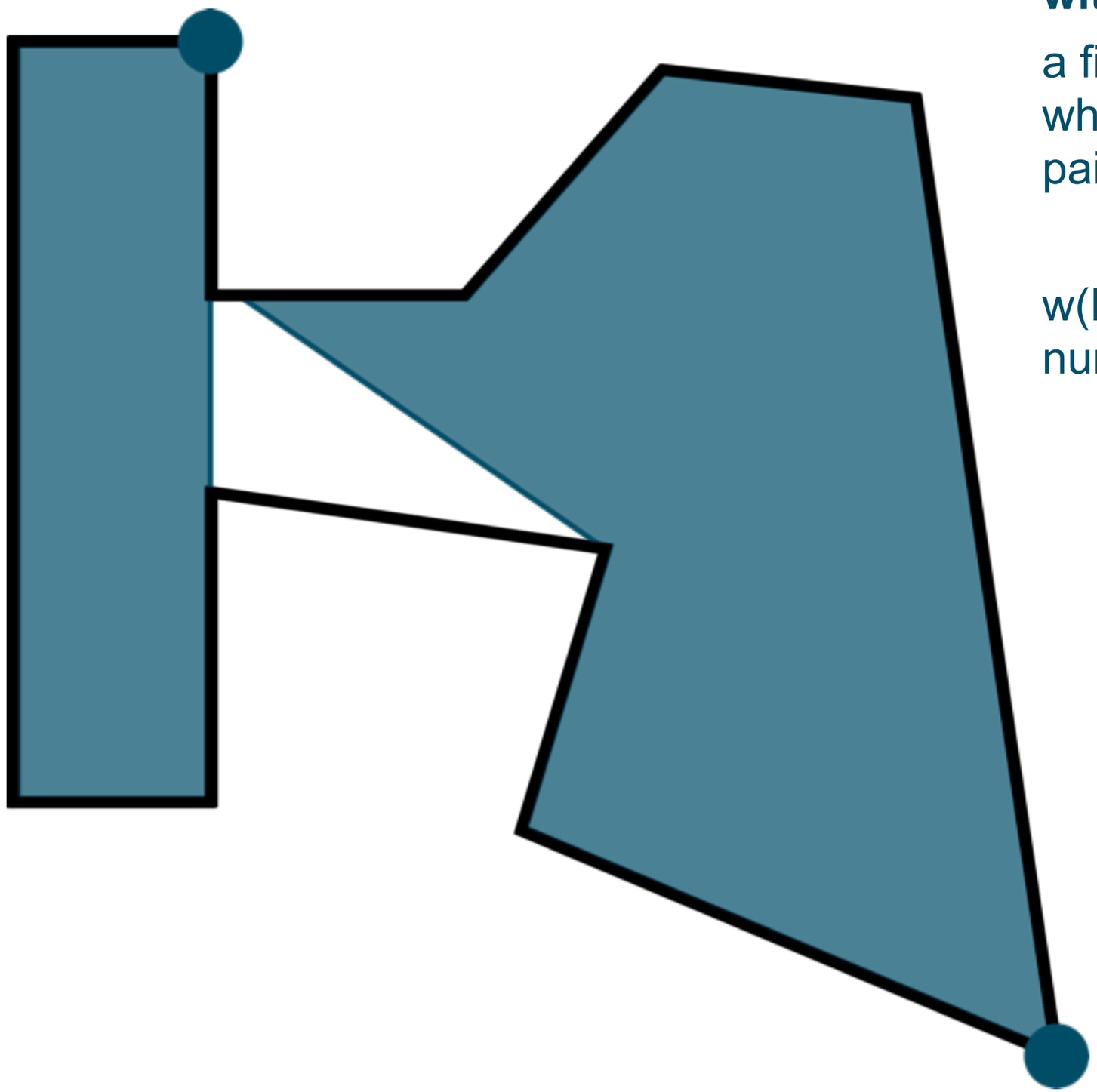

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$w(P)$: independence witness number of P

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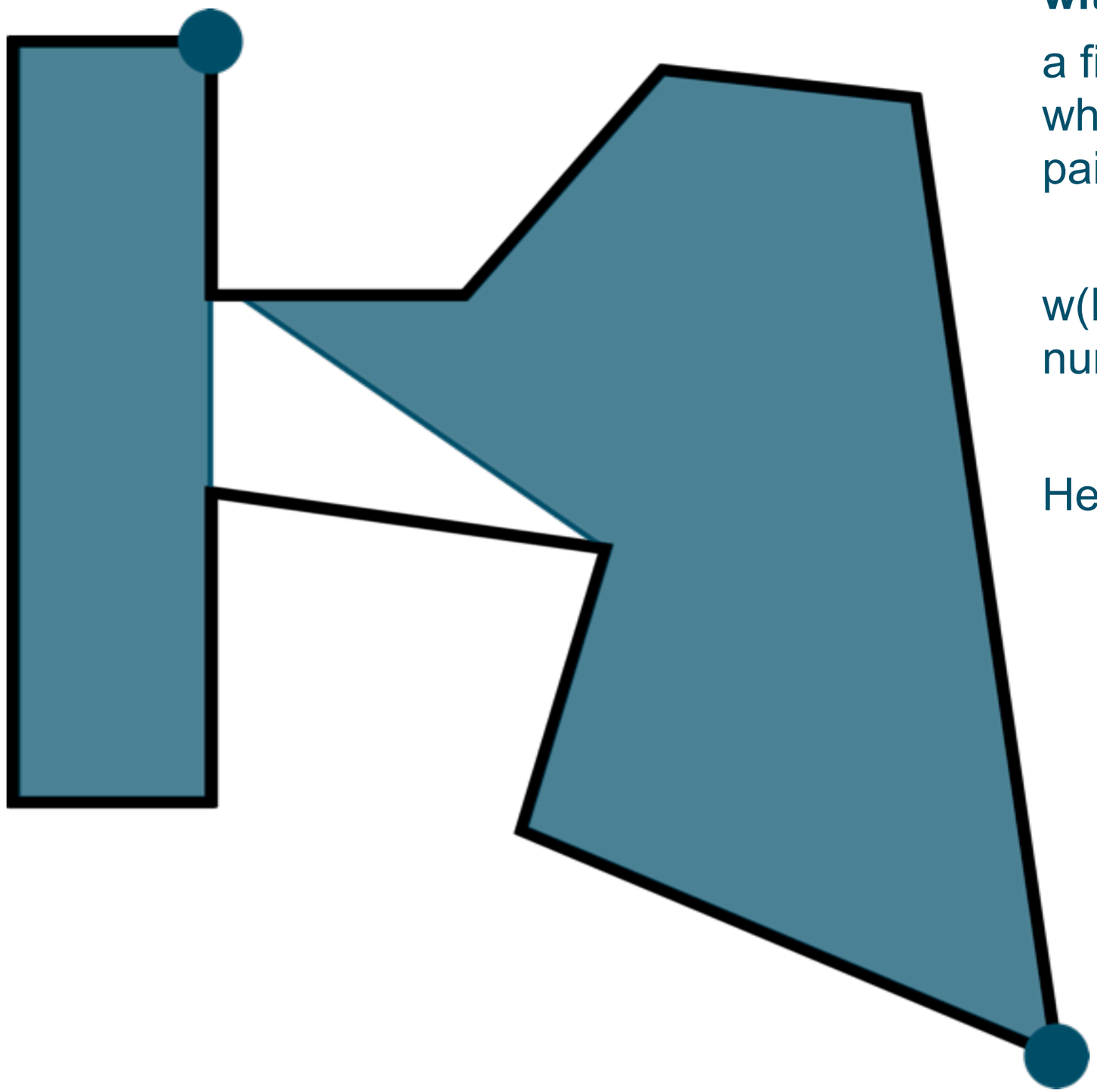

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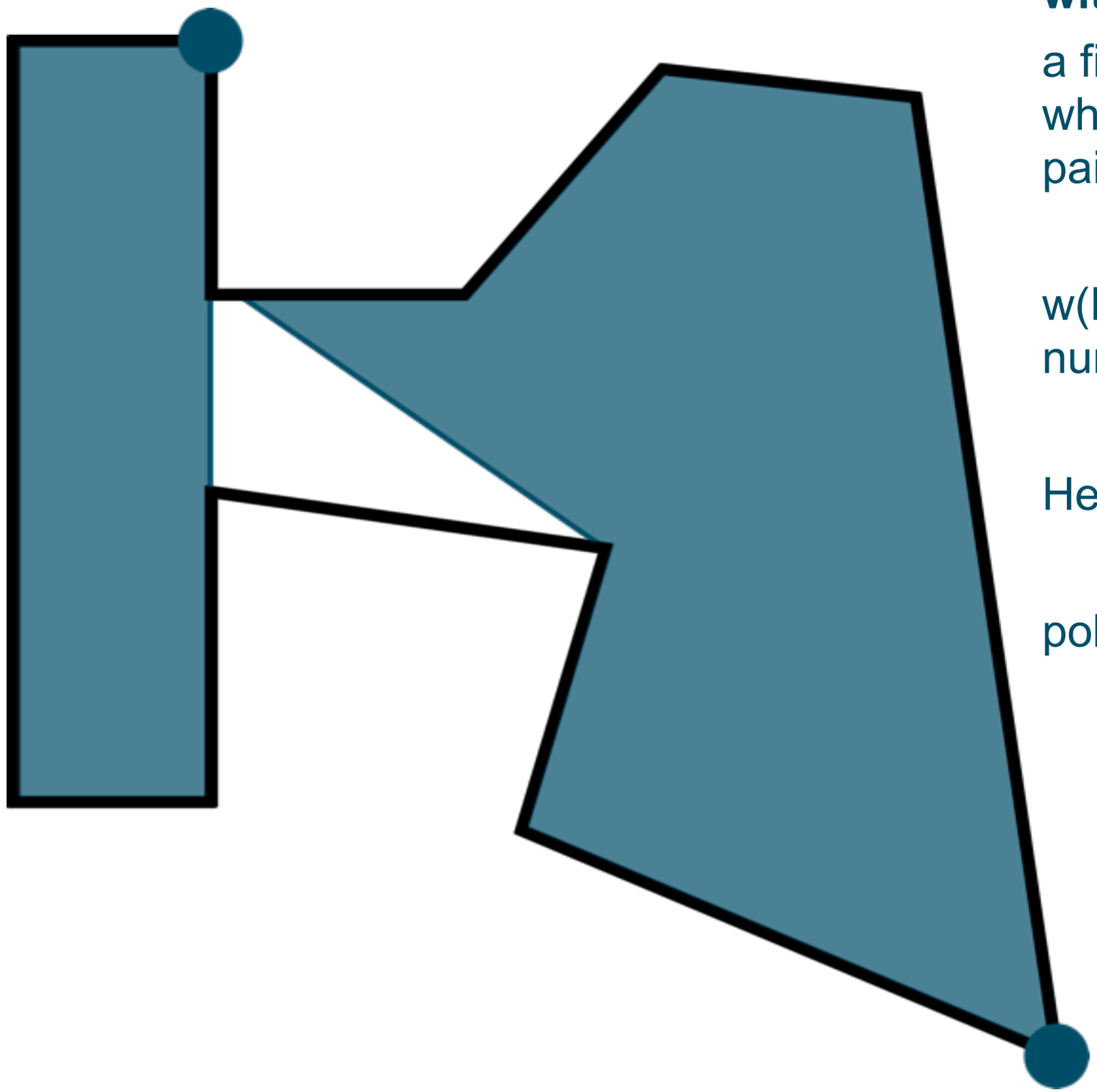
Here: w(P)=2

The Art Gallery Problem, Guards and Witnesses



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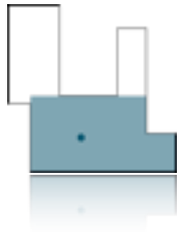
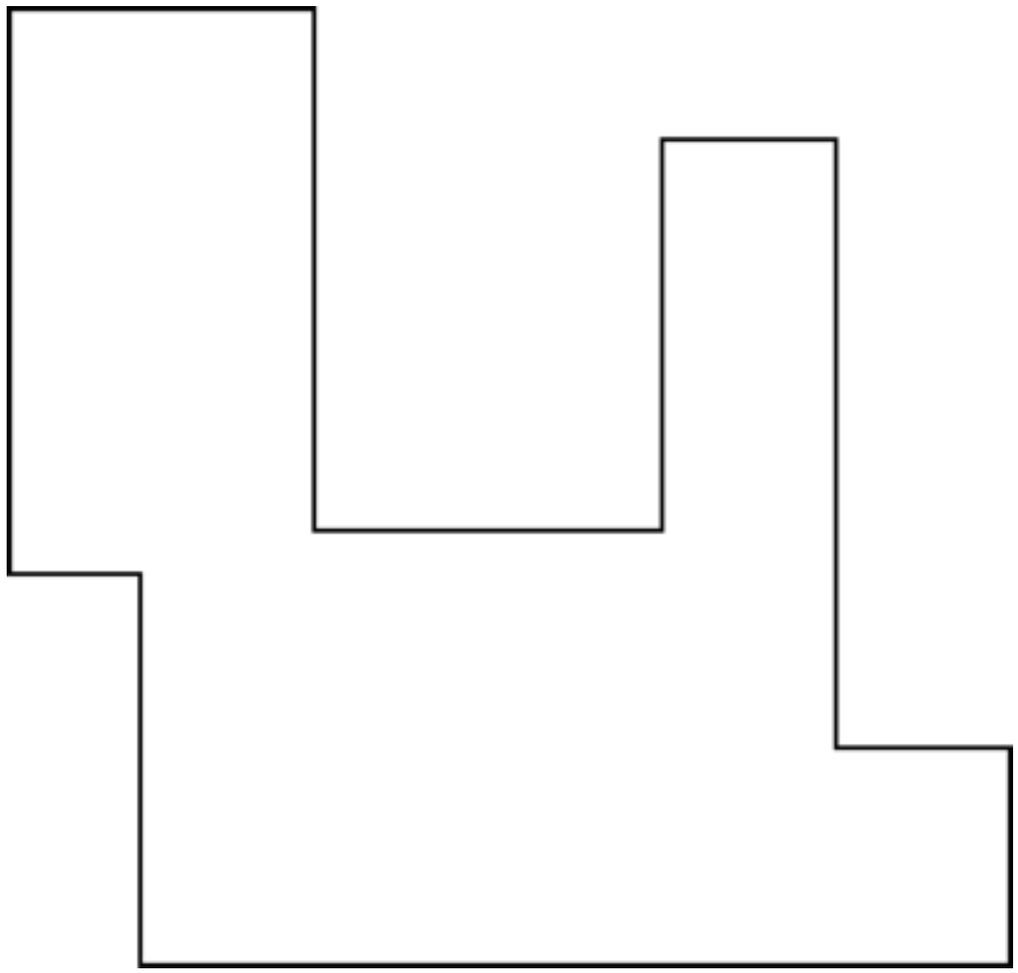
polygon is **perfect** if $w(P)=g(P)$!



Rectangular Vision

Rectangular vision



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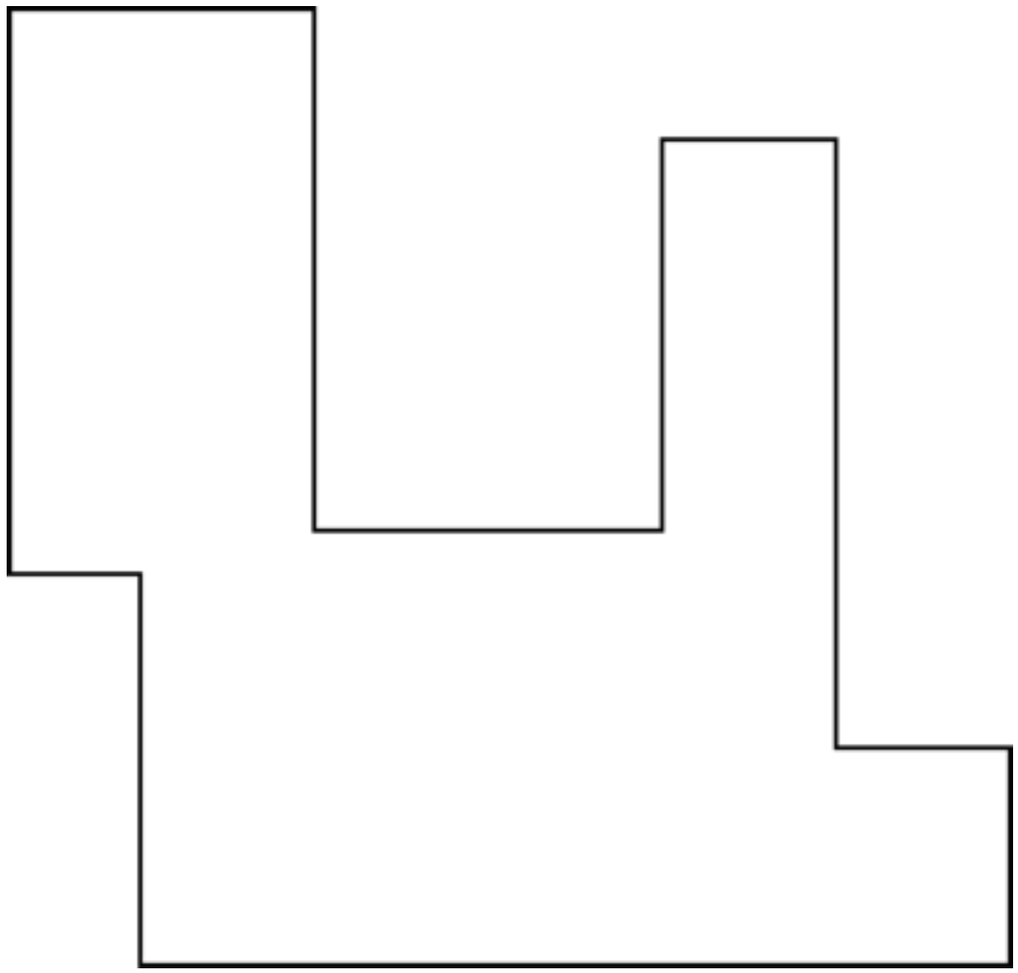

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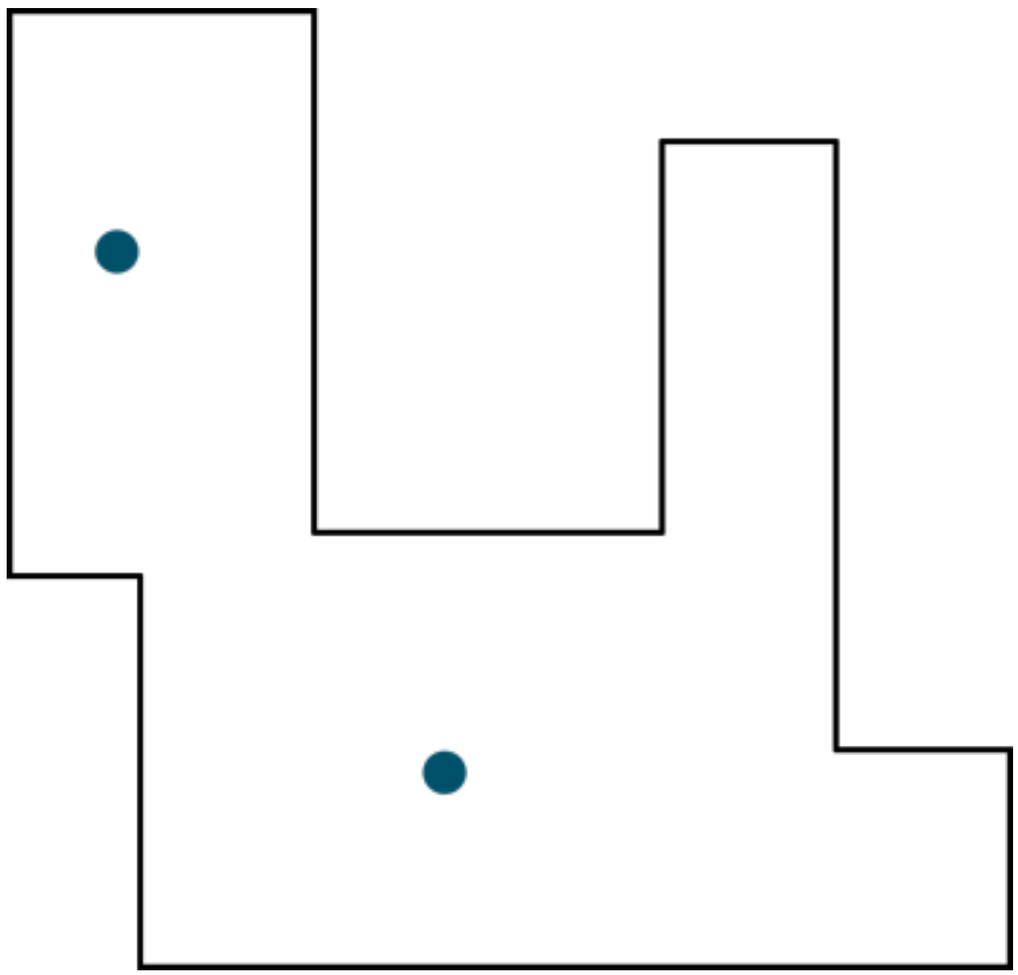

- ordinary visibility:
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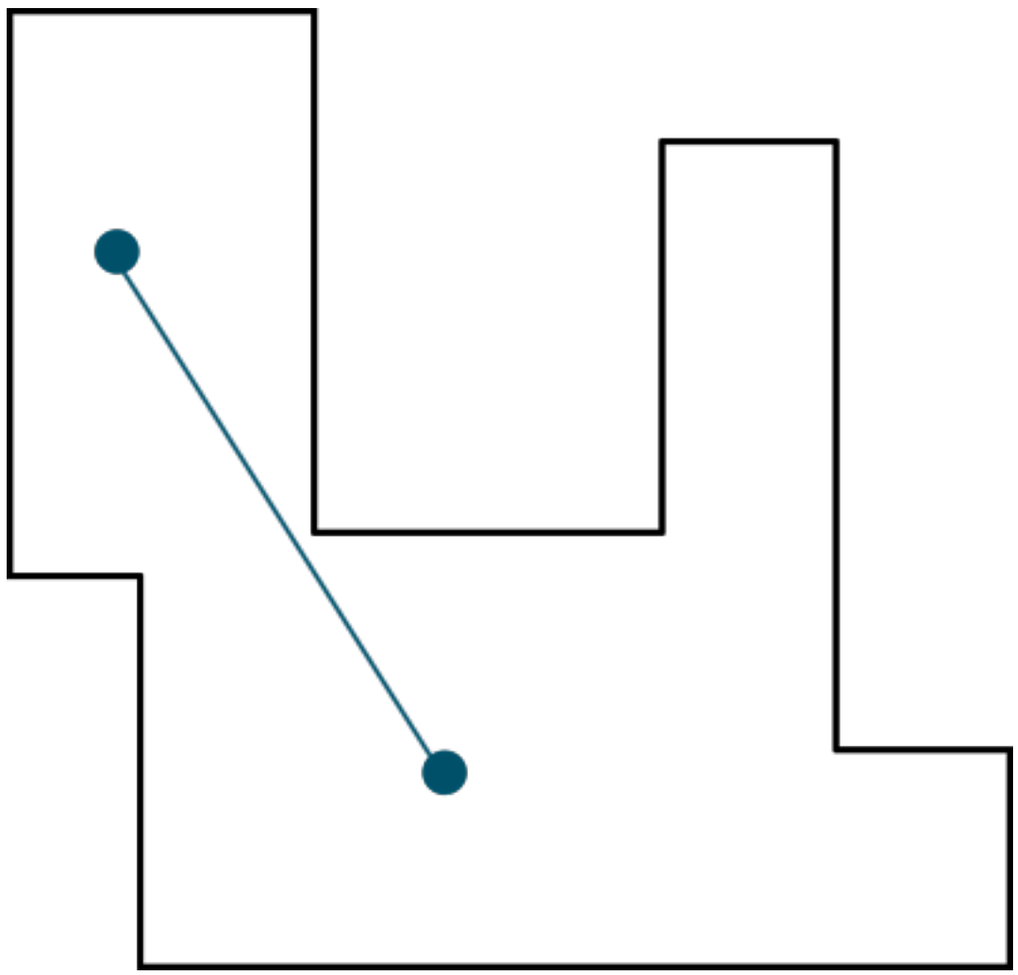

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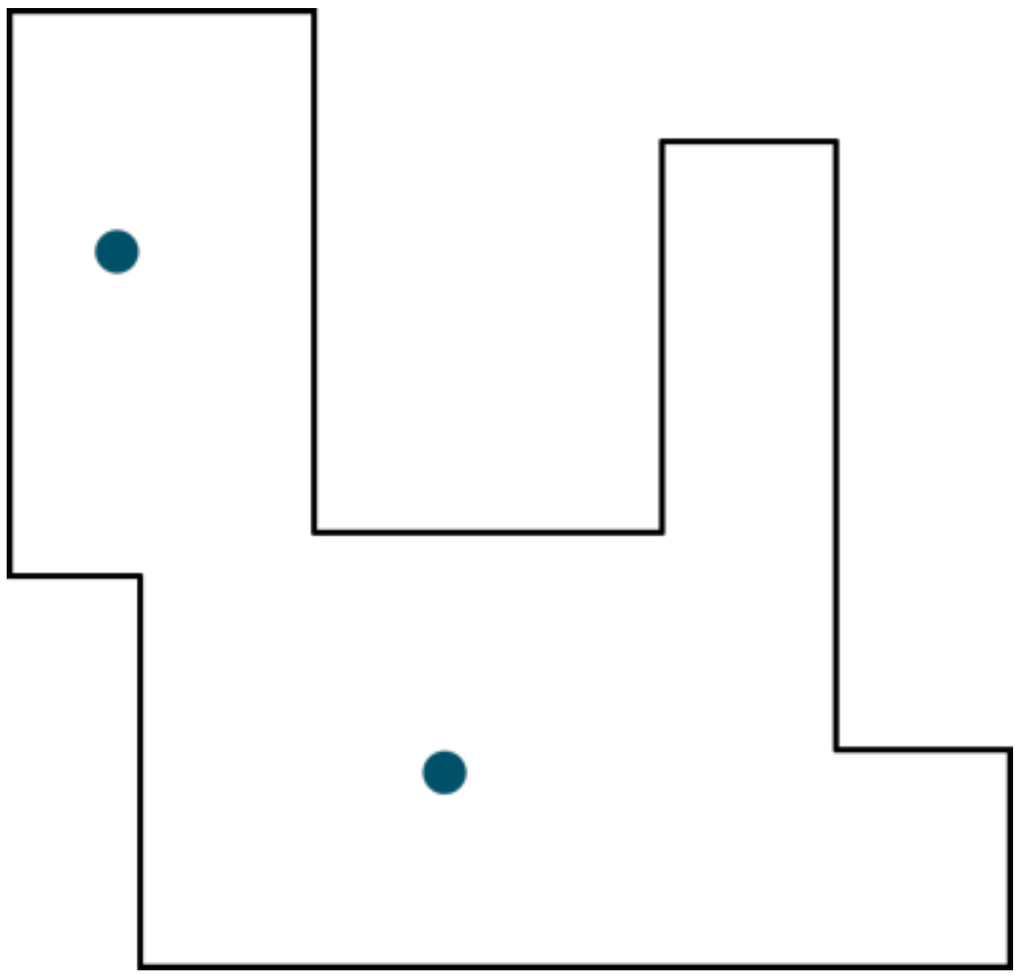

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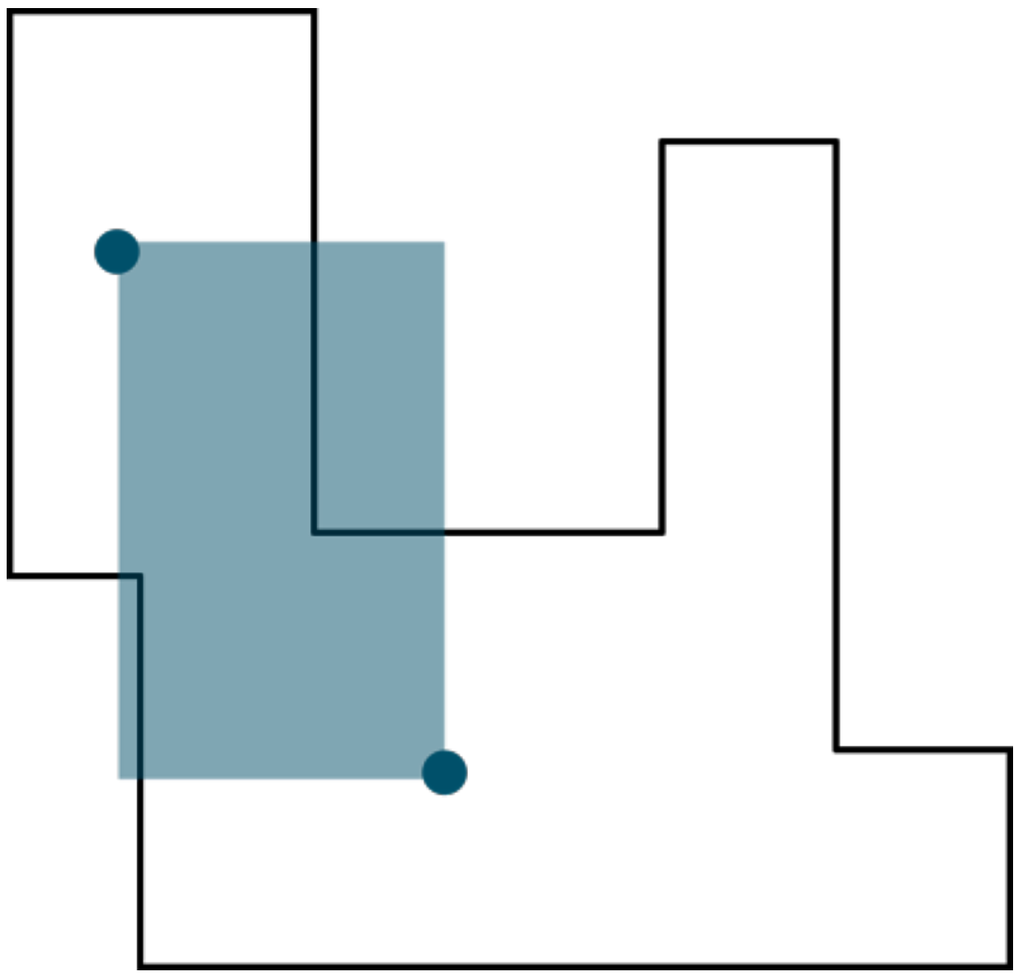

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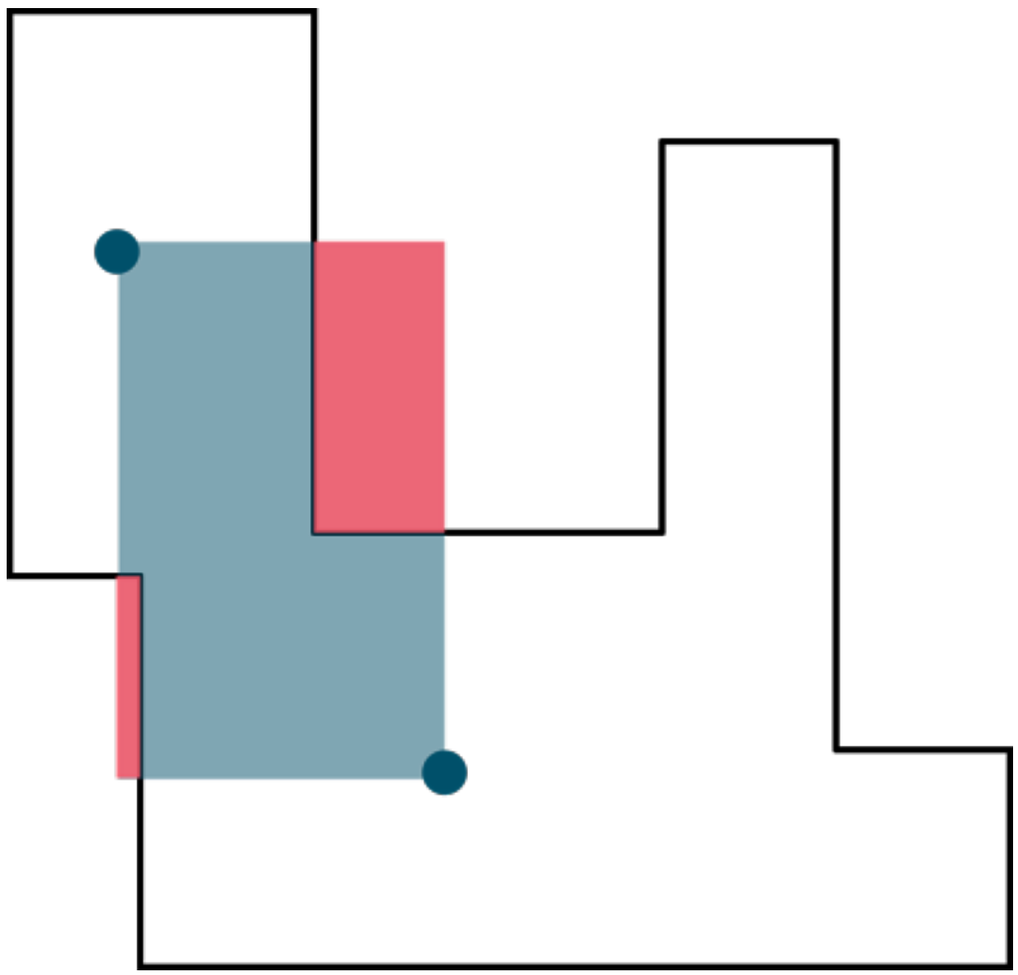

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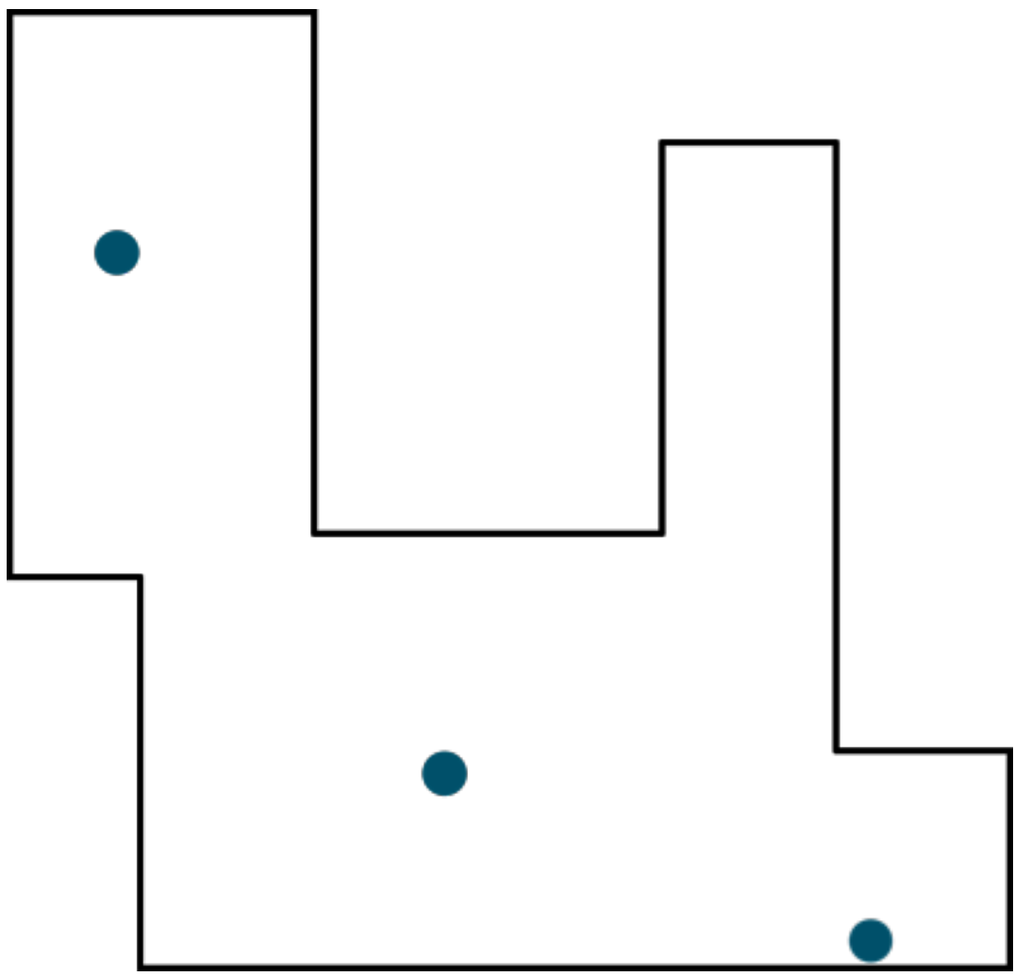

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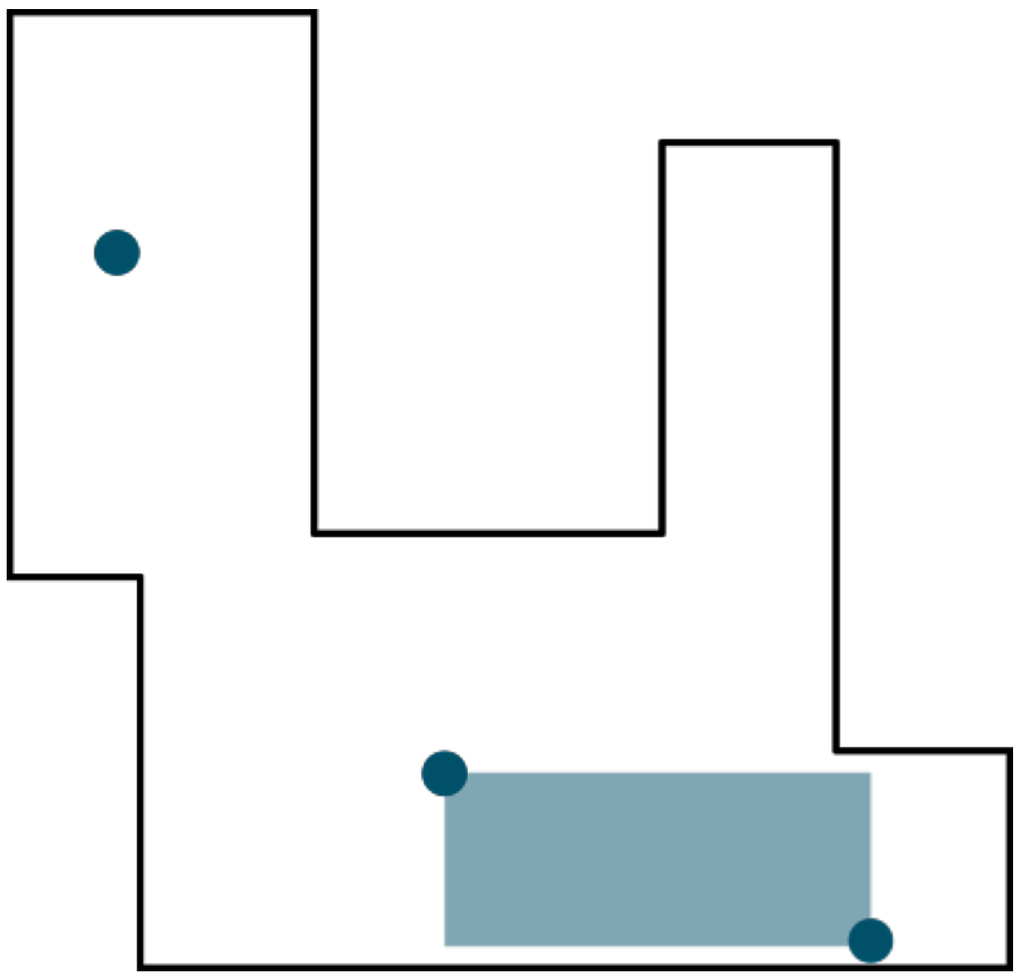

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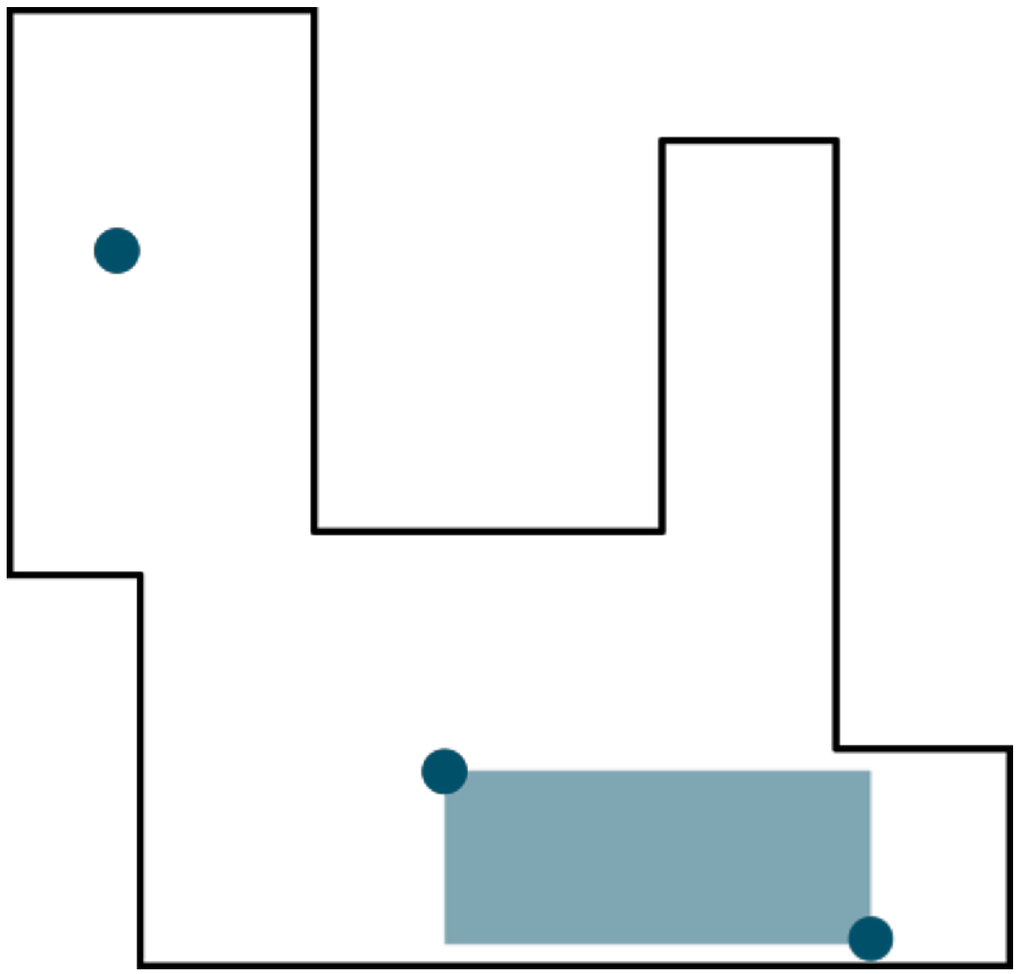

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Here: simple rectilinear polygons.

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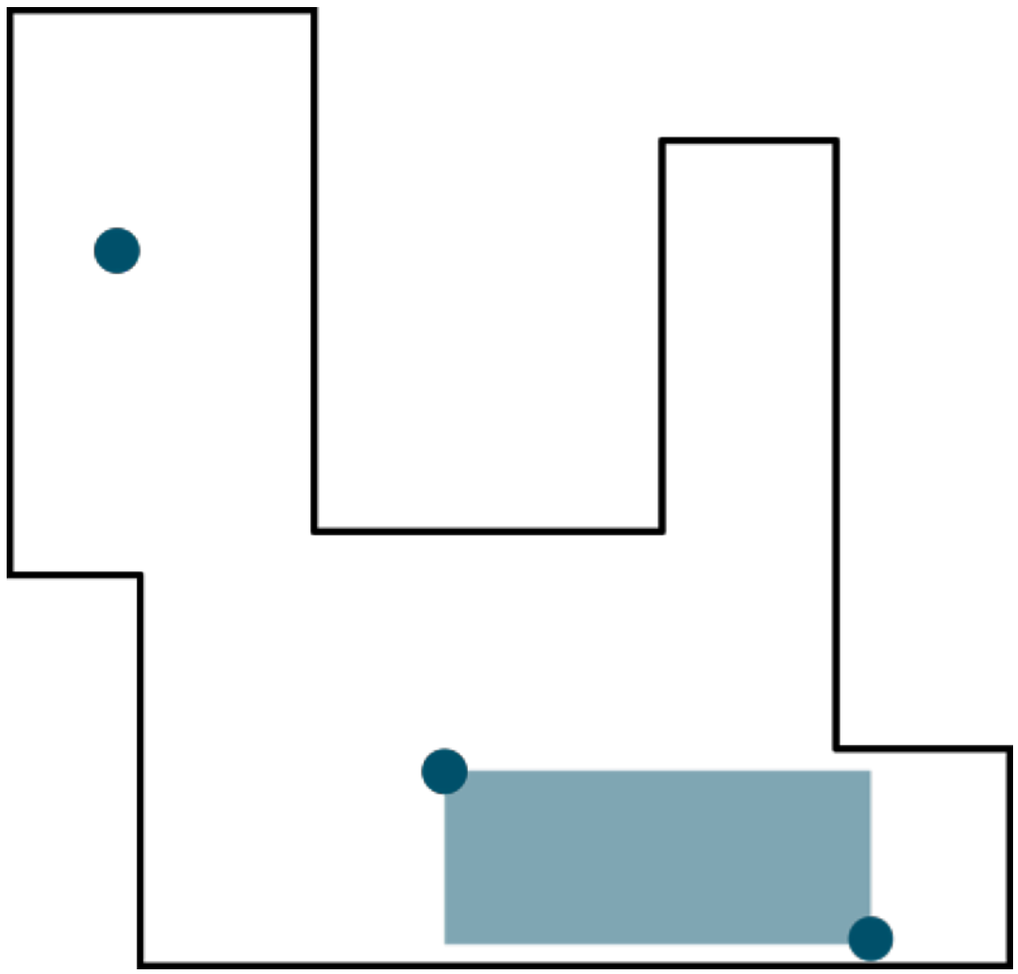

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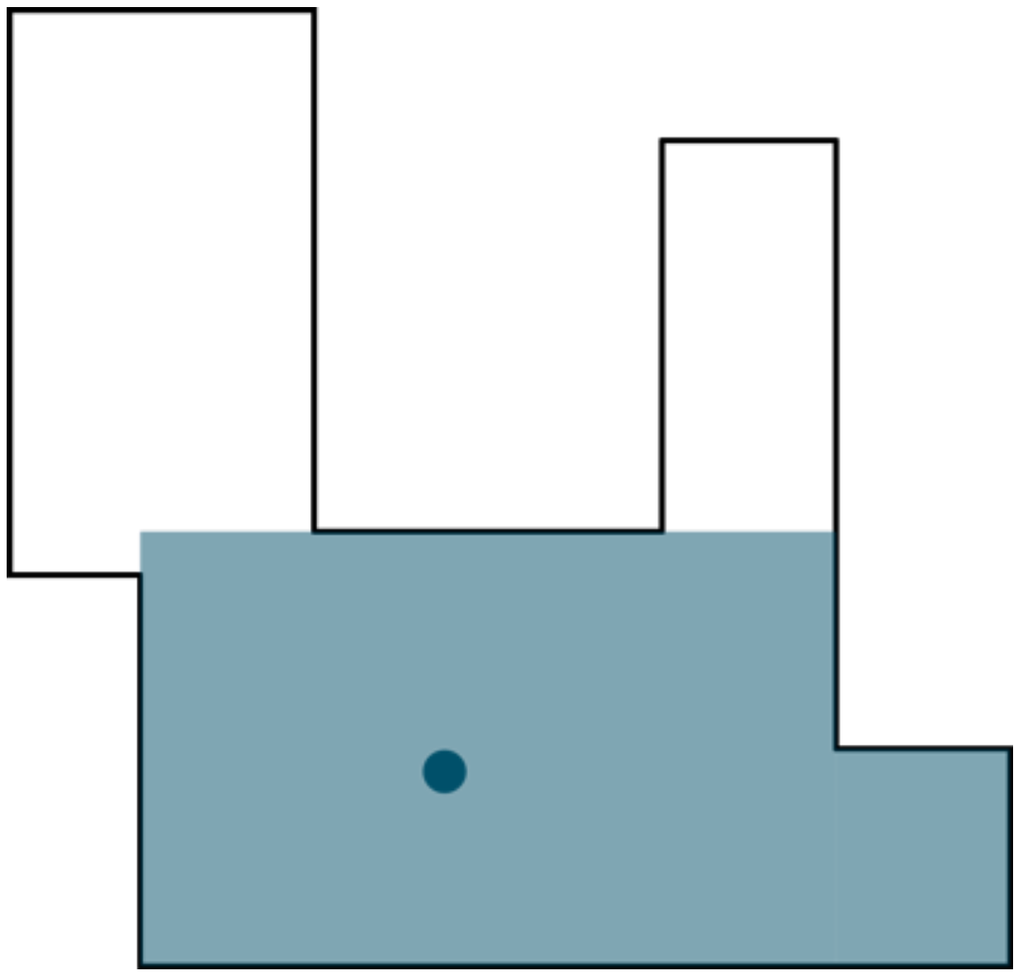
Rectangular vision



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- visibility polygon of a point p in P: VP(p)

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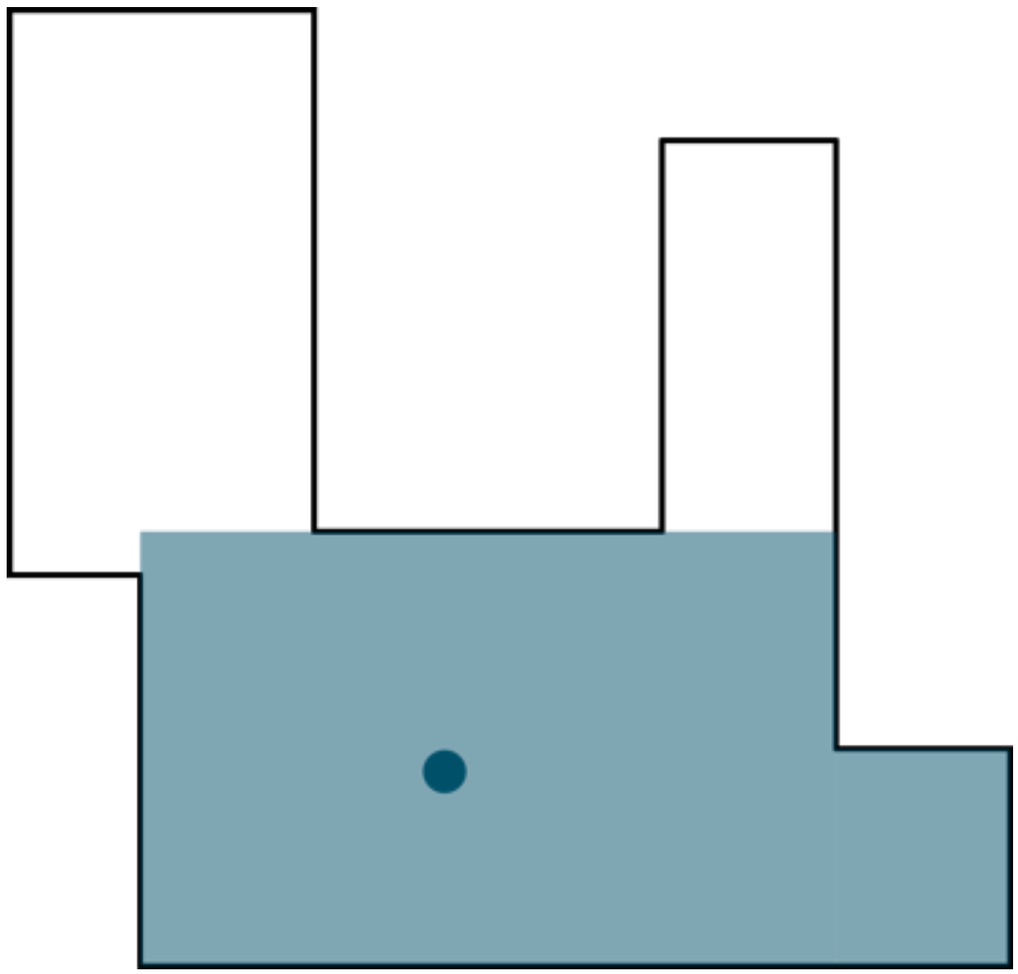

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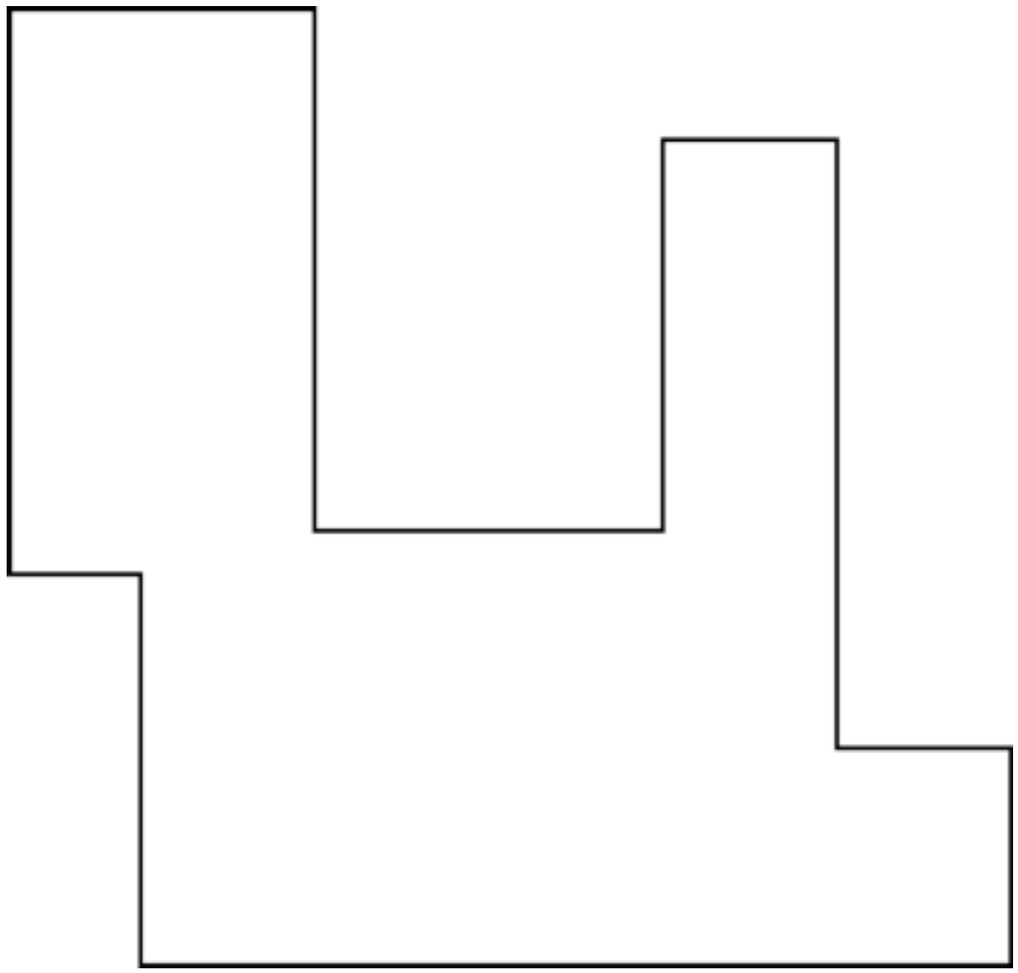
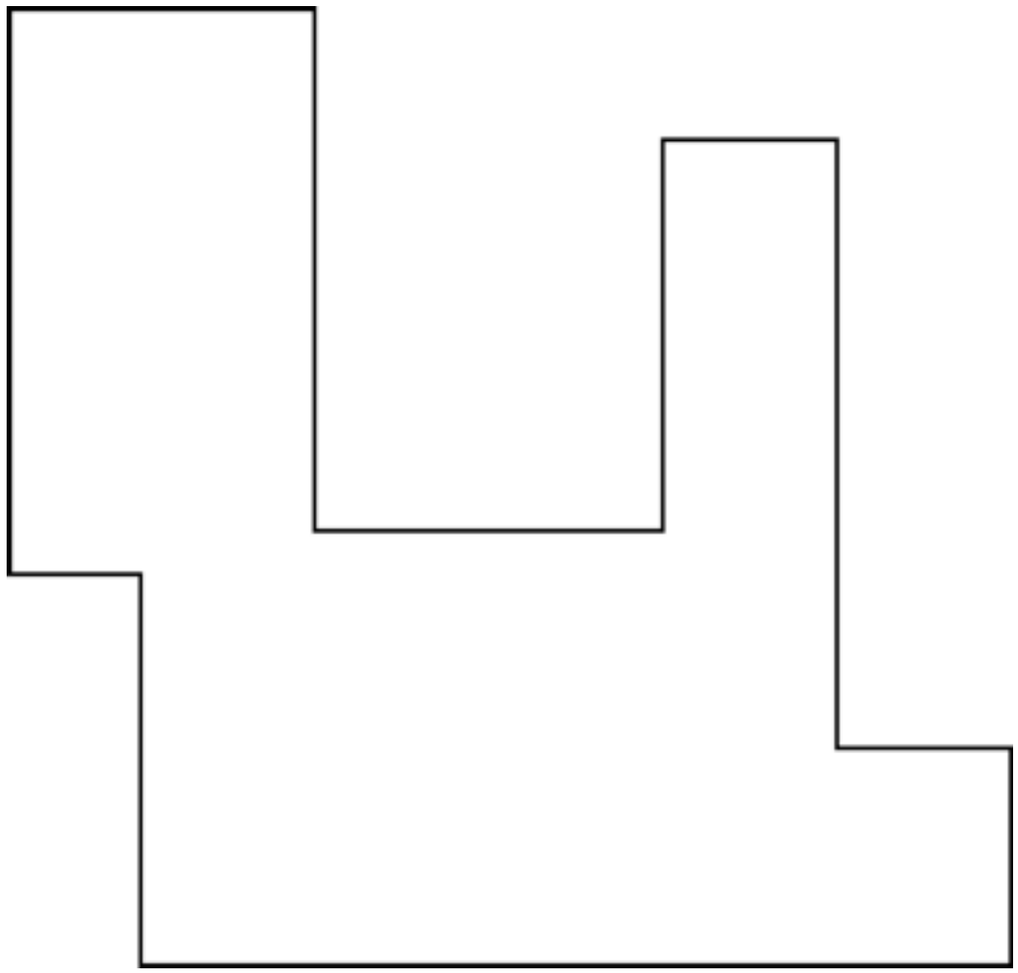


Rectangular vision

Known: for simple rectilinear polygon g(P) can be computed in polynomial time - $O(n^{17})$:

[Worman&Keil, 2007]



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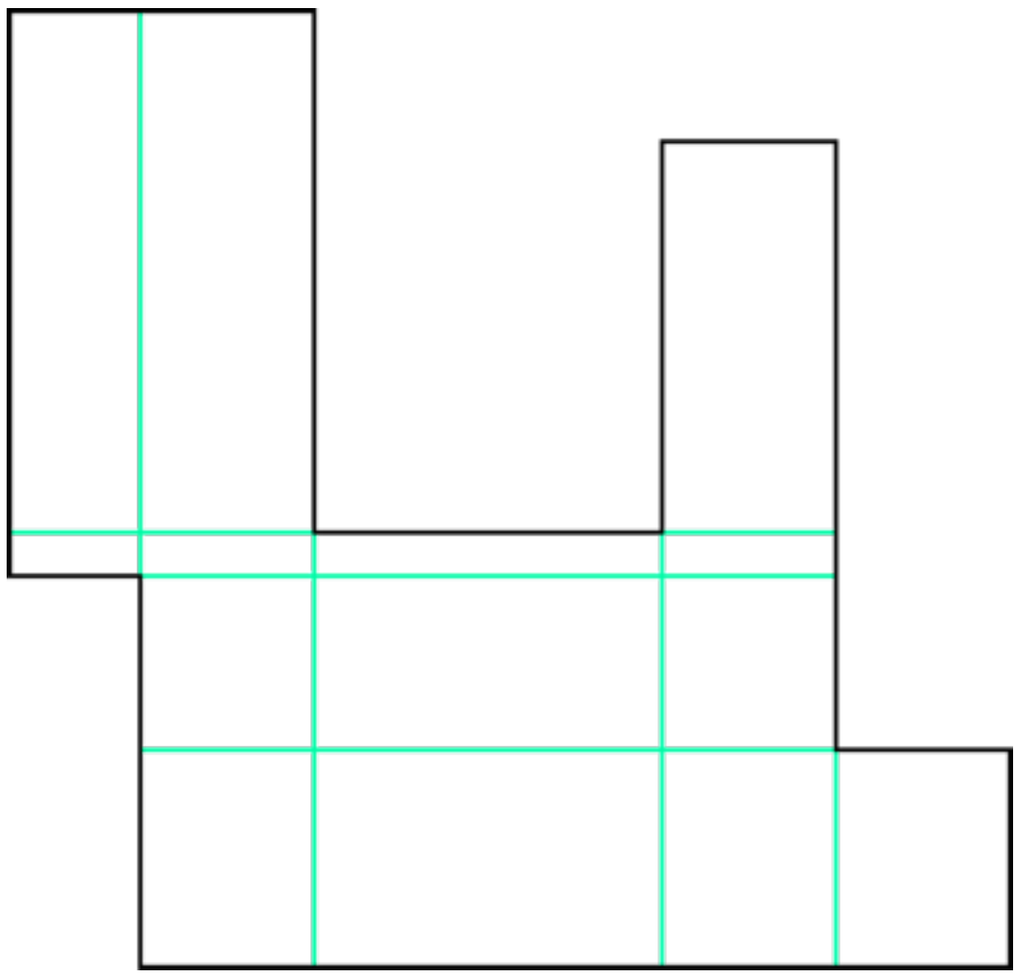

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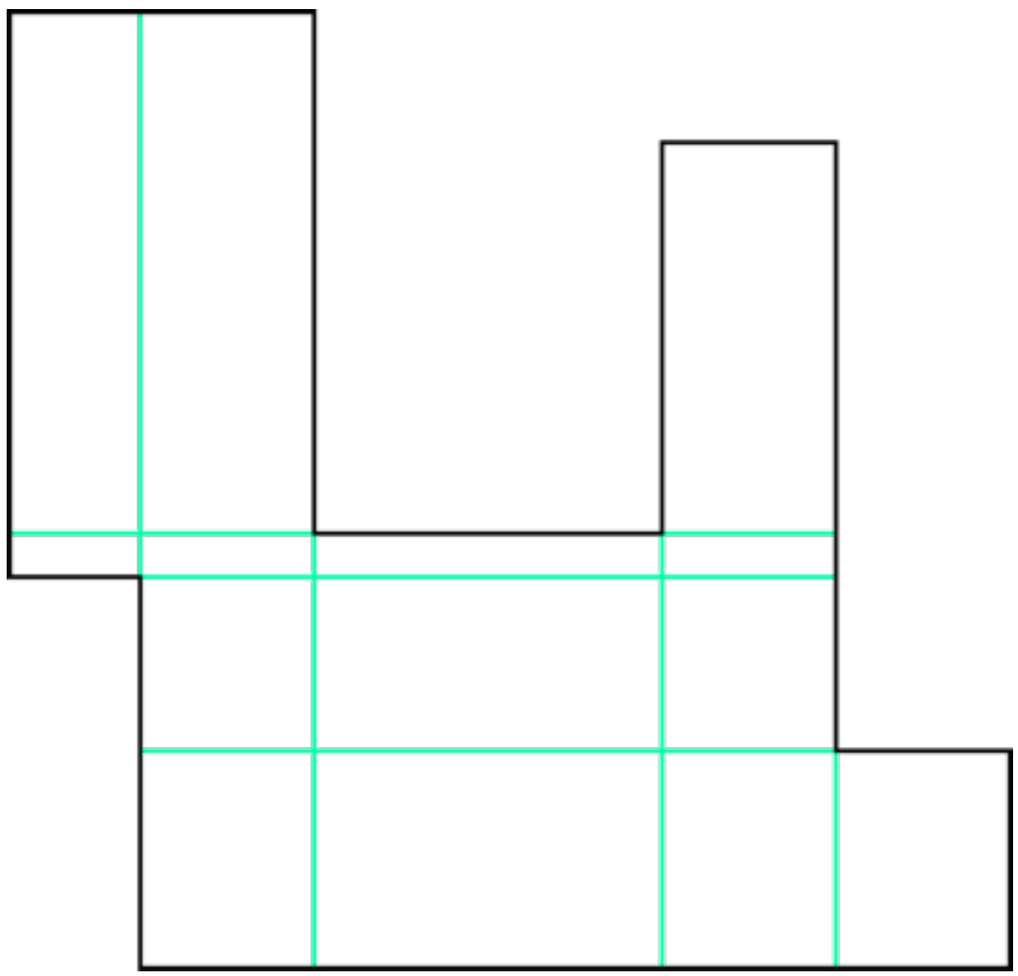

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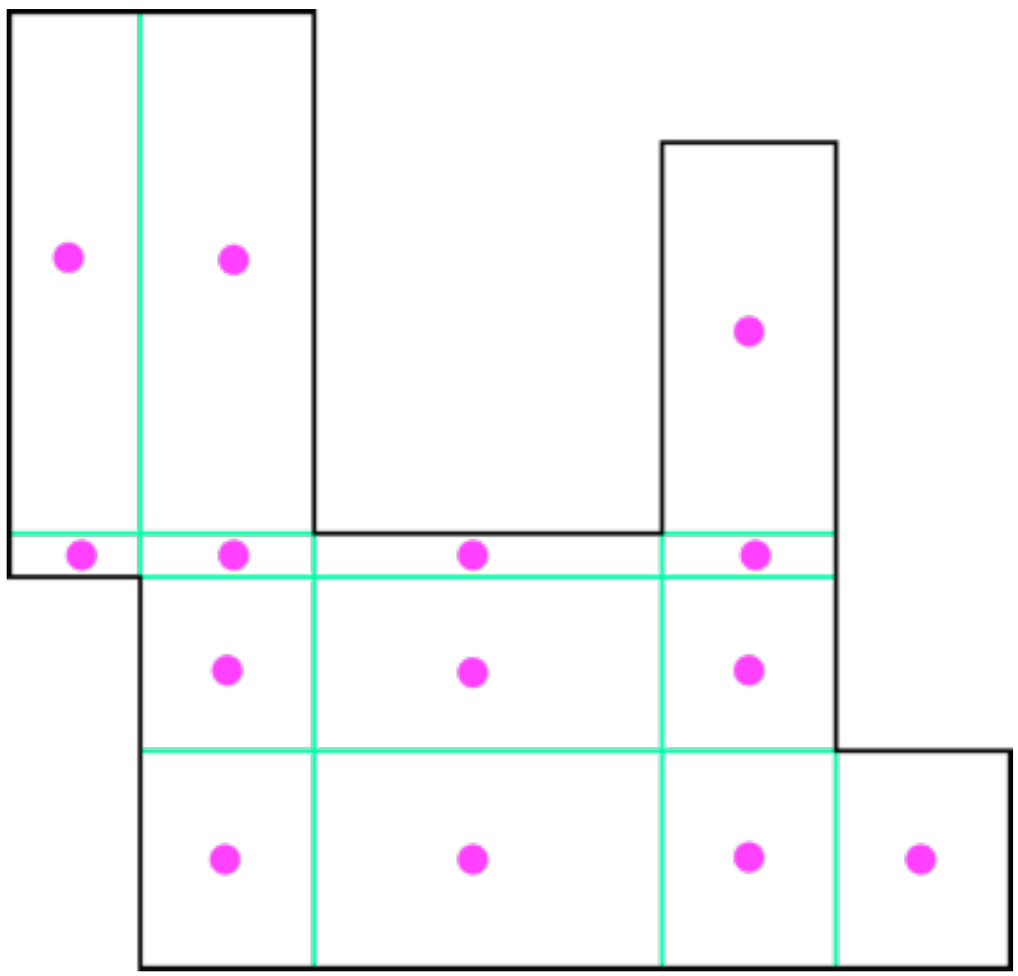

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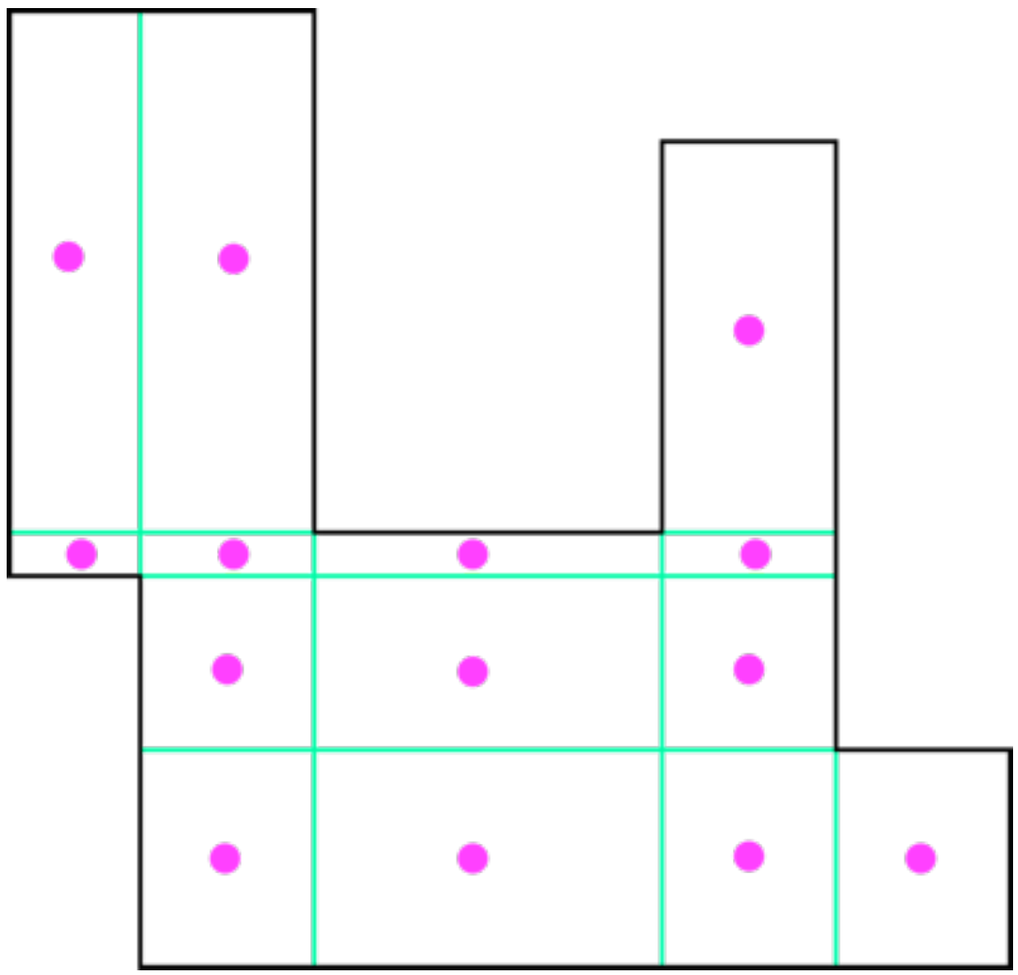

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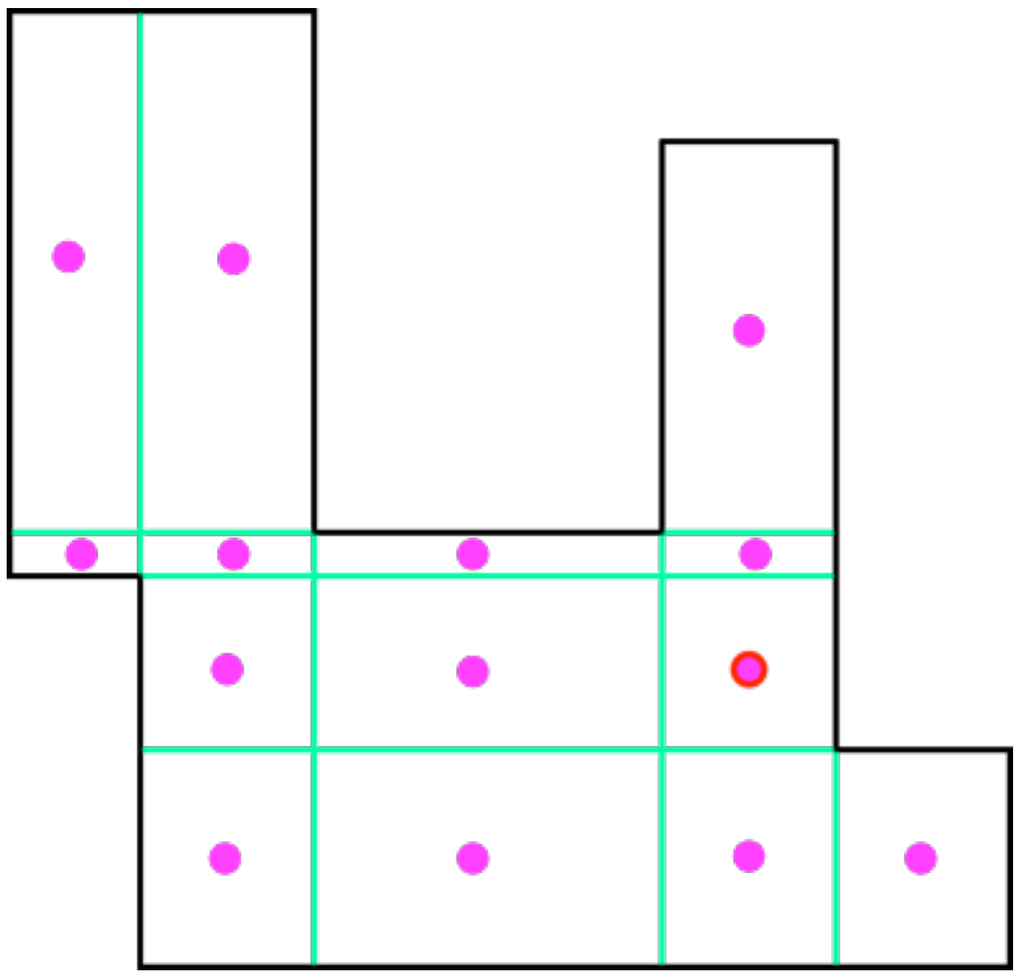

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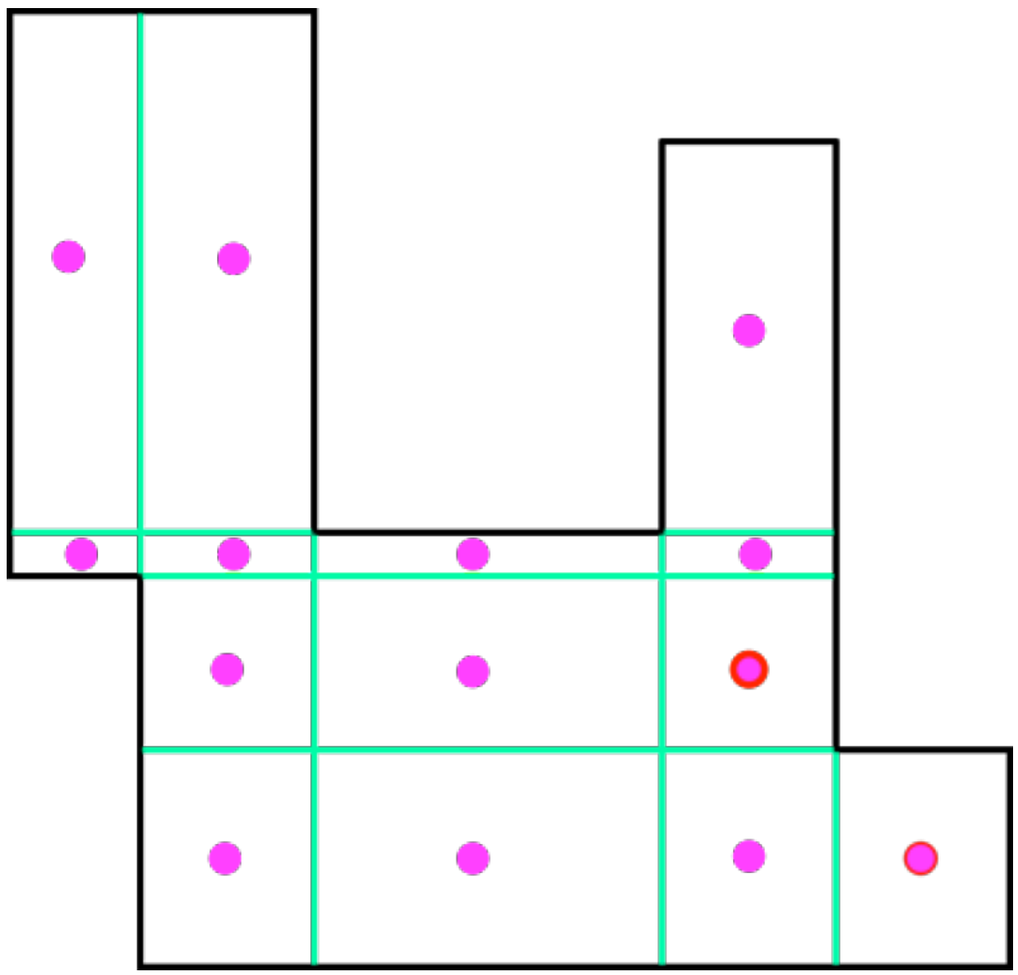

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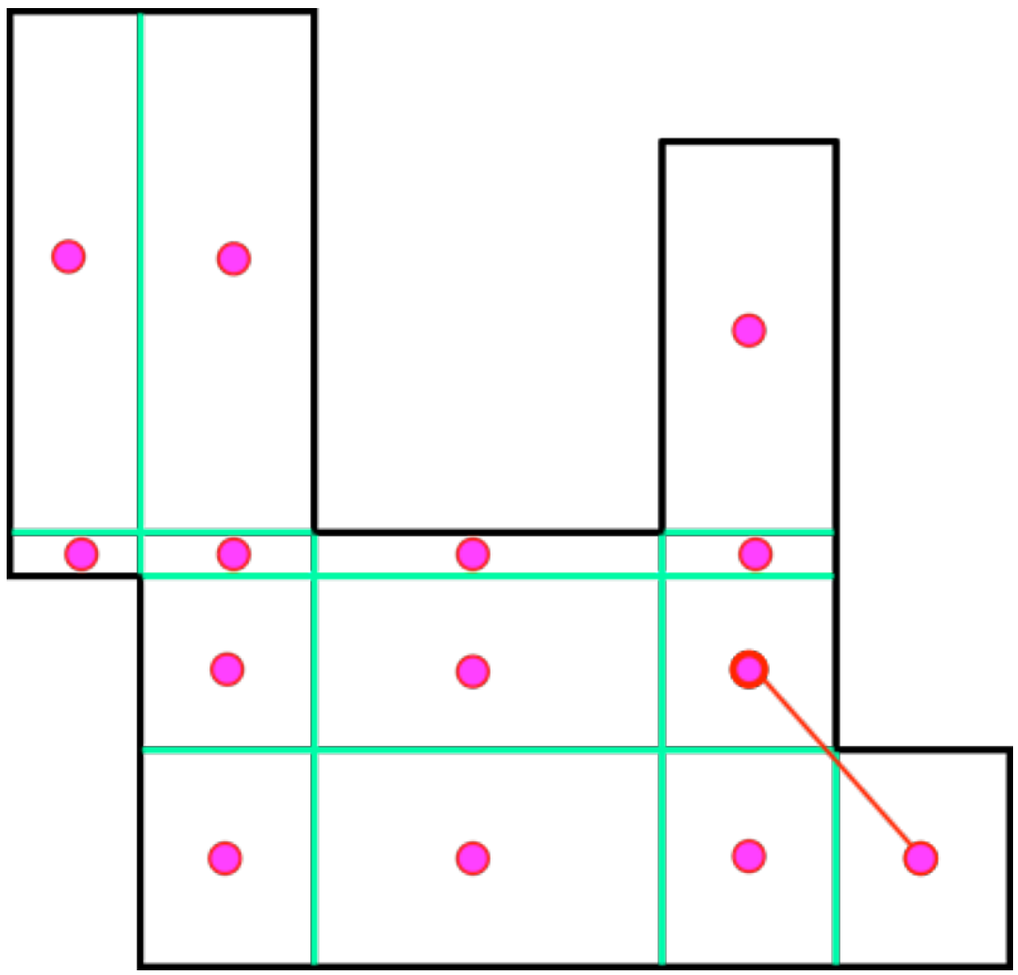

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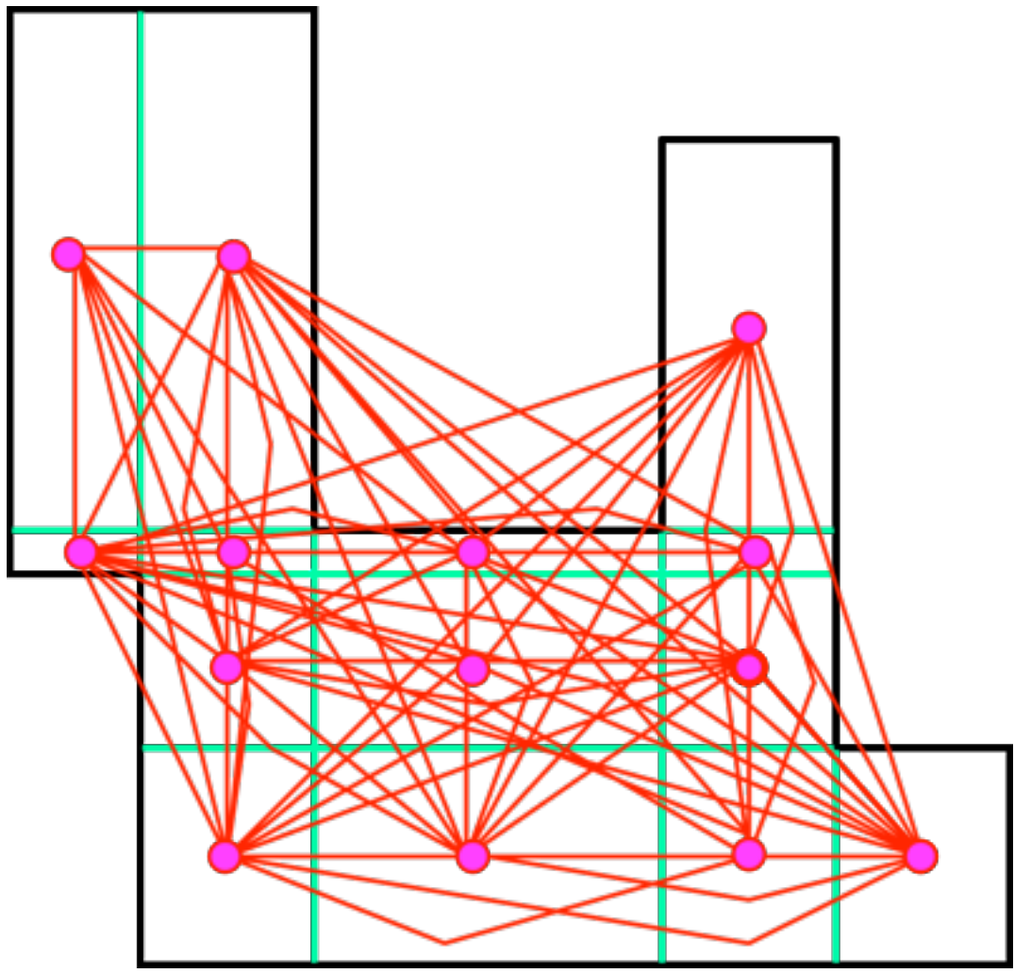

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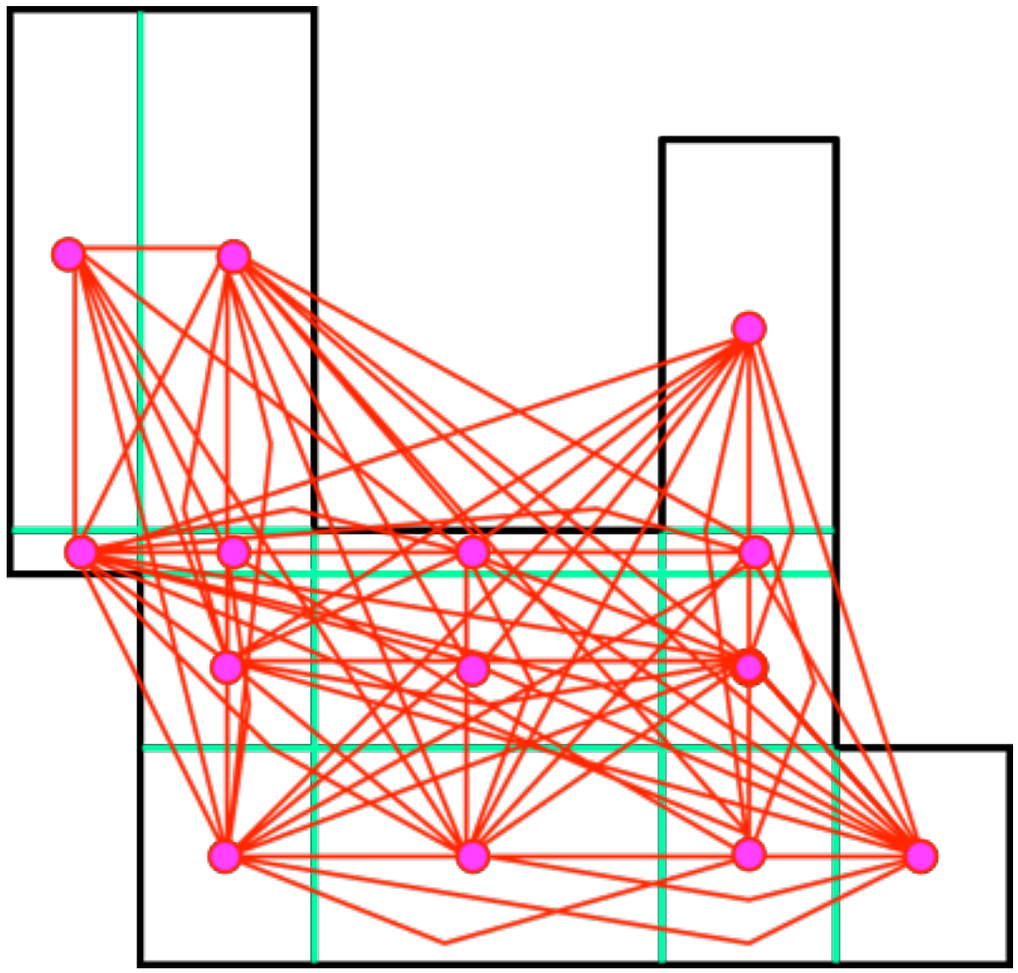

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- Theorem: Given a simple orthogonal polygon P and its RVG(P), there is a star containing a set of regions U iff the members of U form clique in RVG(P).

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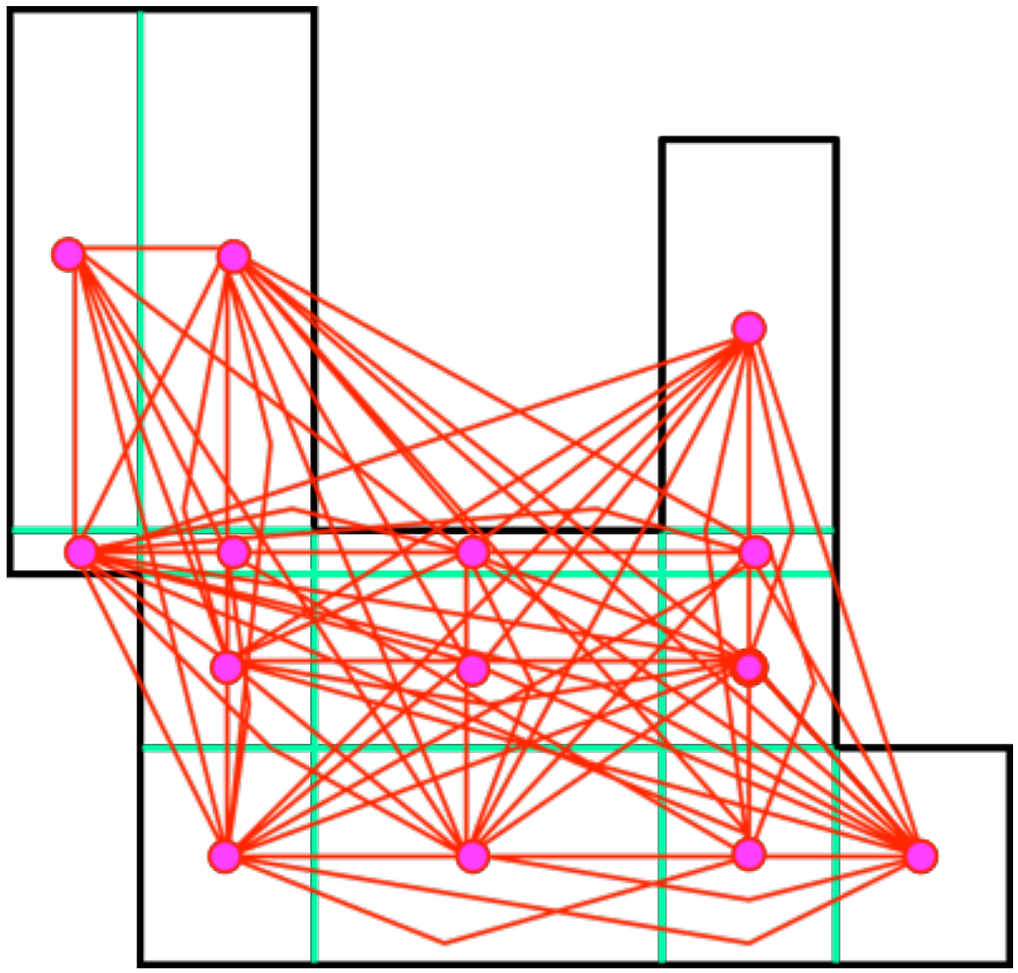

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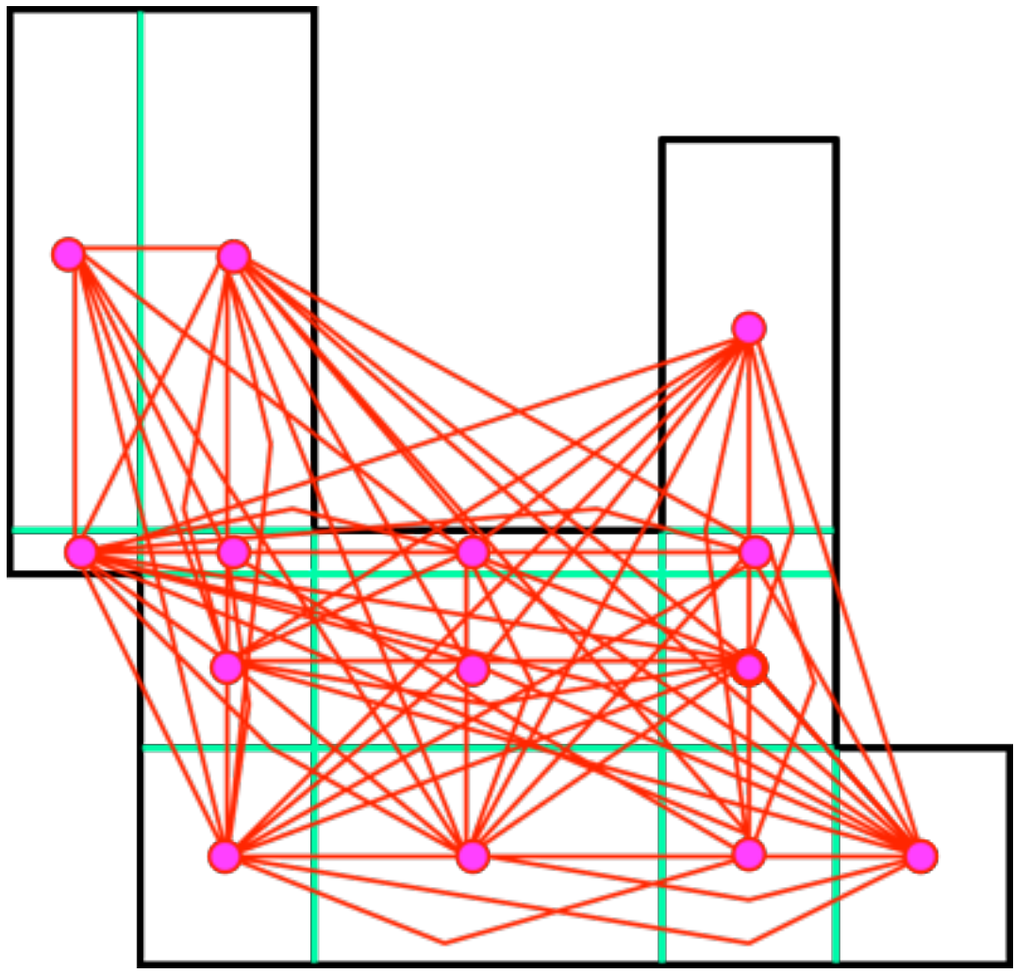

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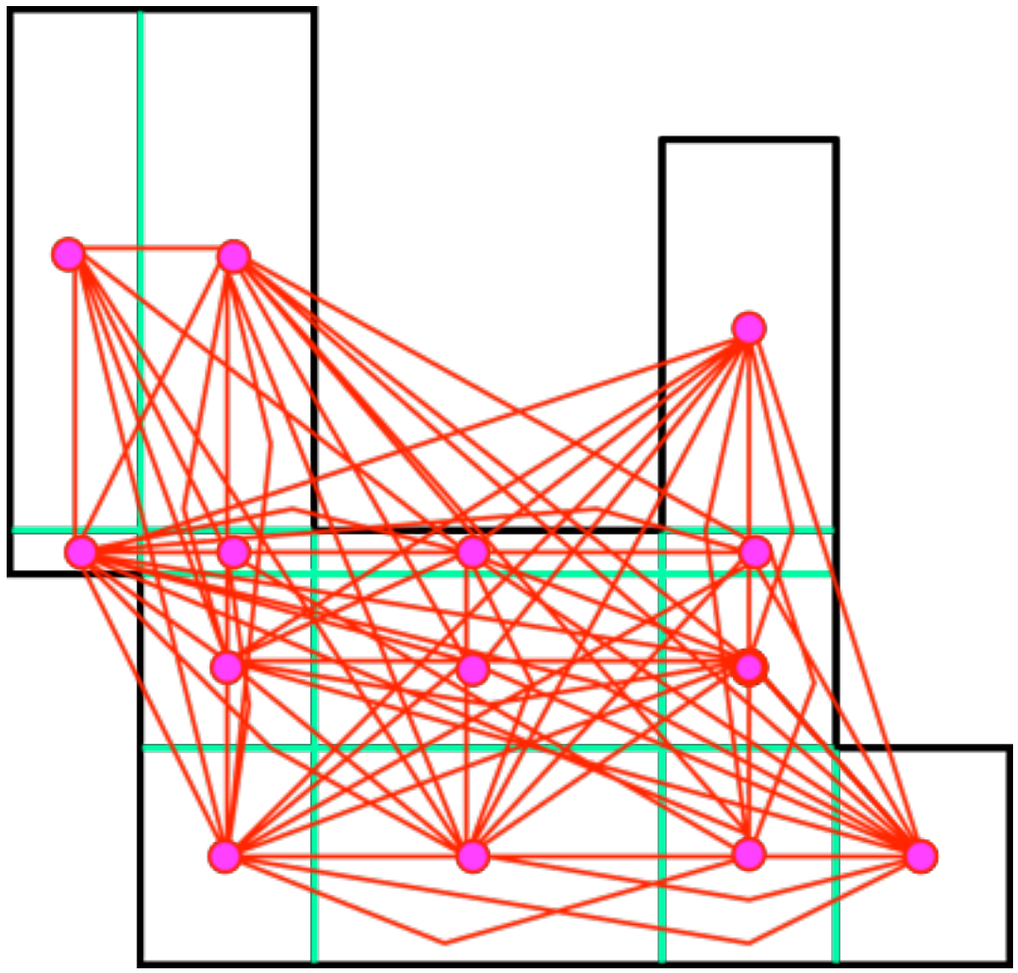

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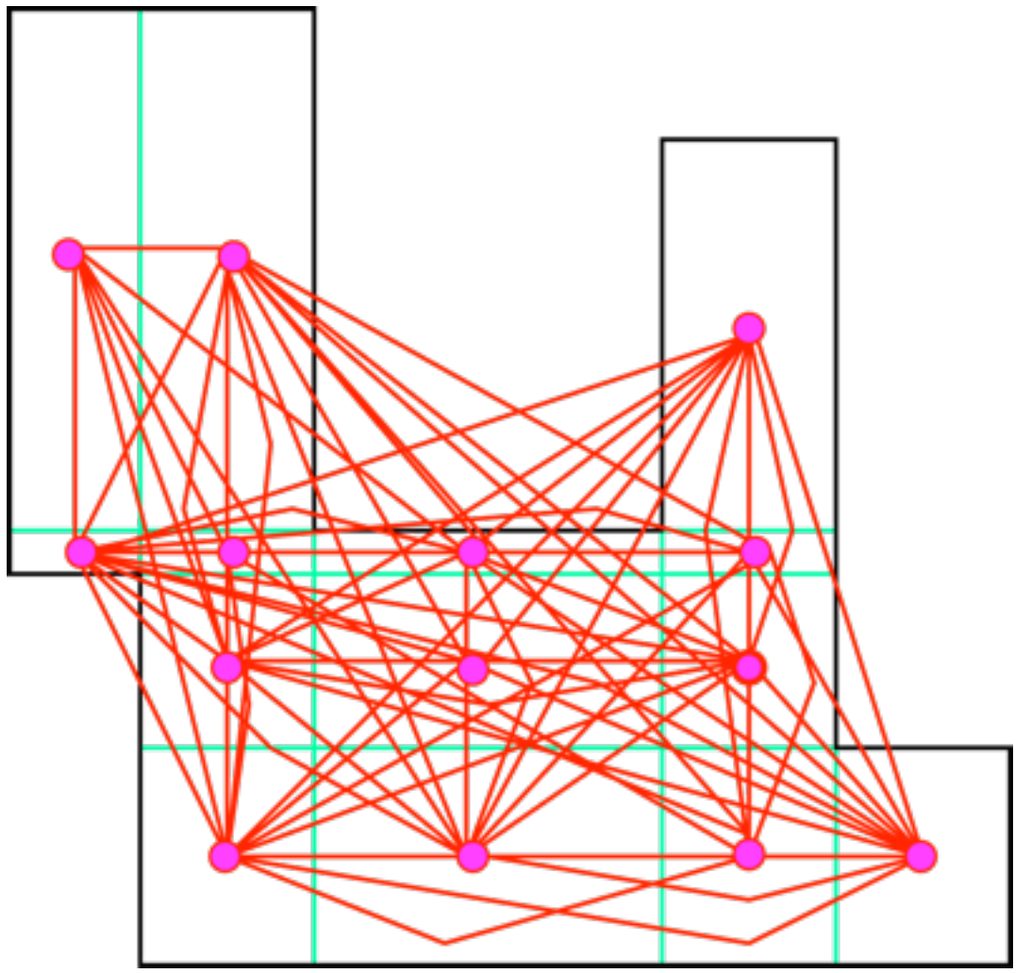
  - two nodes u and v are adjacent iff there is a star that contains both u and v
- Theorem: Given a simple orthogonal polygon P and its RVG(P), there is a star containing a set of regions U iff the members of U form clique in RVG(P).
- Worman and Keil show that the class of region visibility graphs over all simple orthogonal polygons $\mathcal{RVG}$ are perfect graphs:
  - this allows for the computation of a minimum clique cover in polynomial time
  - proof: graphs in $\mathcal{RVG}$ have no large holes and no large odd antiholes (>=5)

Rectangular vision

Rectangular vision

- Computing a max independent witness set in P is equivalent to computing a max independent set in RVG(P)



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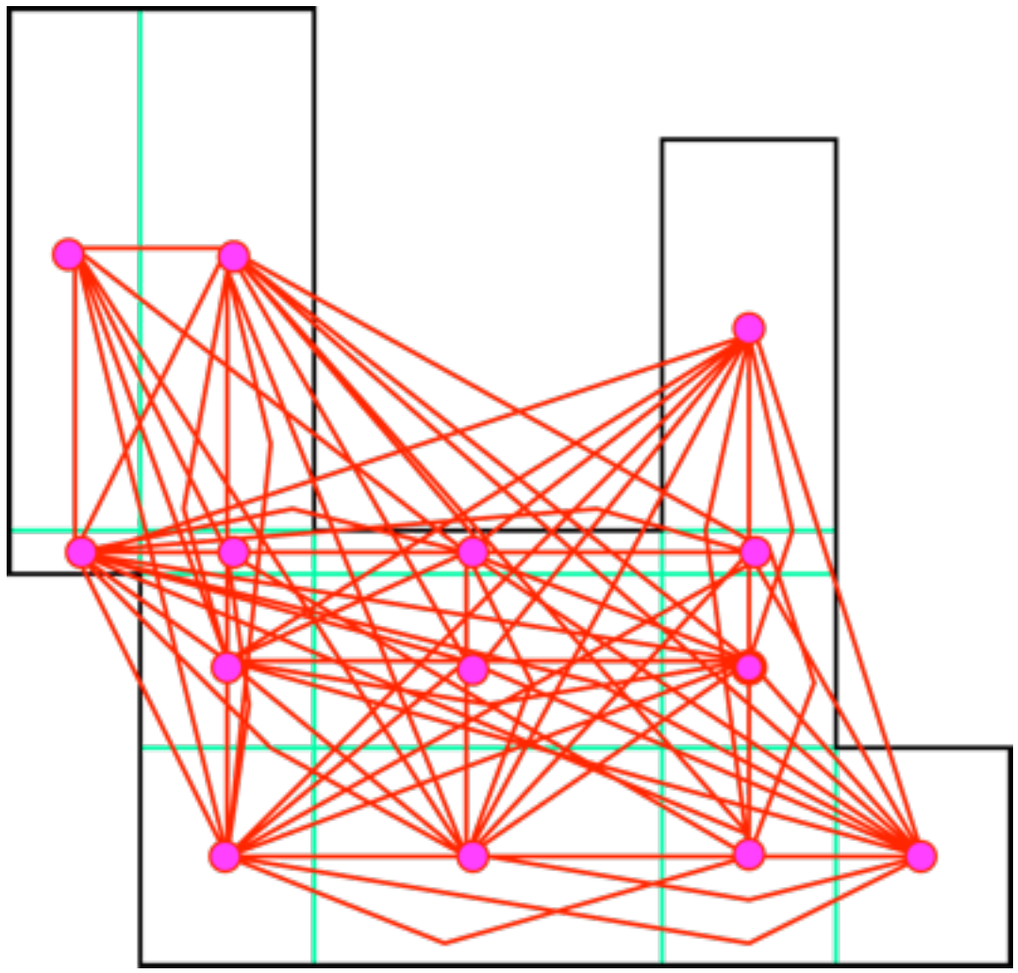

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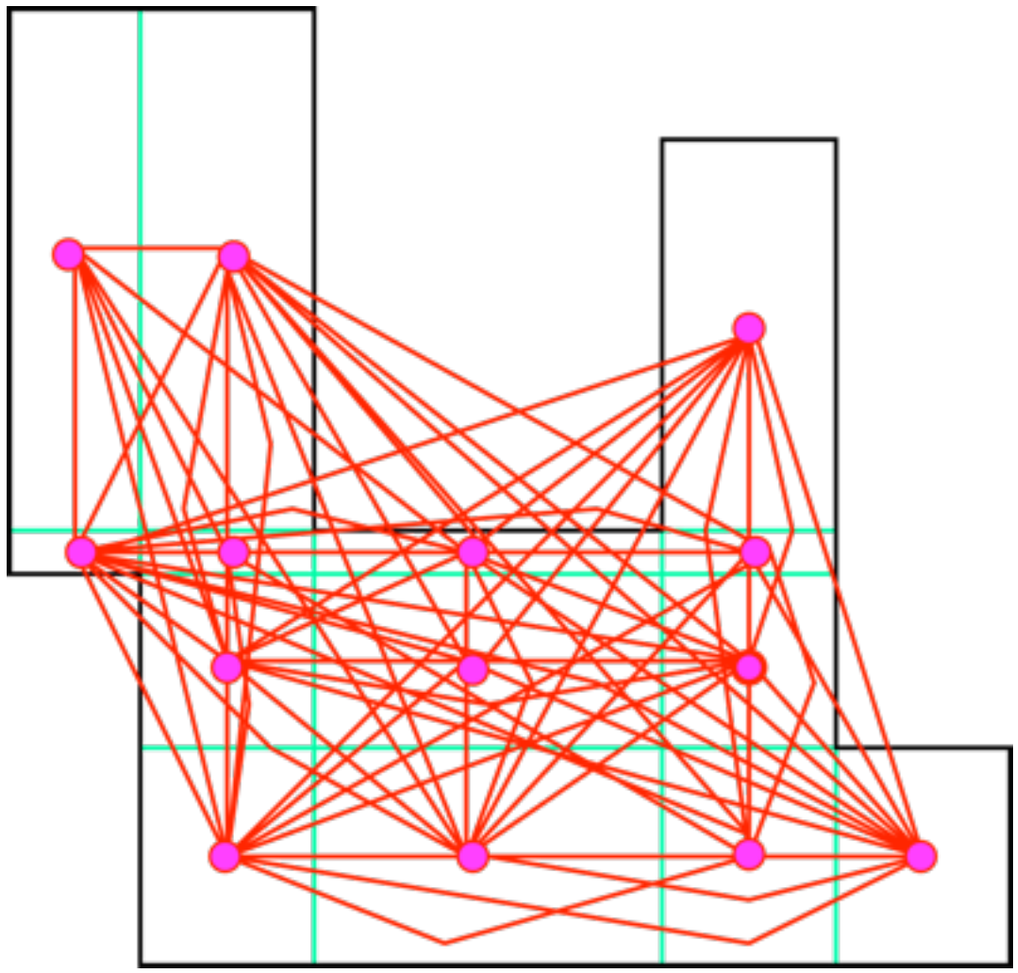

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- we give a direct geometric proof (maybe simpler)

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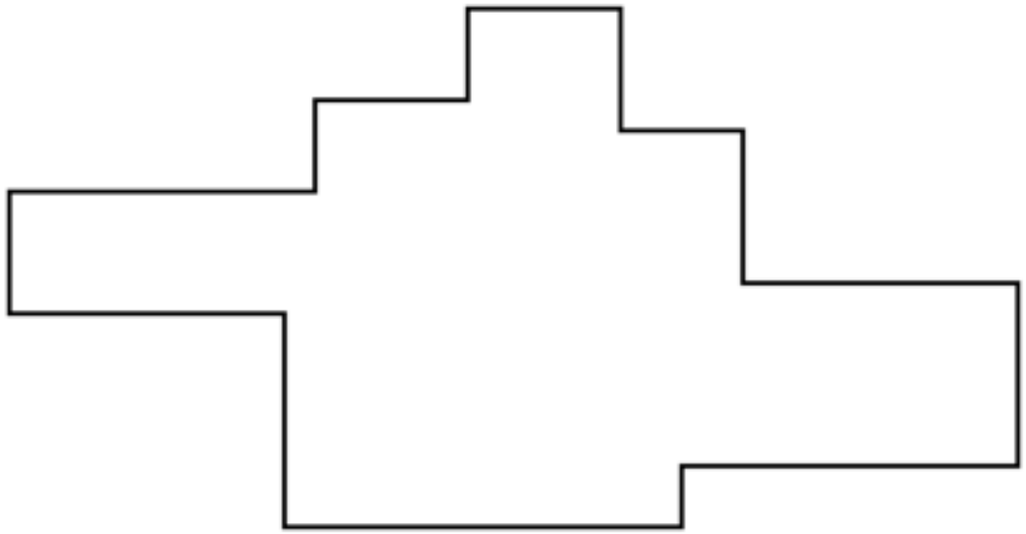

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Given three points $w, p, q \in P$. Then:
$VP(w) \cap VP(p) \neq \emptyset$
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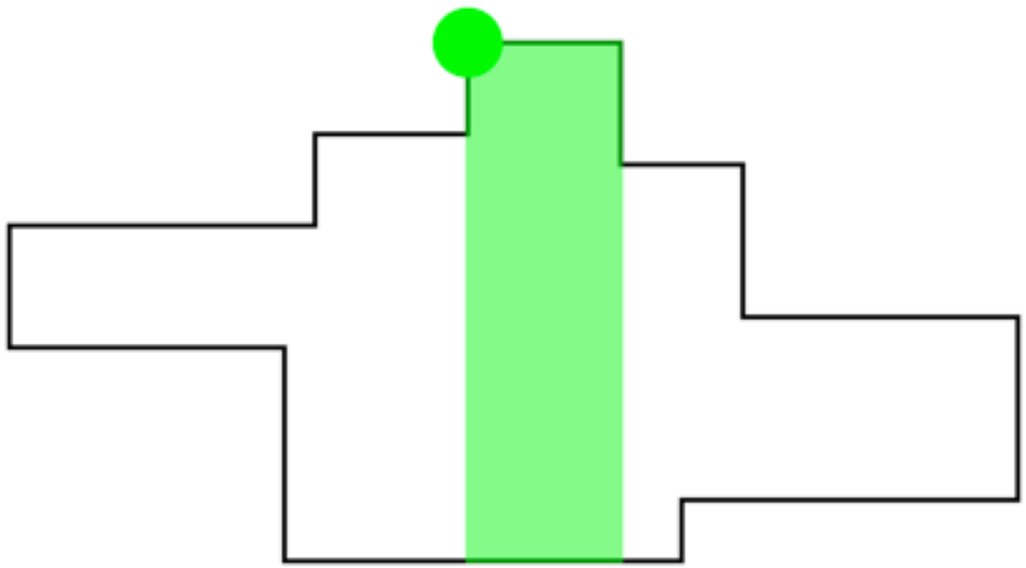

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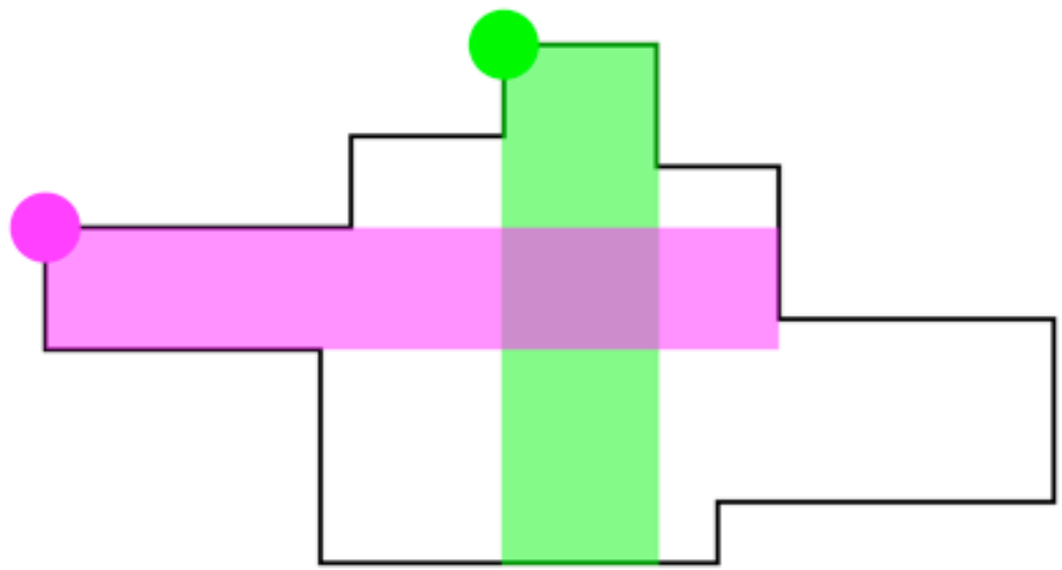

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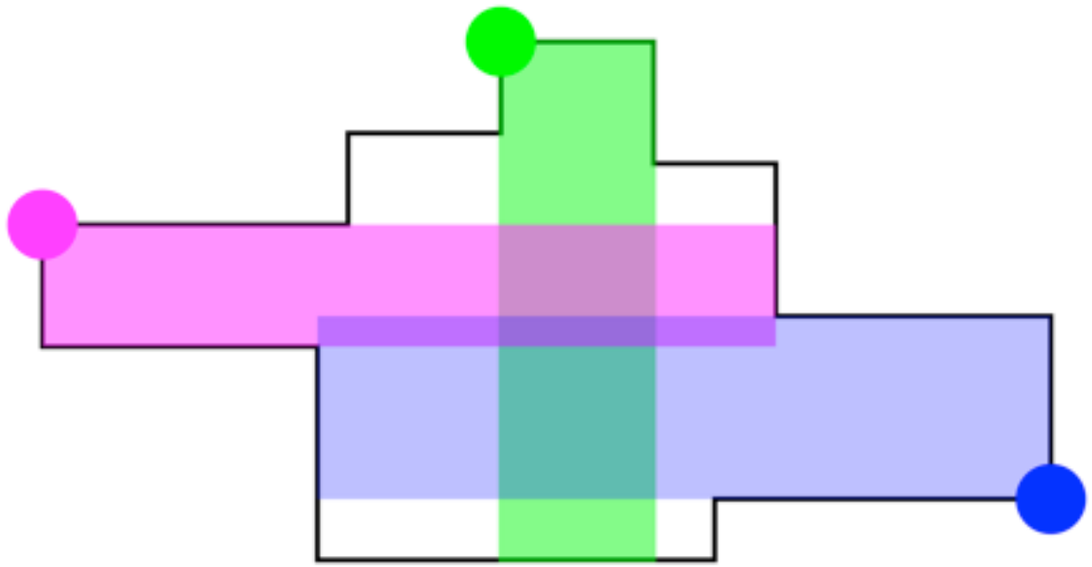

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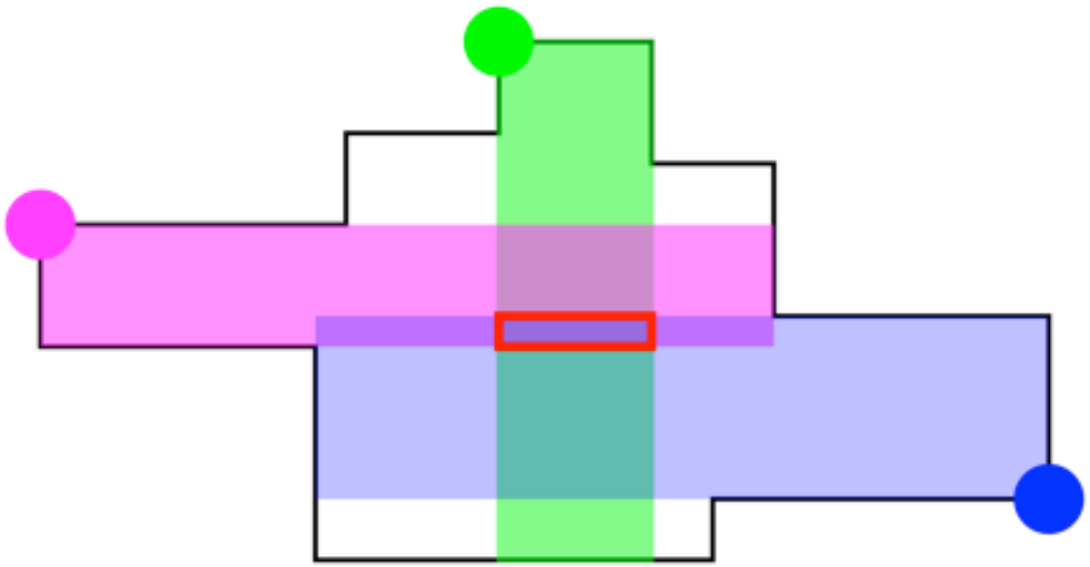

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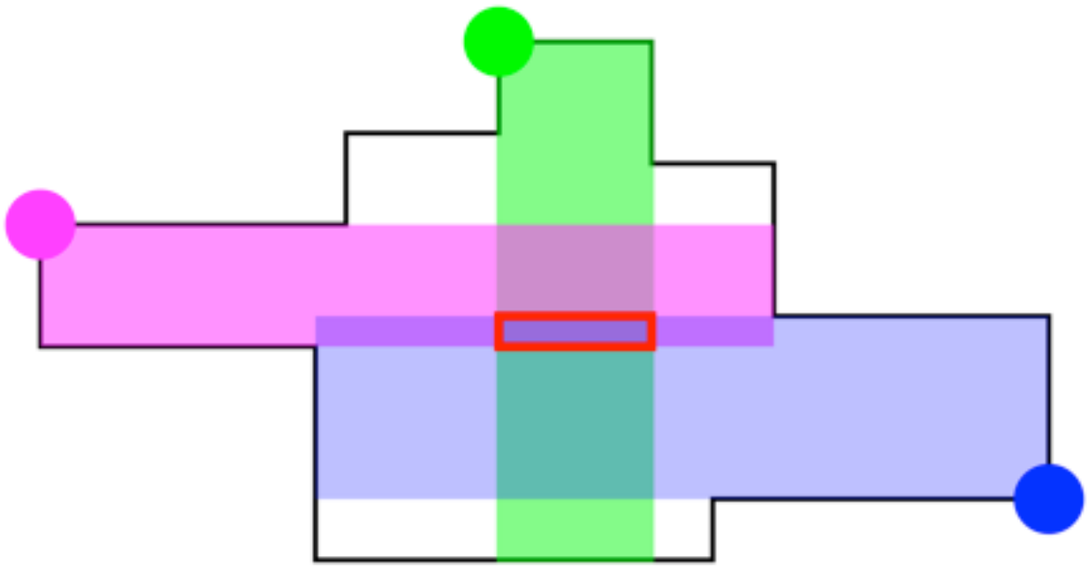

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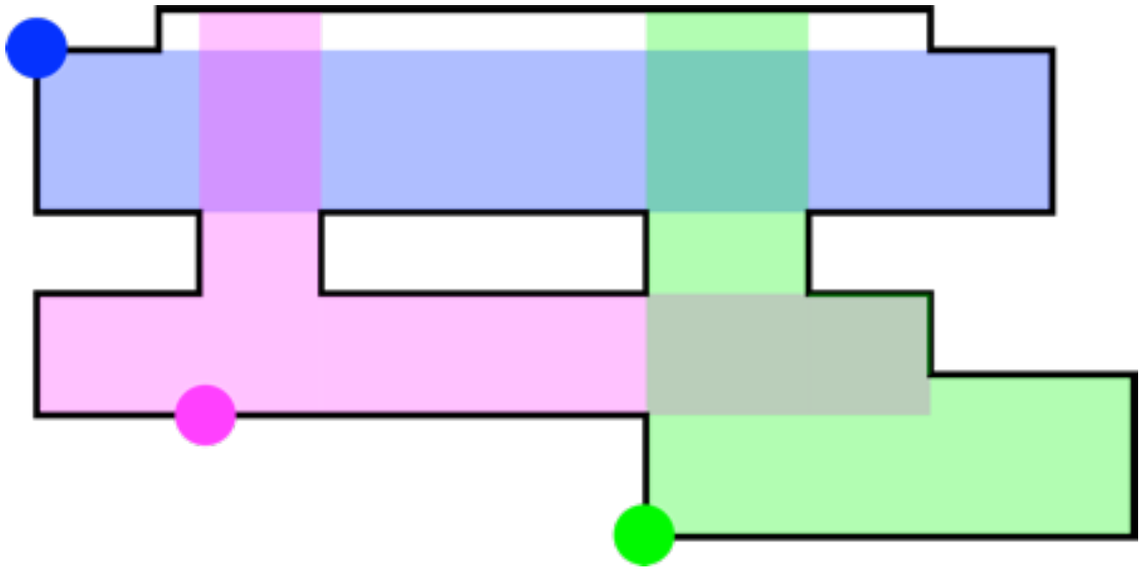
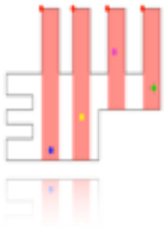


Not true for polygons with holes!



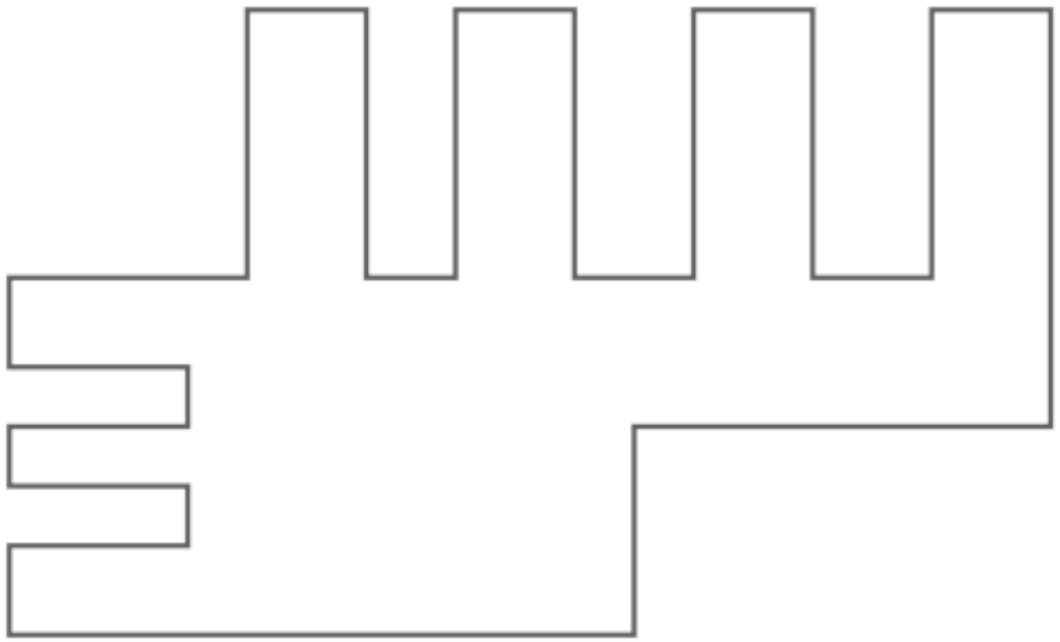
Perfectness

Perfectness

Theorem 1:

Simple rectilinear polygons are perfect under rectangular vision: $w(P)=g(P)$.

Perfectness



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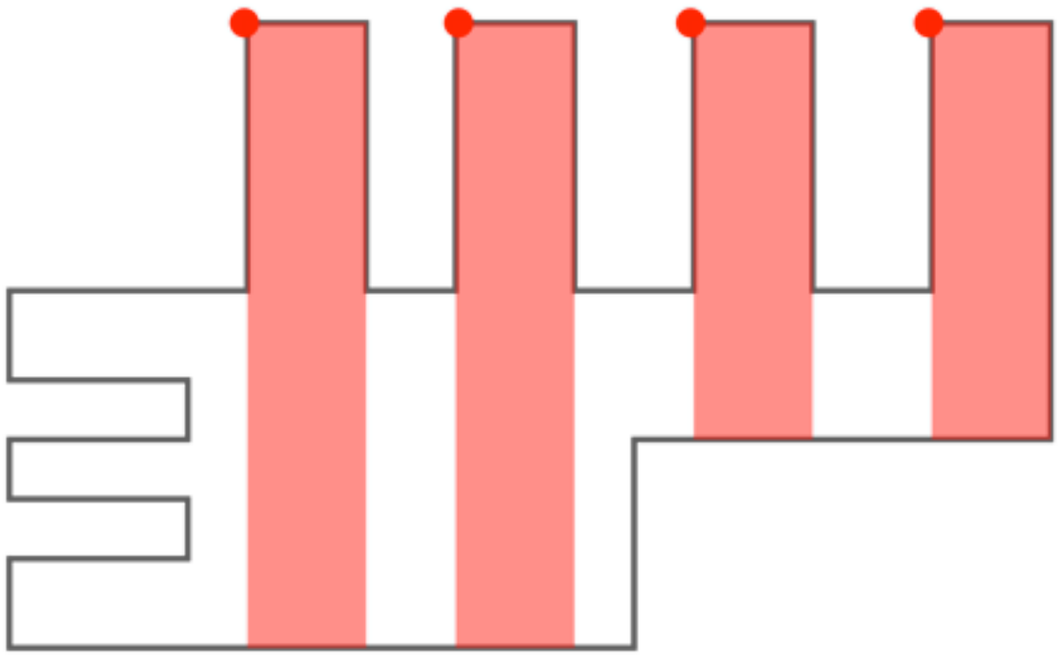

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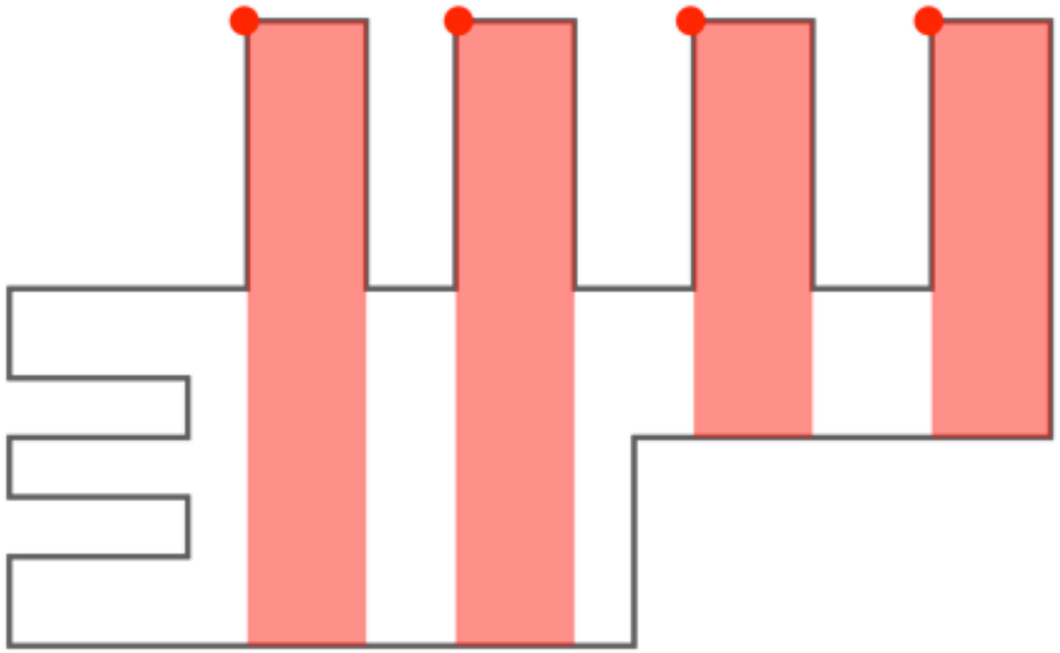

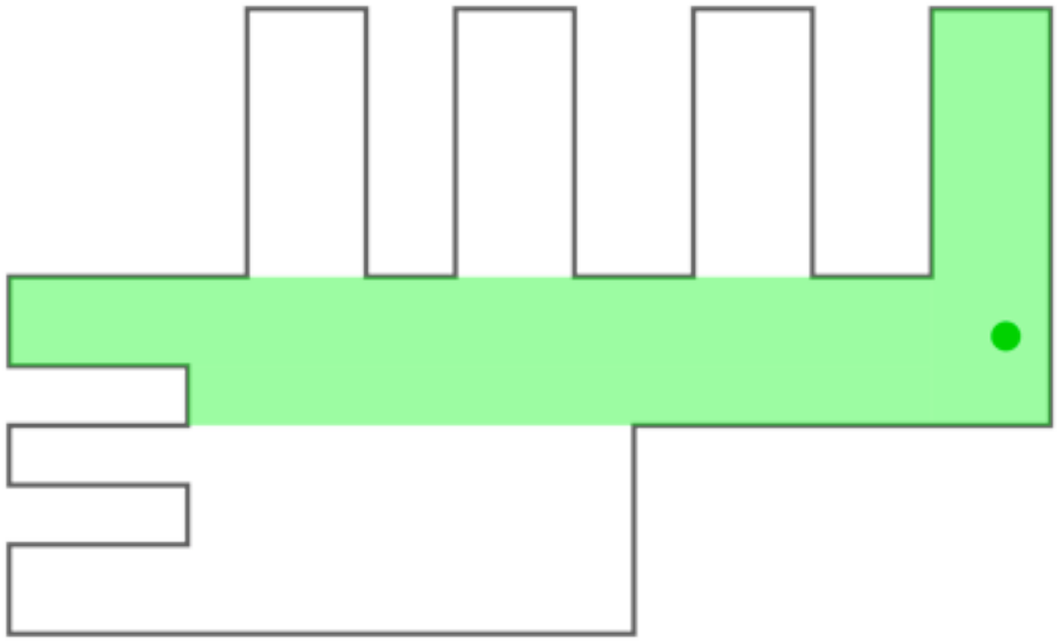

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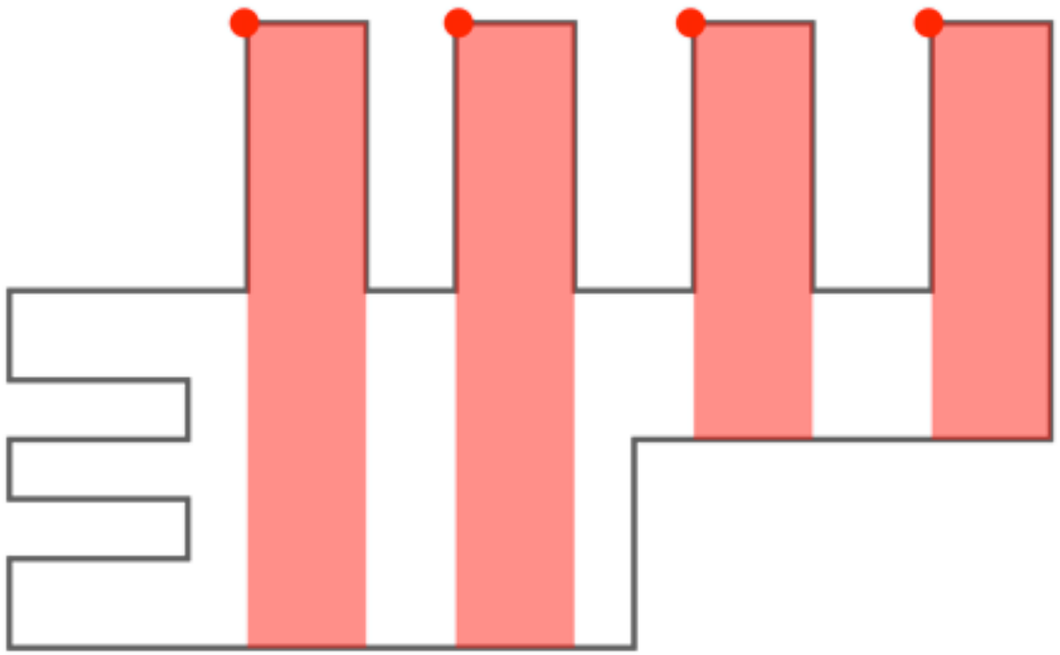

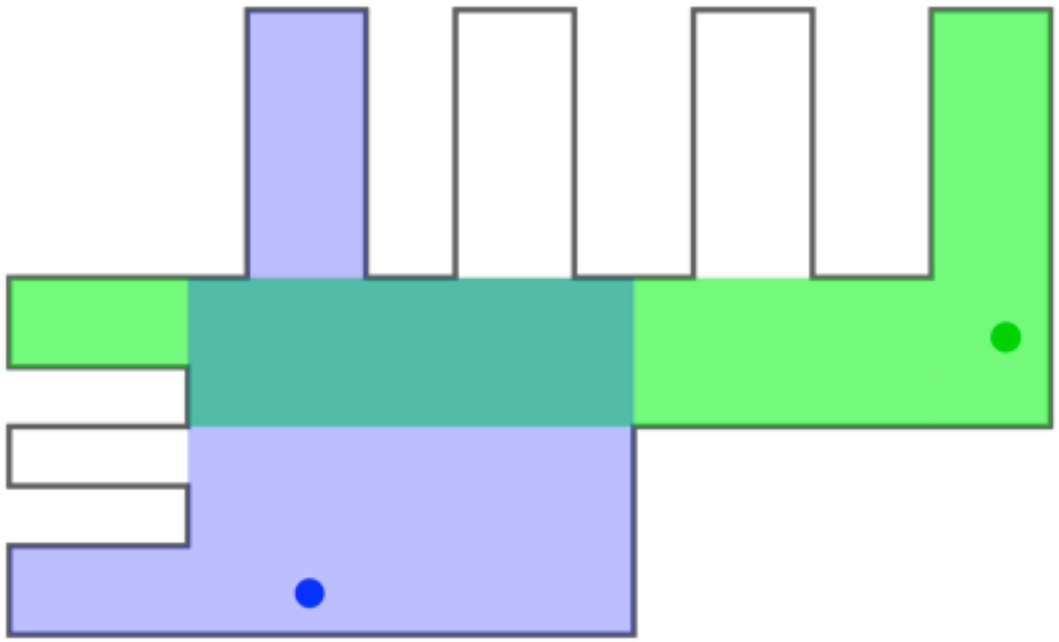

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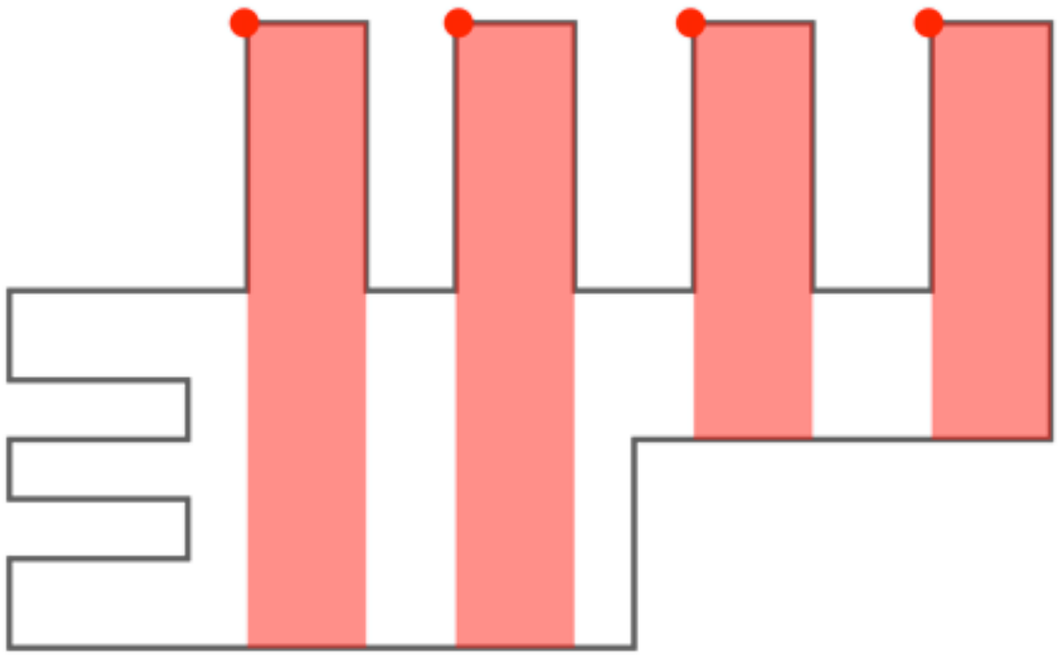

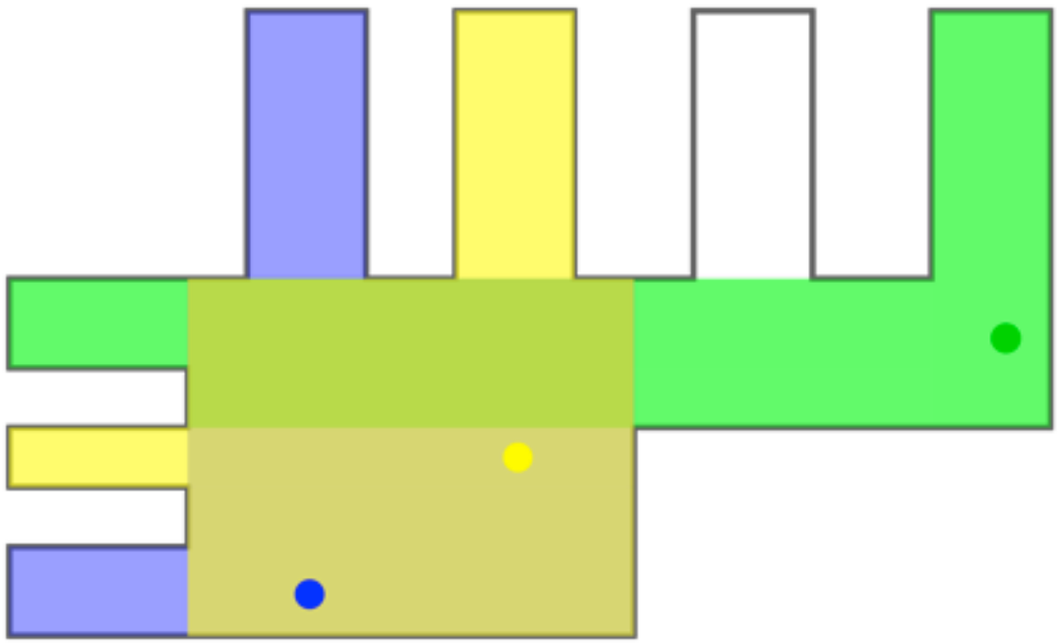

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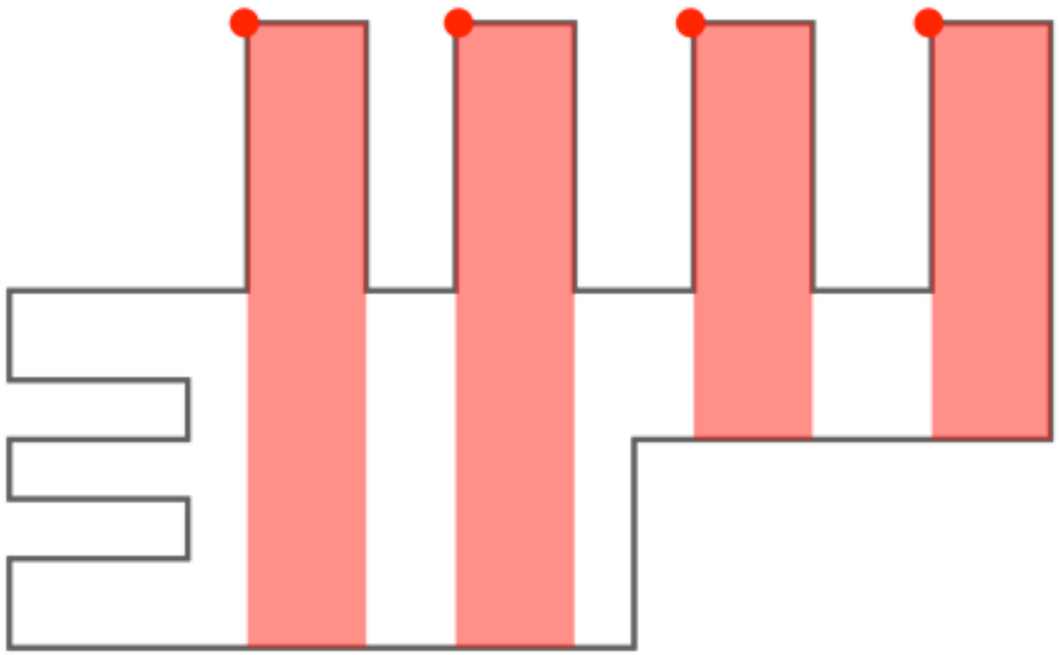

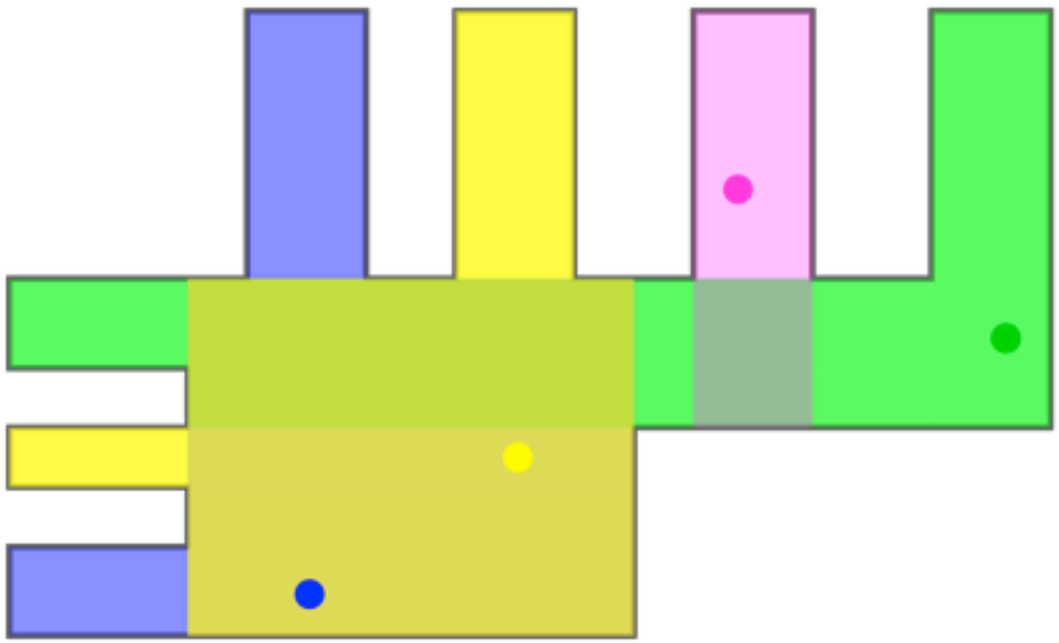

Perfectness

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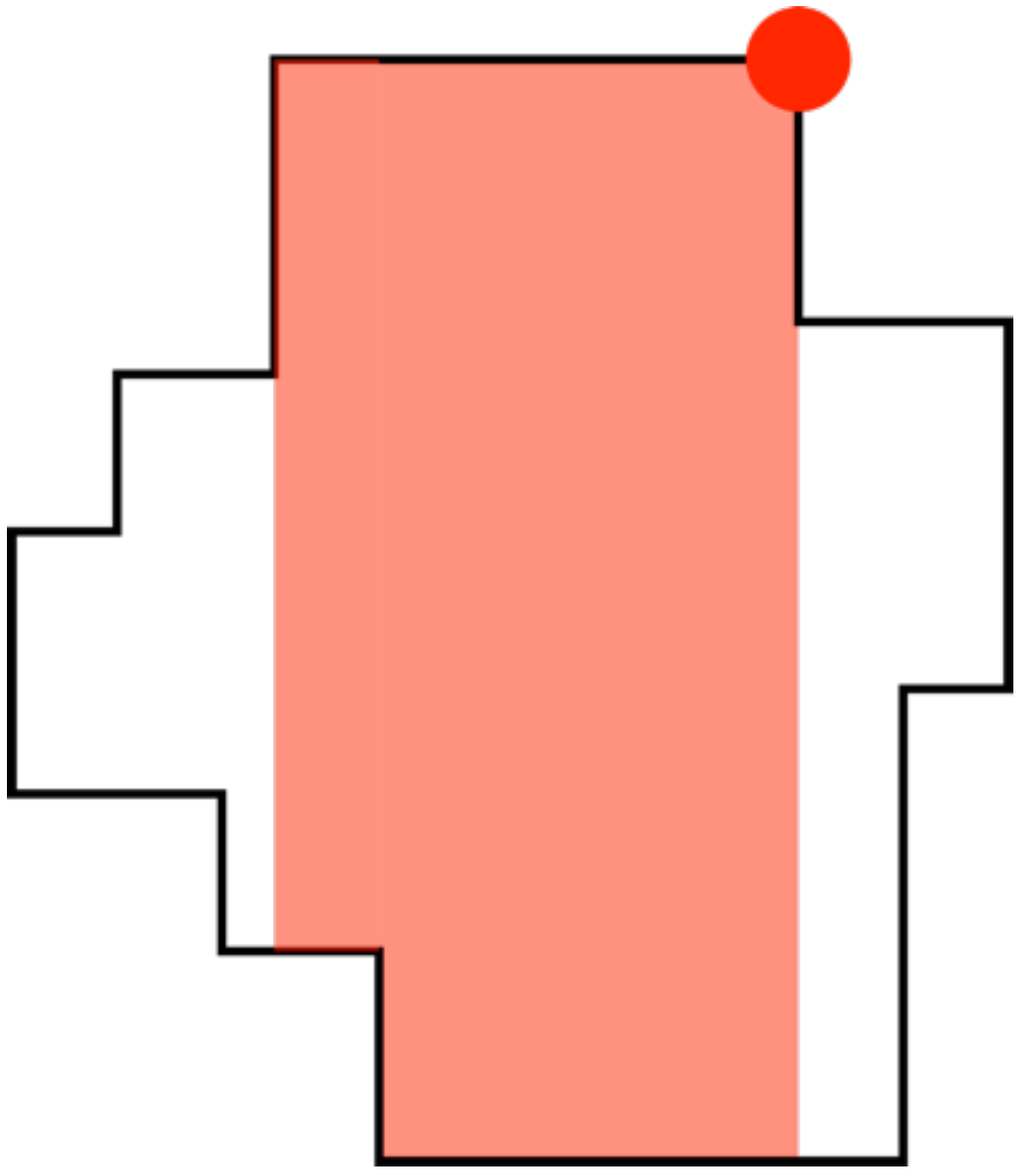
Proof by induction on the number, k, of witnesses:

Perfectness



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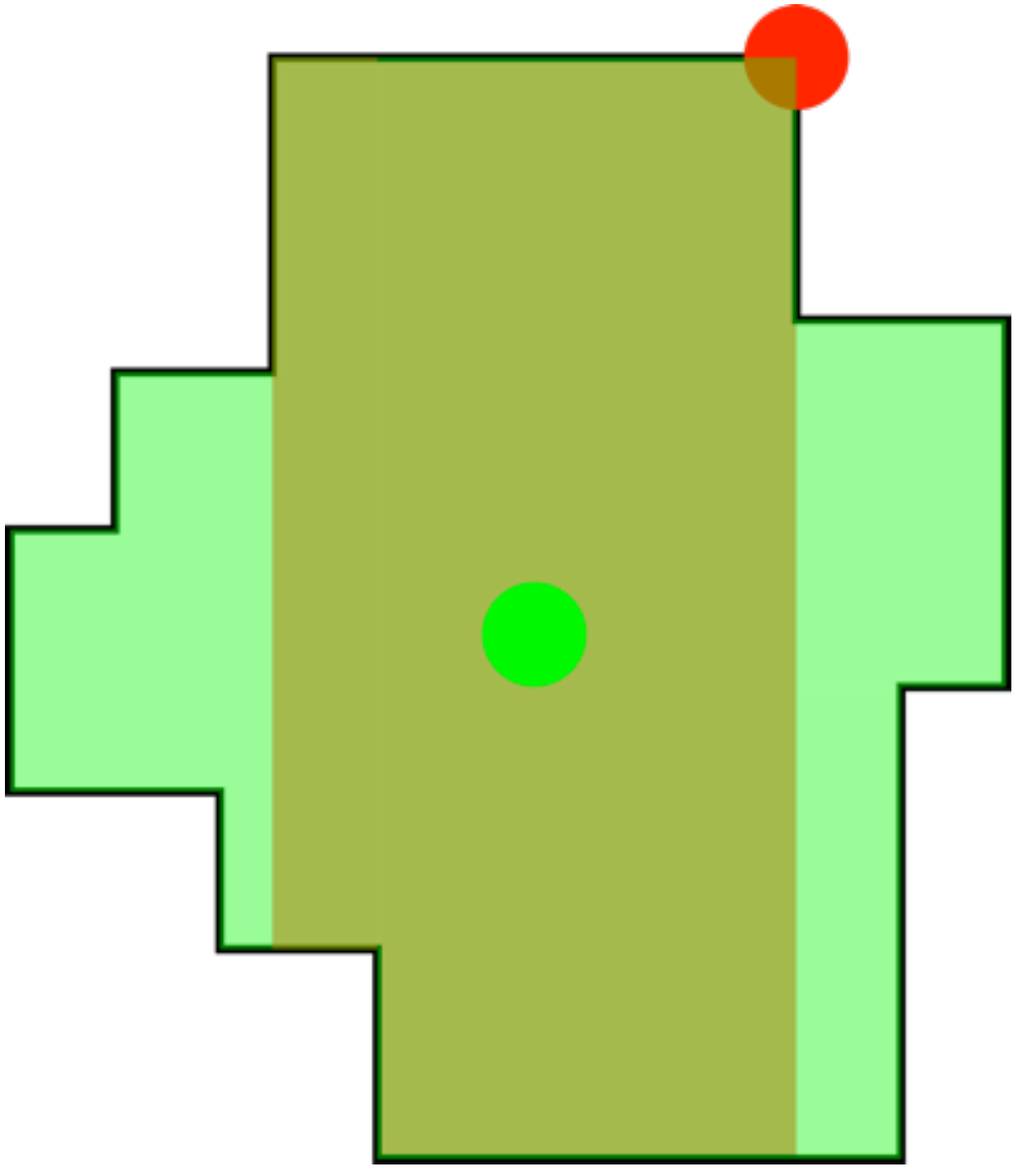

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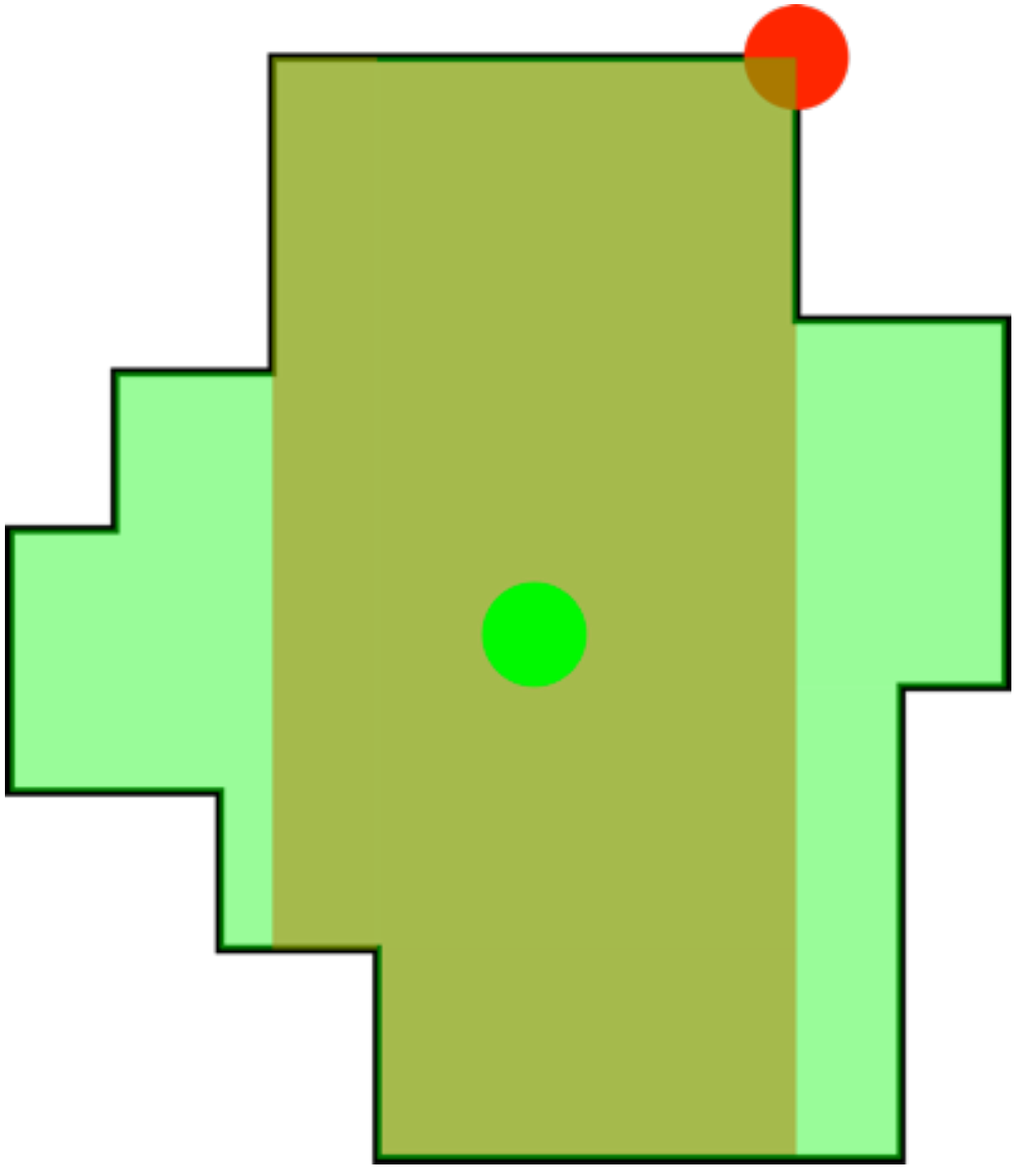

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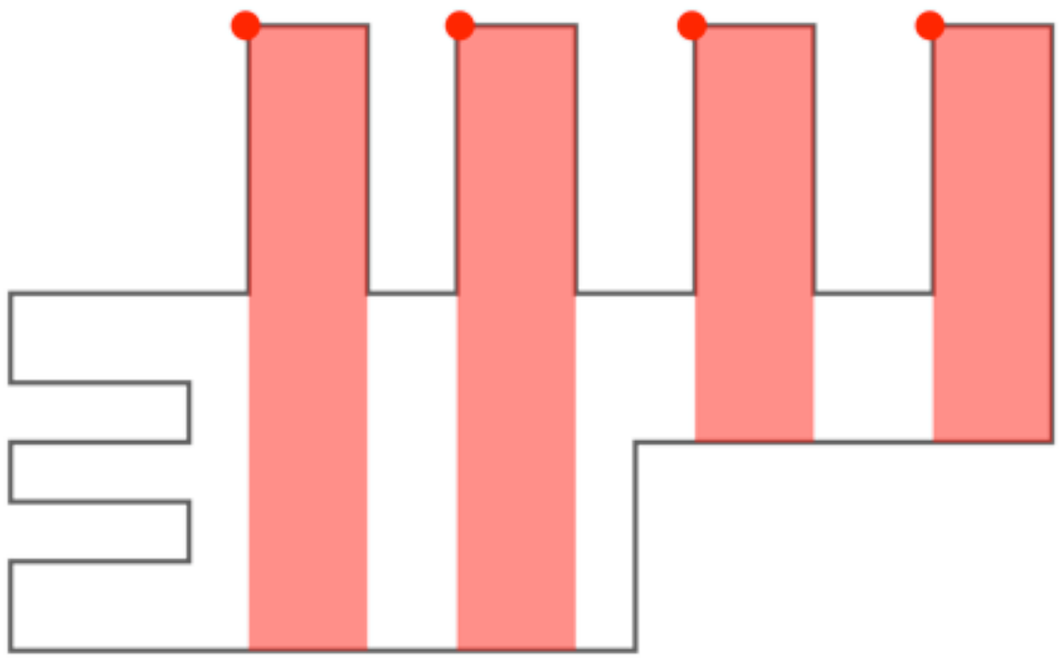

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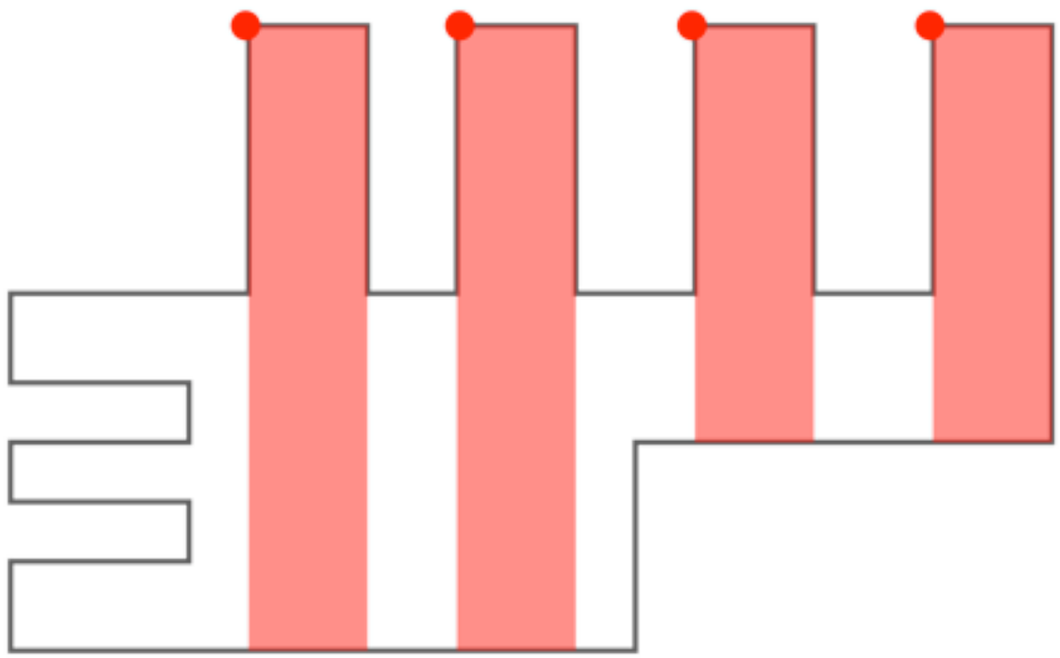

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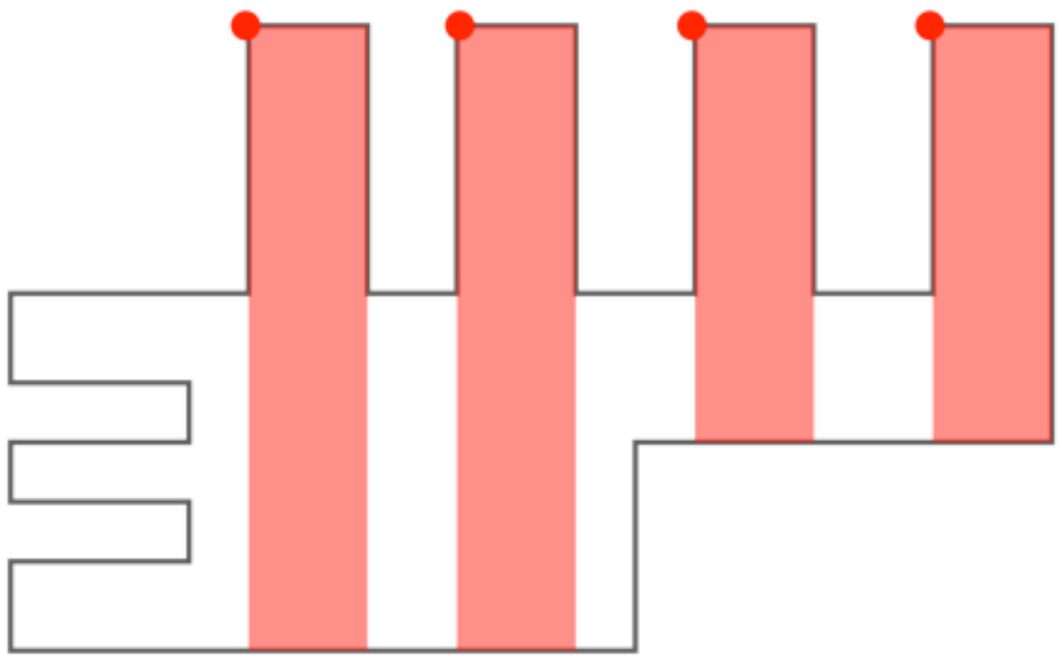
Perfectness



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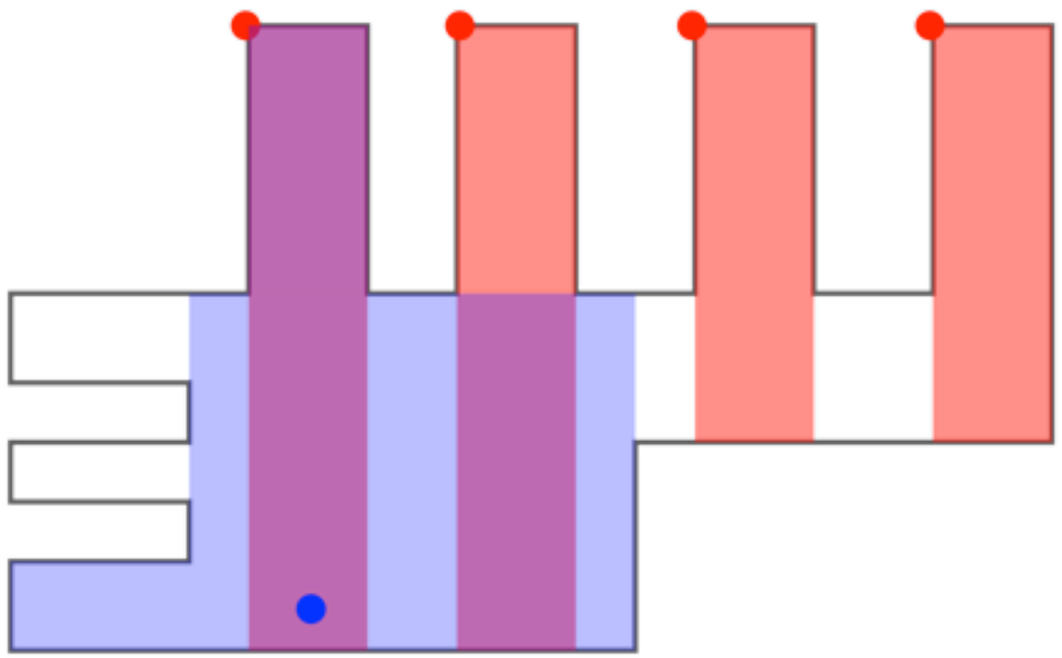
Perfectness



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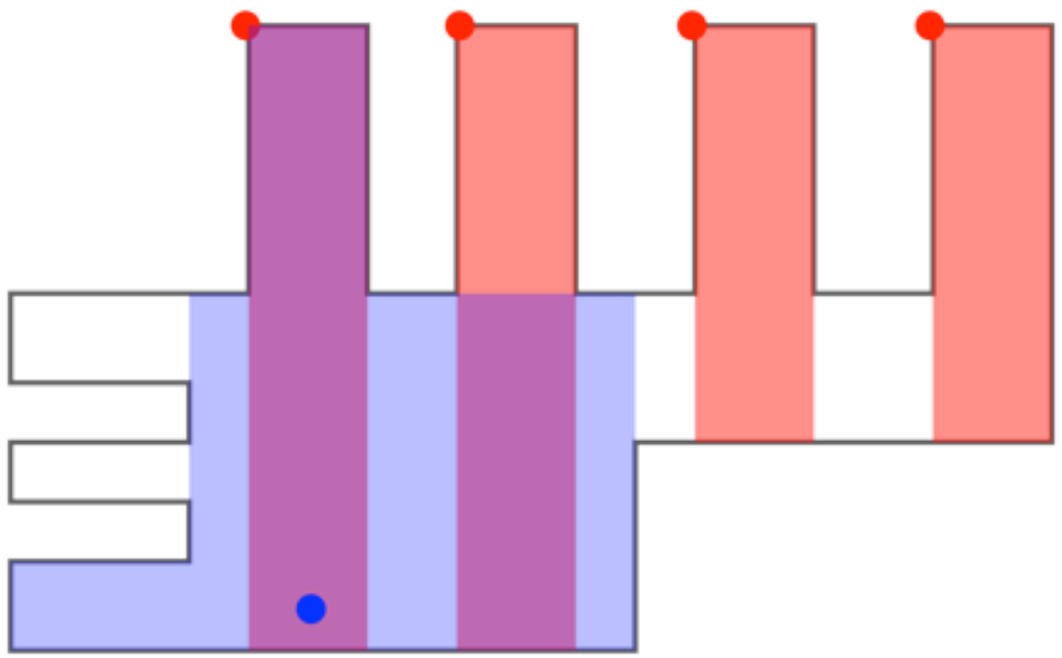

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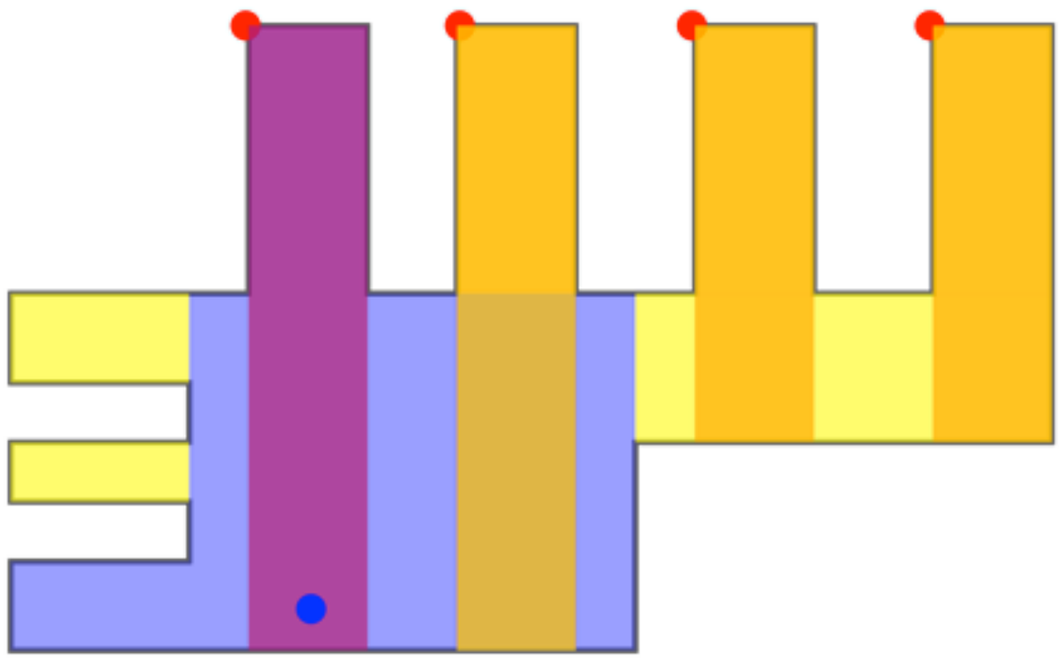

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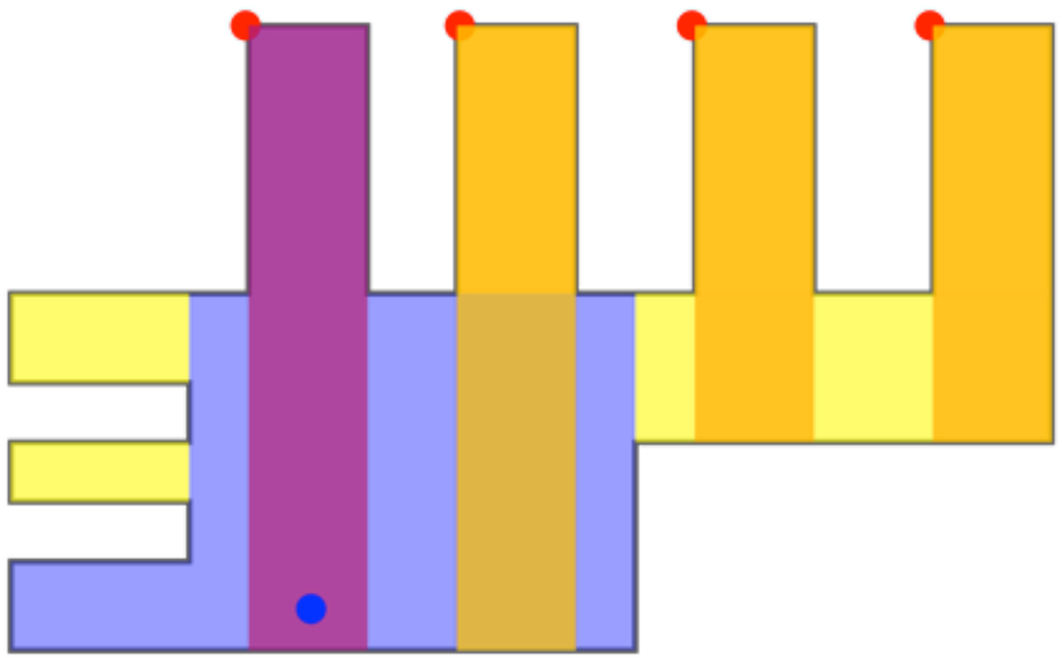

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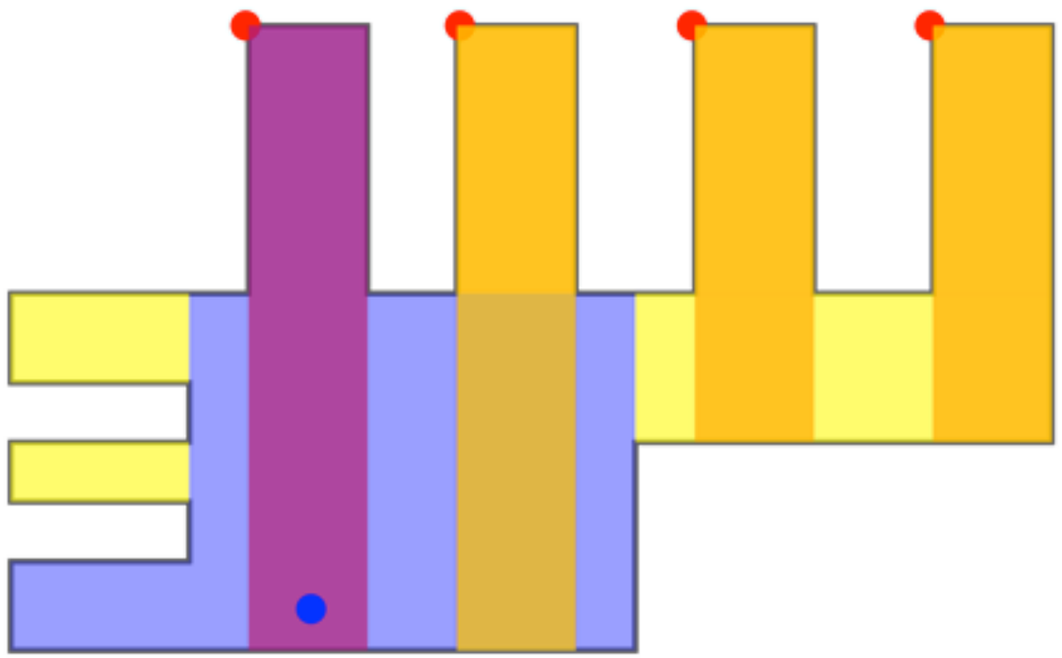

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How to construct these polygons?

Perfectness

Perfectness

We start with a specific witness:

Perfectness

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- Claim: There exists an **ear witness**, i.e., a witness w such that all other witnesses $w_1, \ldots, w_k$ are in the same connected component of P‘\VP(w).

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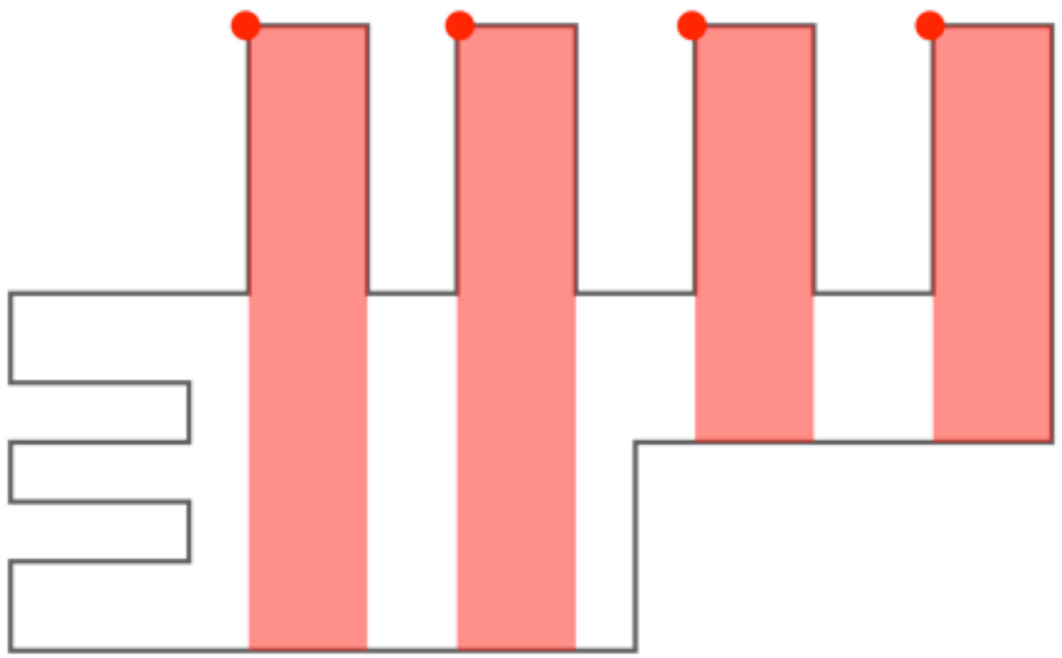
Perfectness



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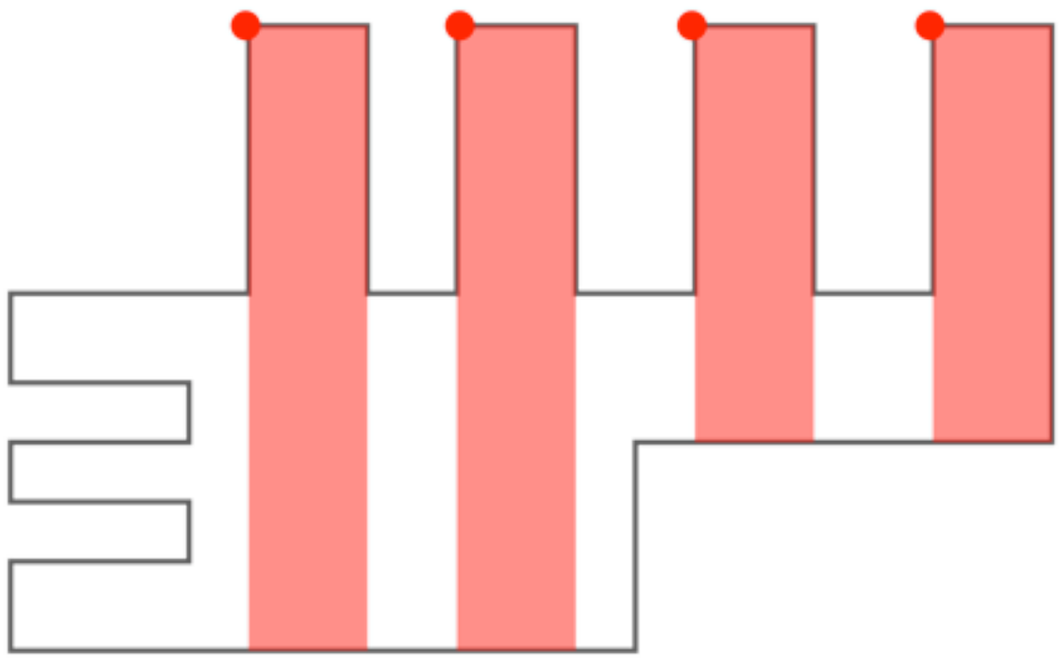
- Proof:

Perfectness



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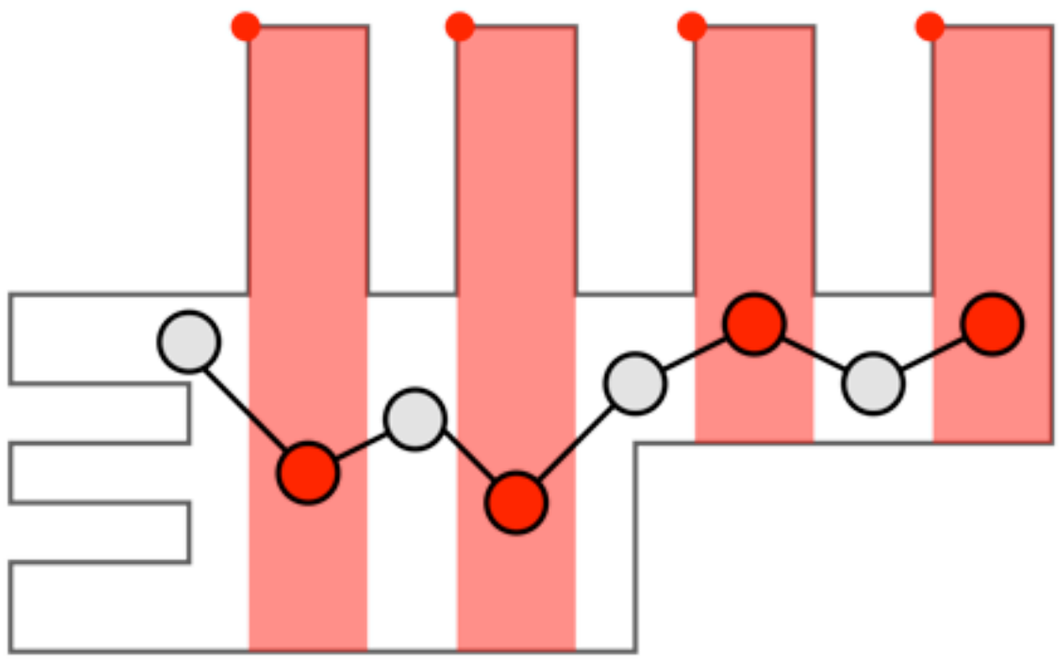
Perfectness



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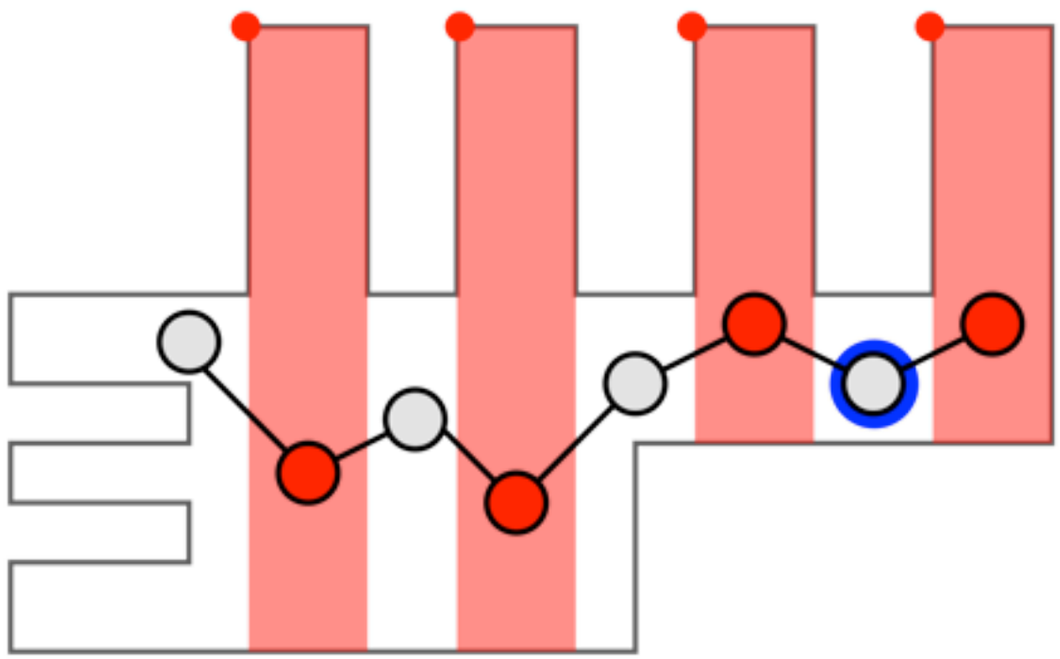

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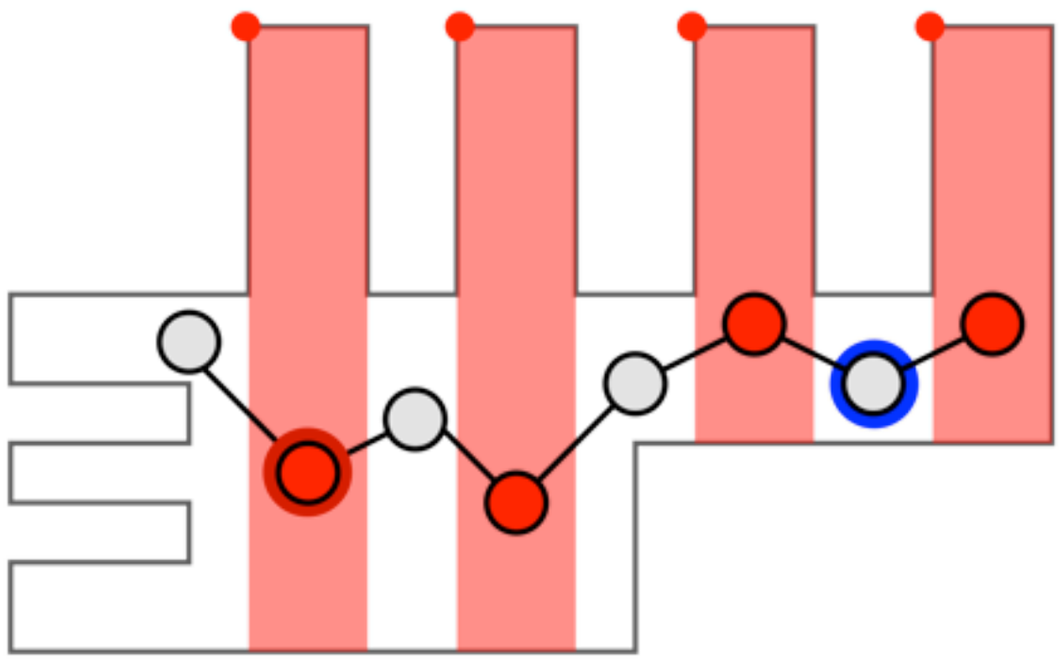
  Define a root arbitrarily,

Perfectness



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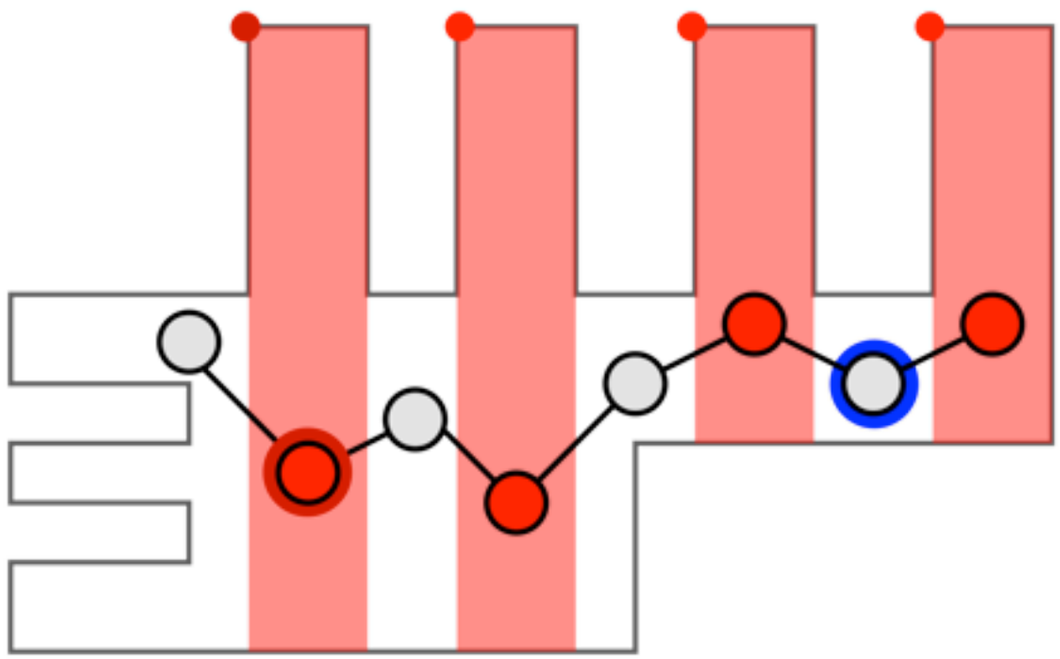

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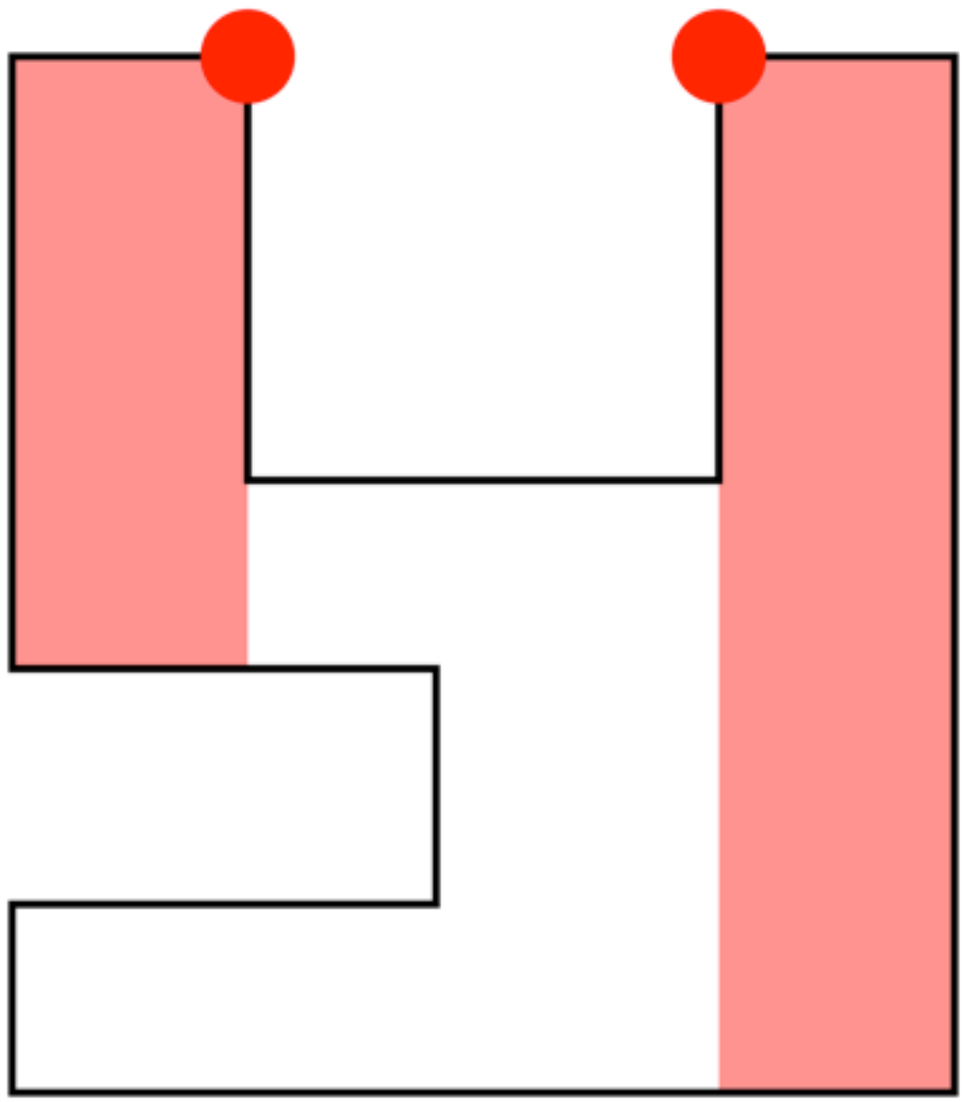

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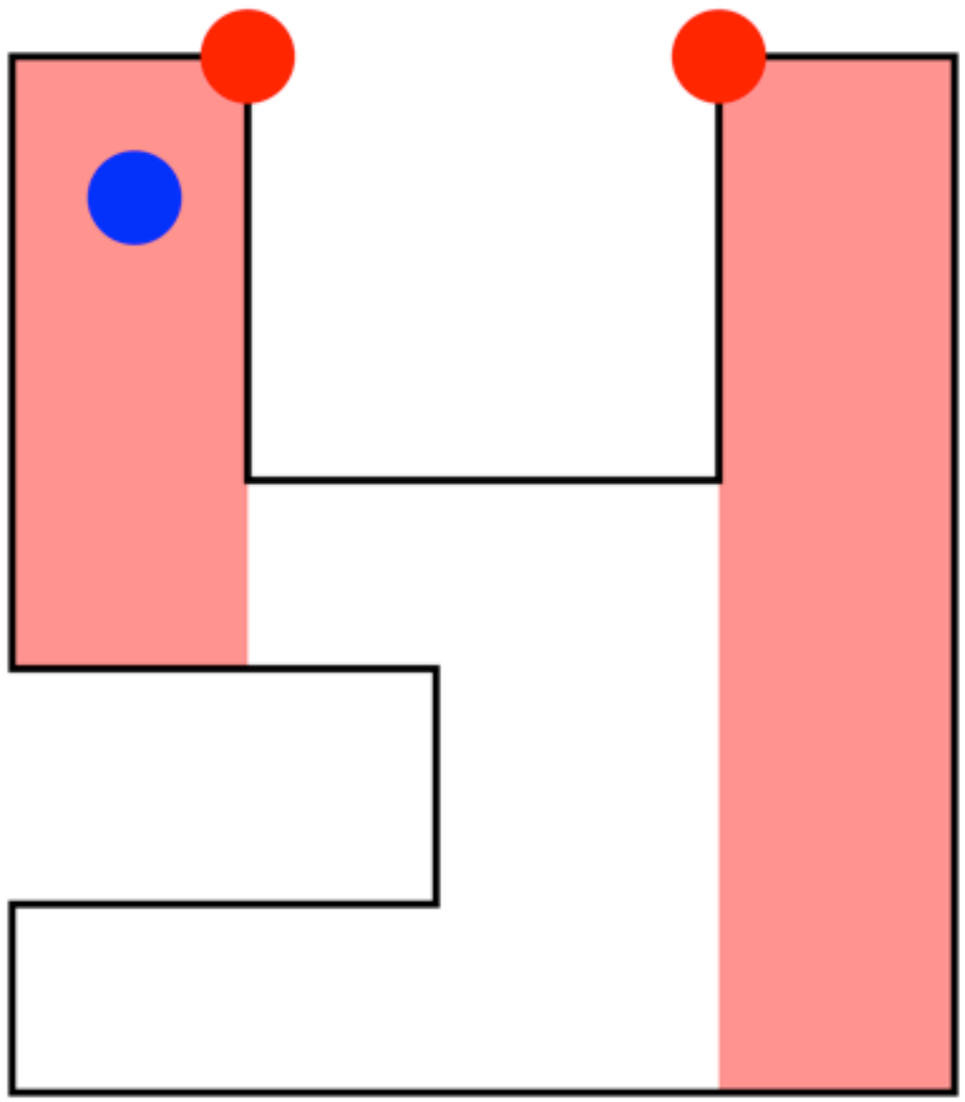

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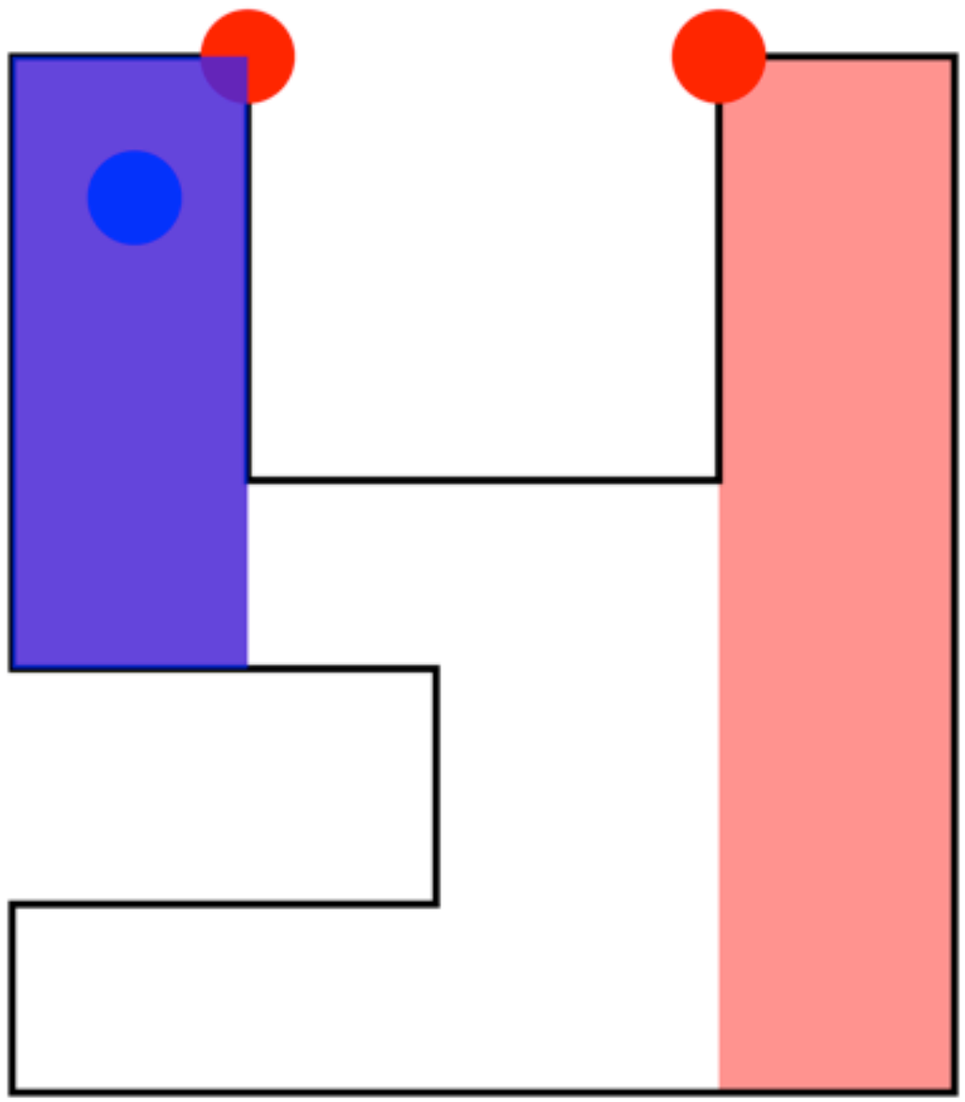

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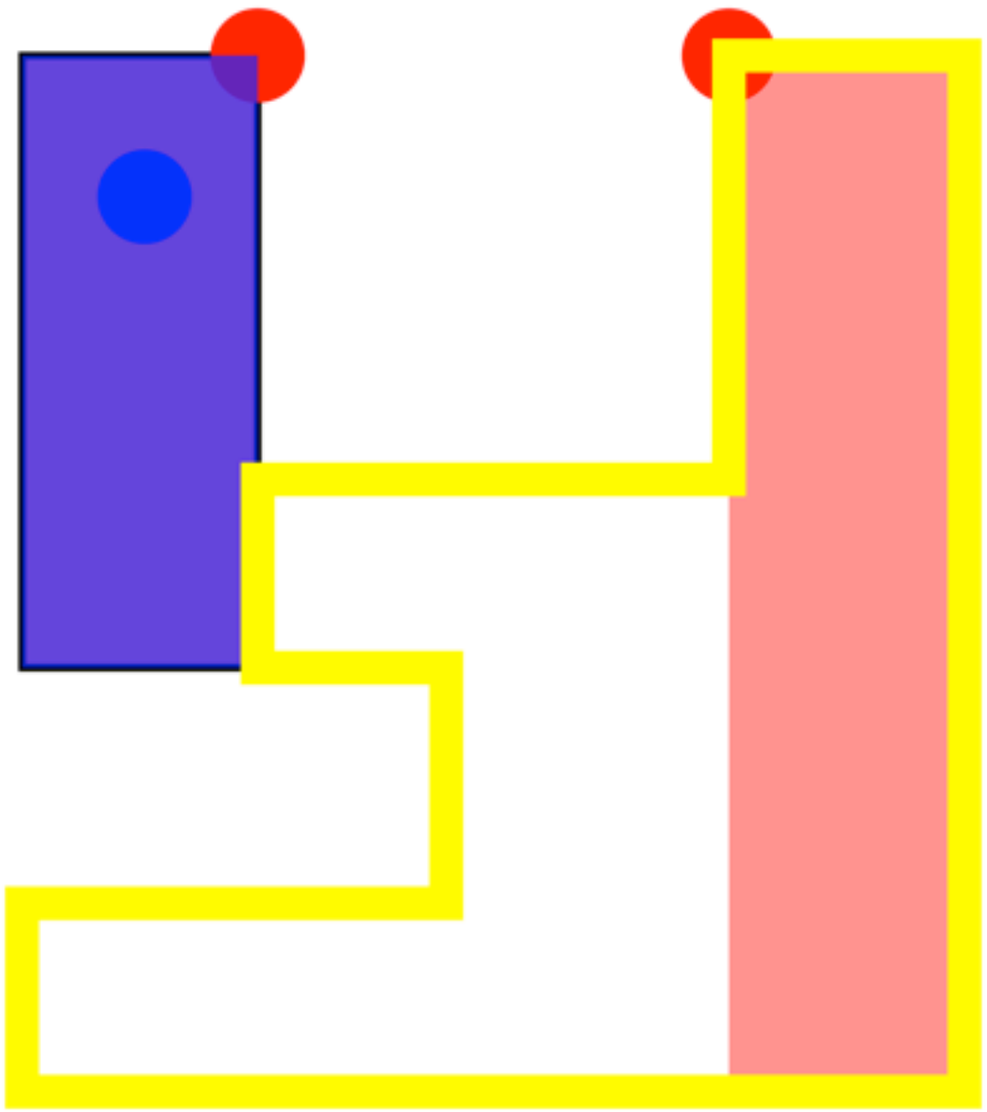

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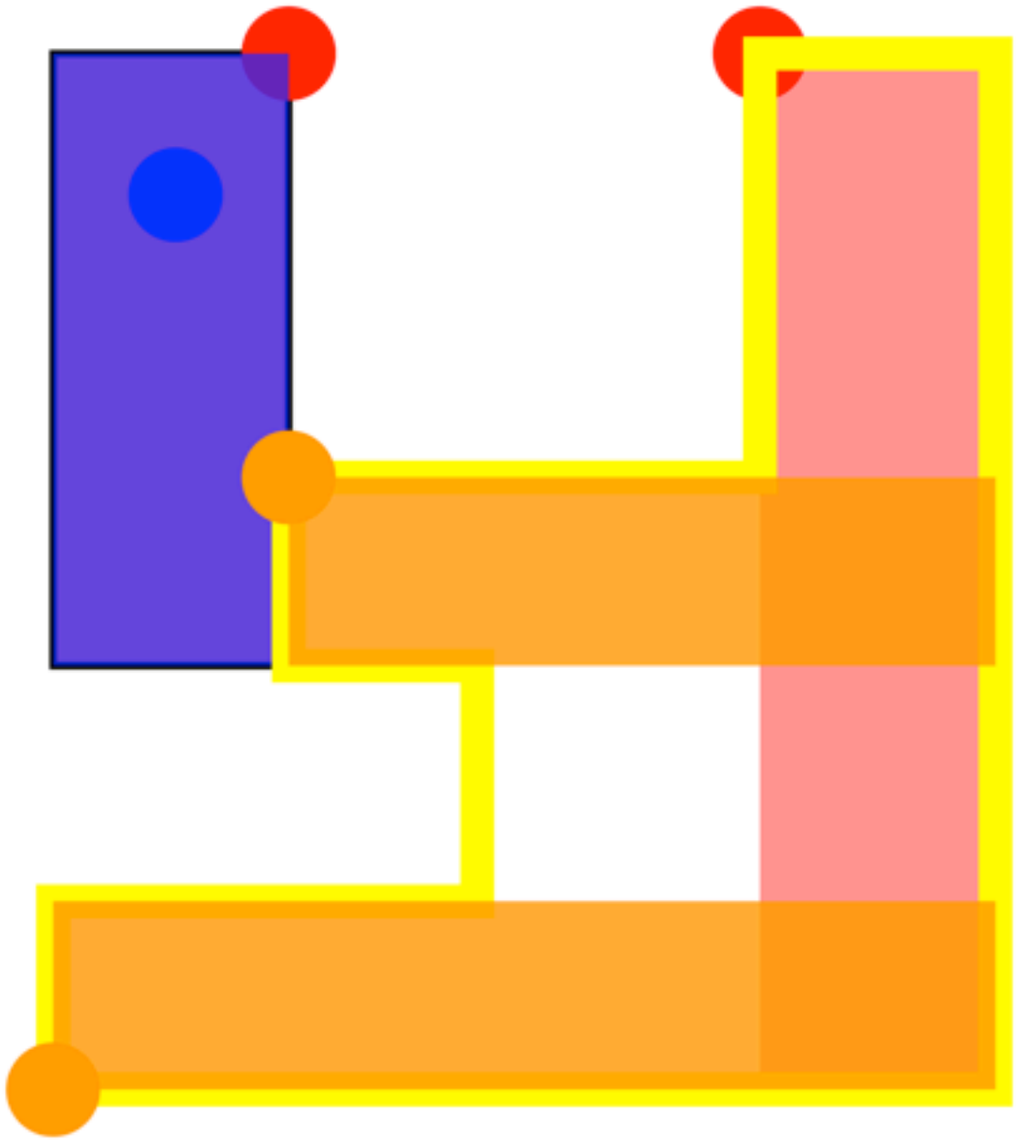

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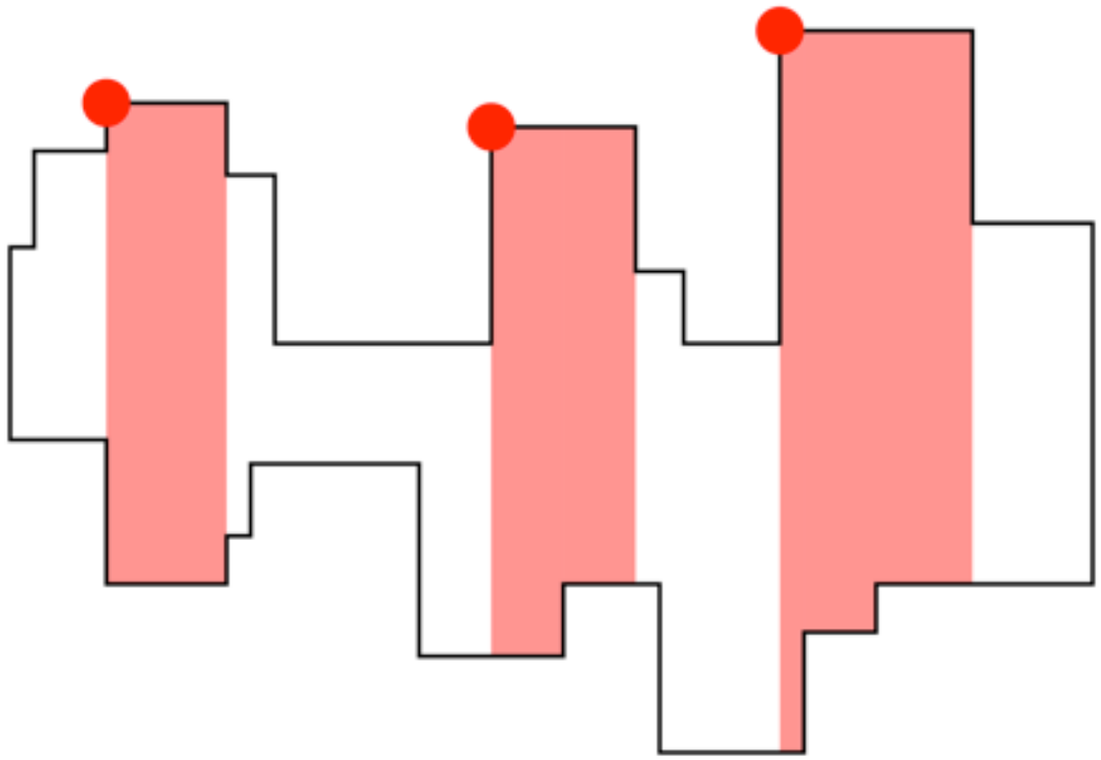

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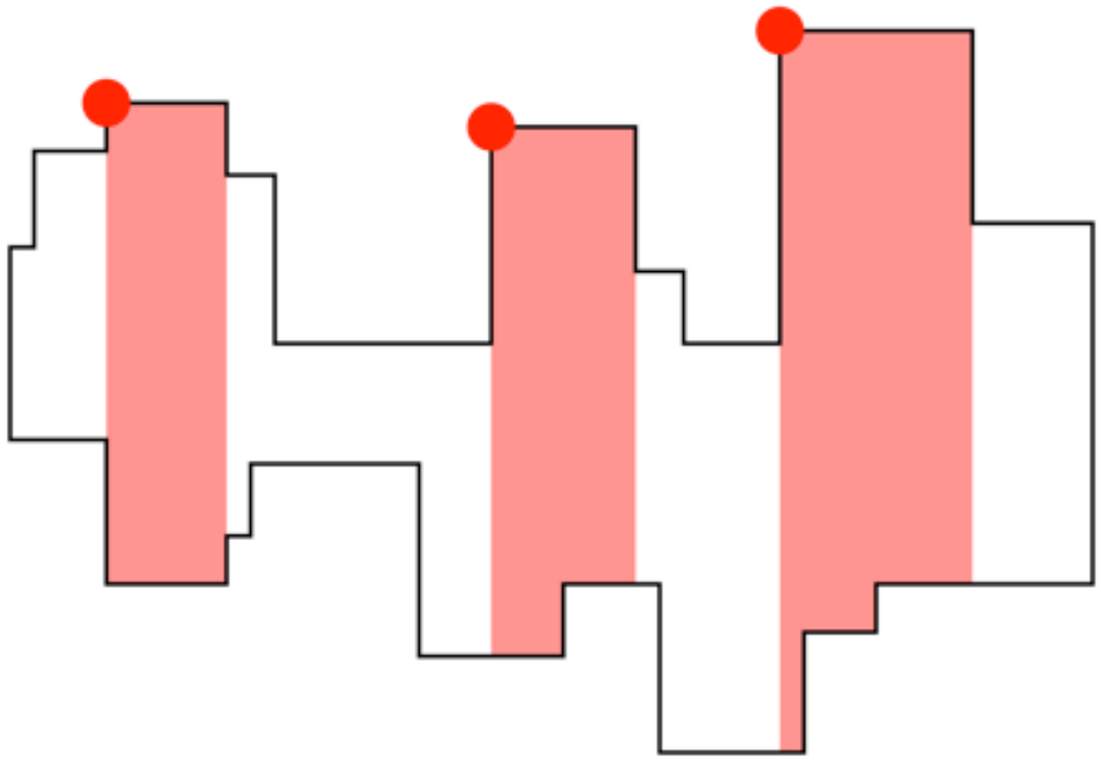

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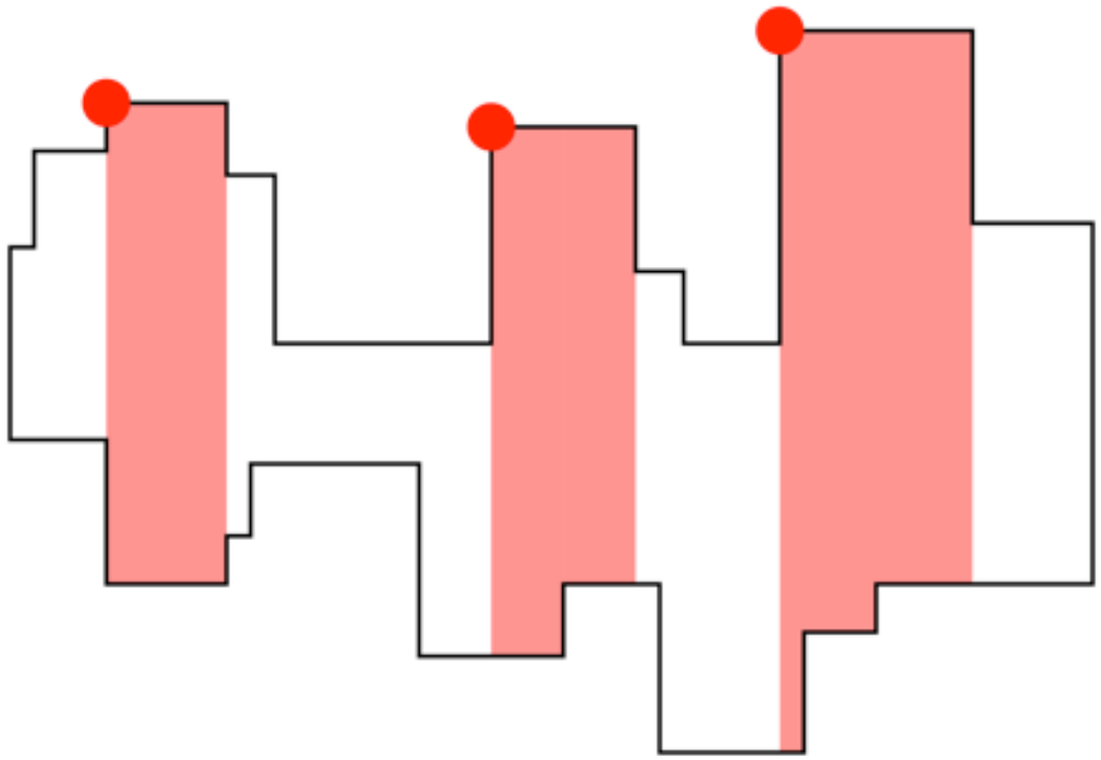

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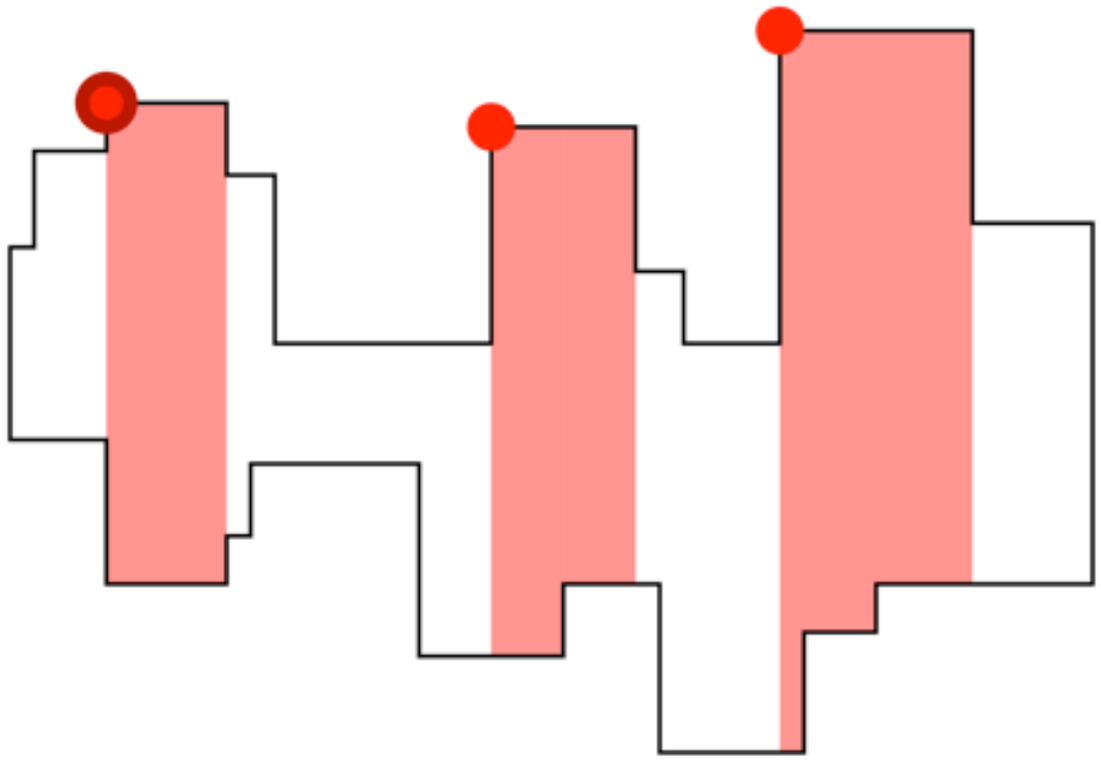

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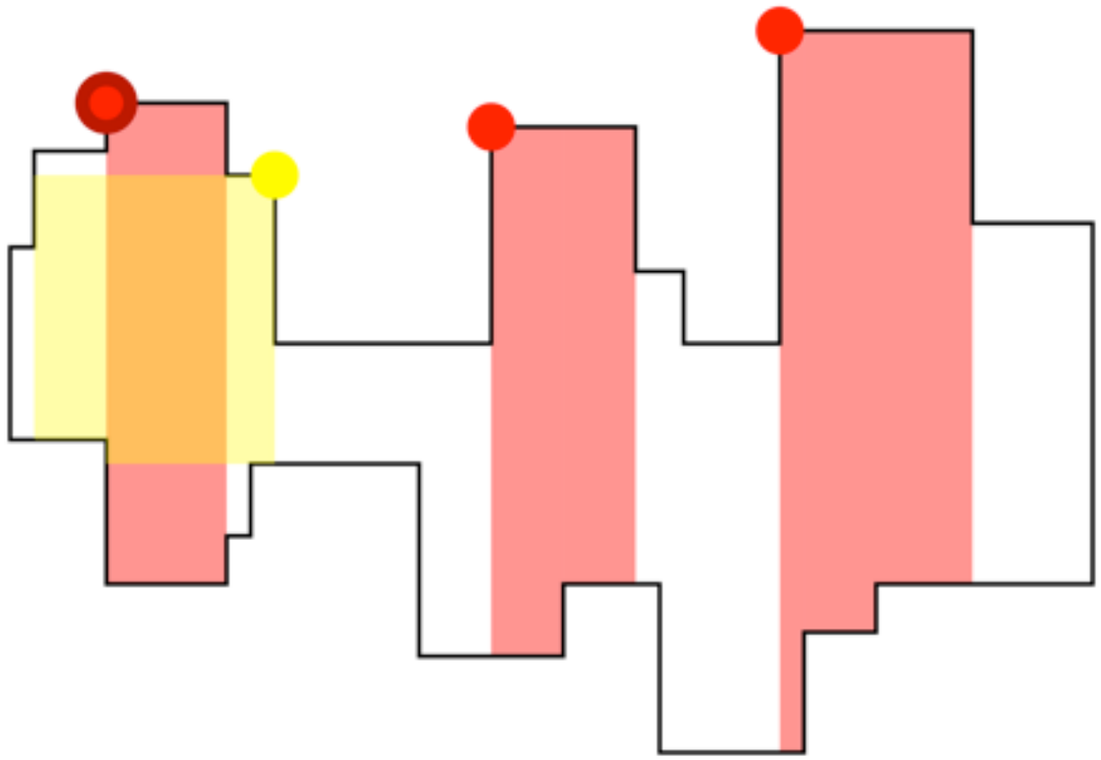

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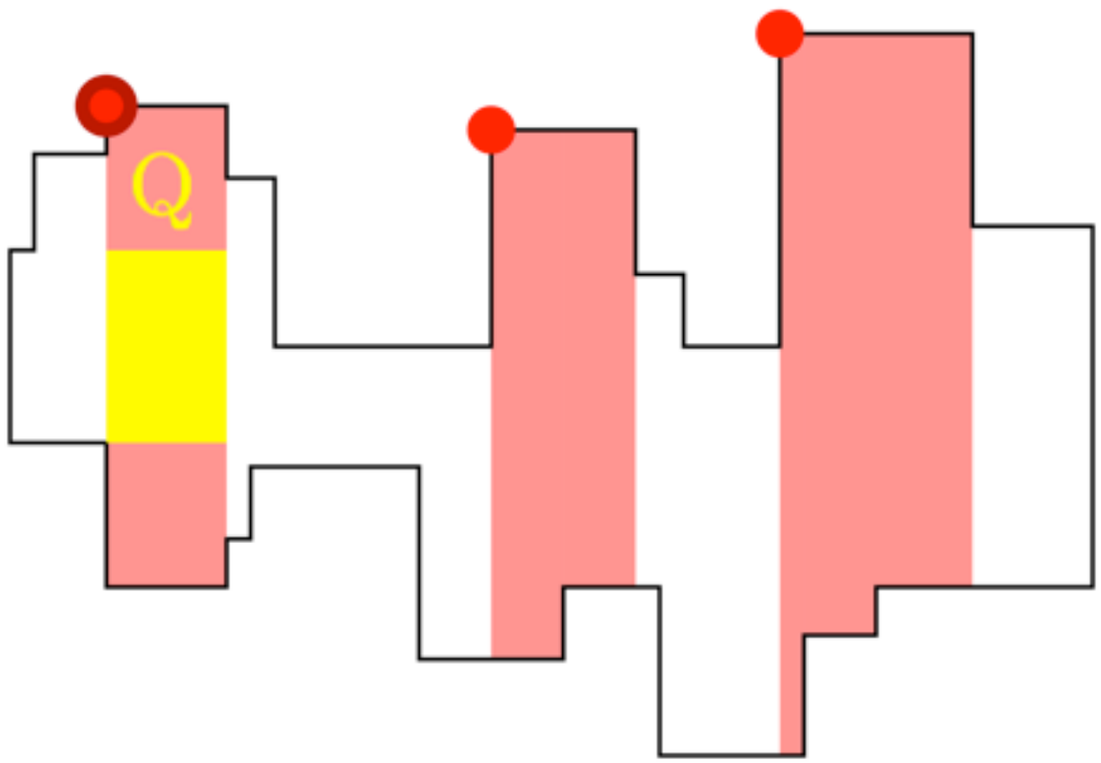

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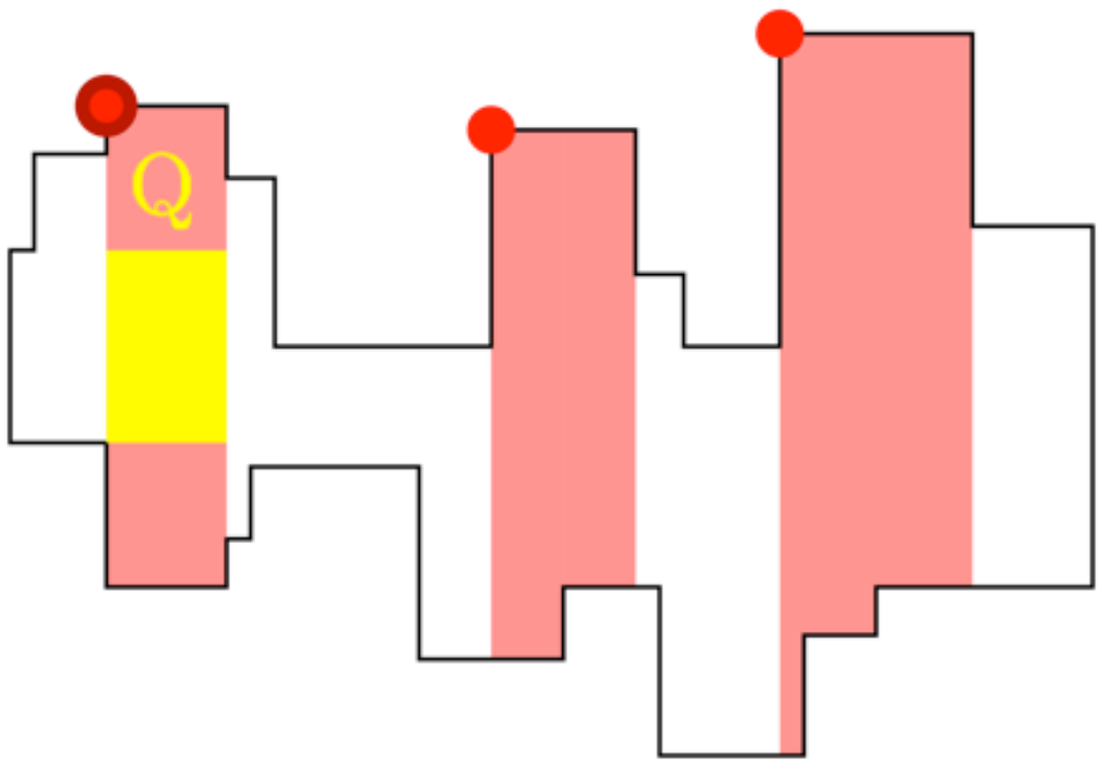

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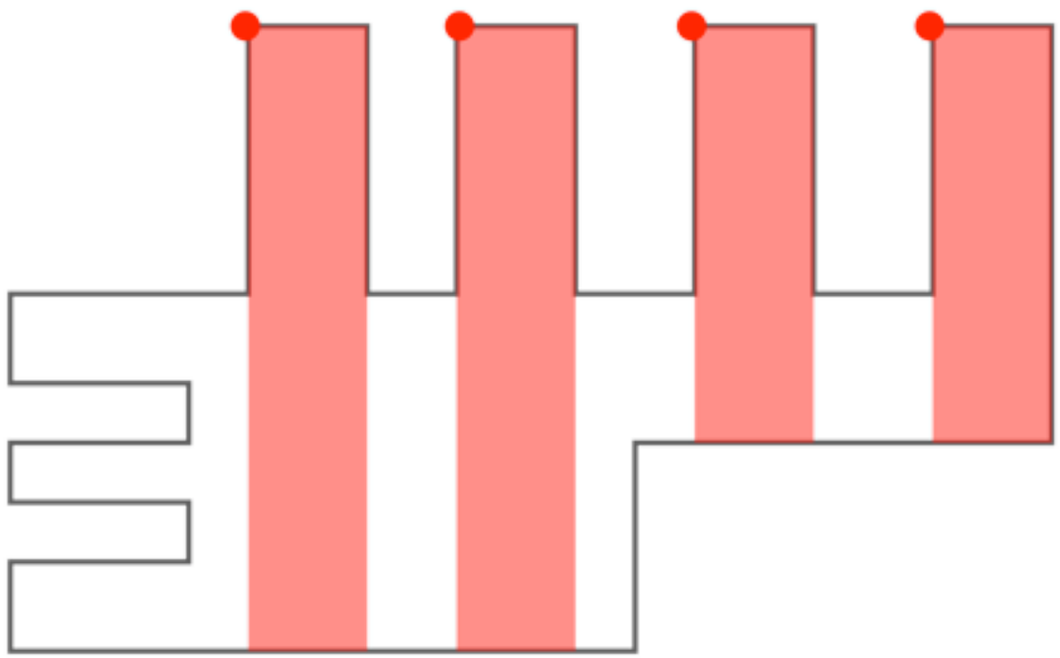

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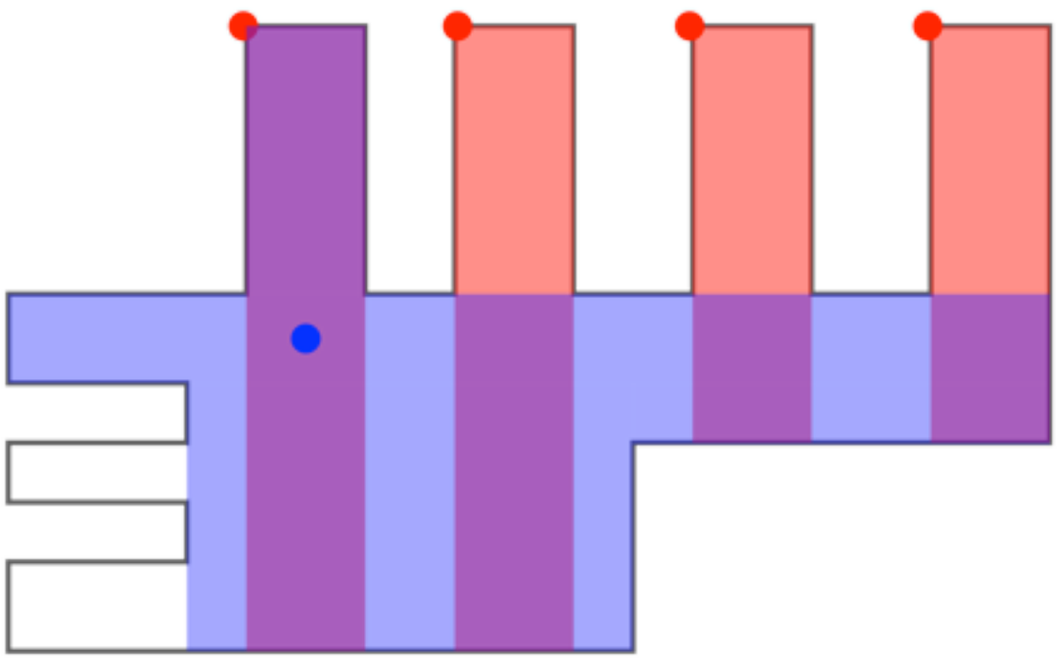

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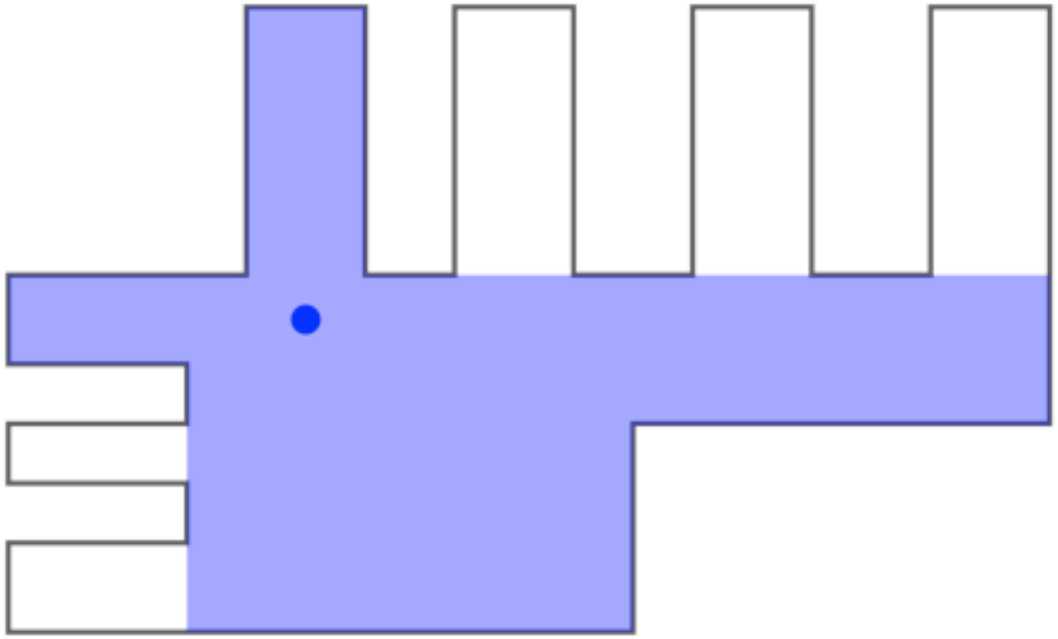

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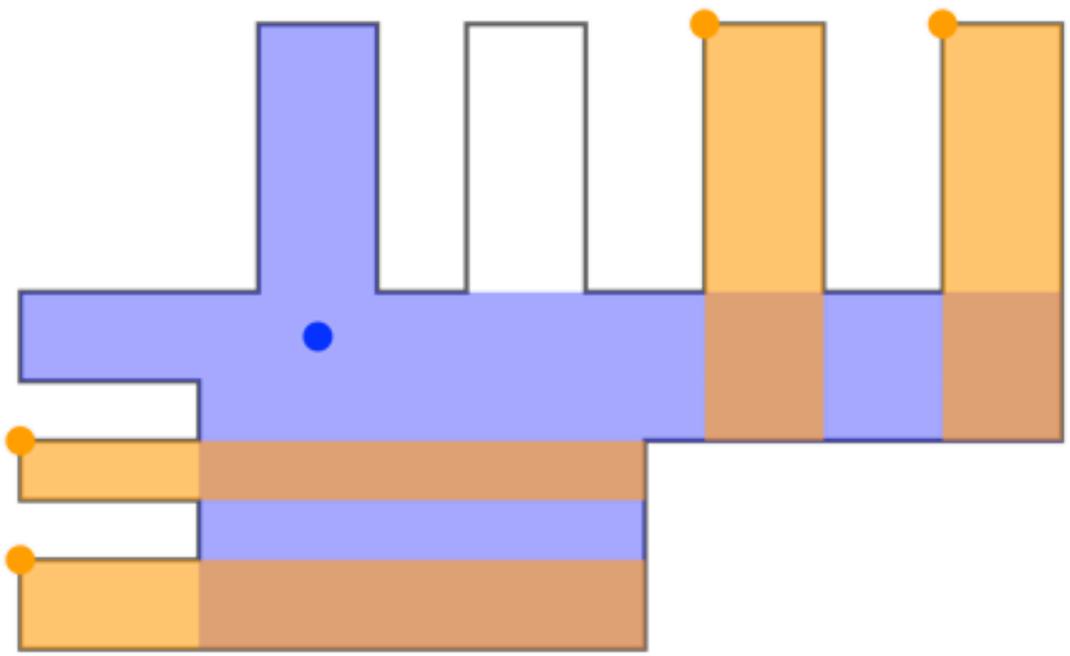

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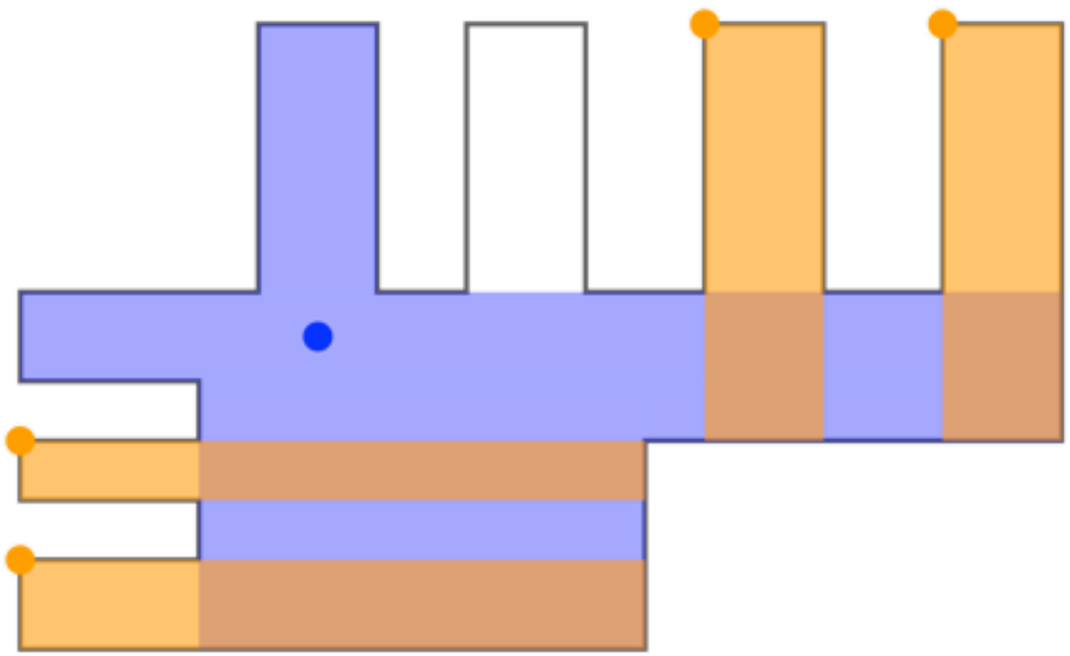

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Intuition: we need to match witness VPs from both CC_k and $P' \setminus \{CC_k \cup VP(w)\}$

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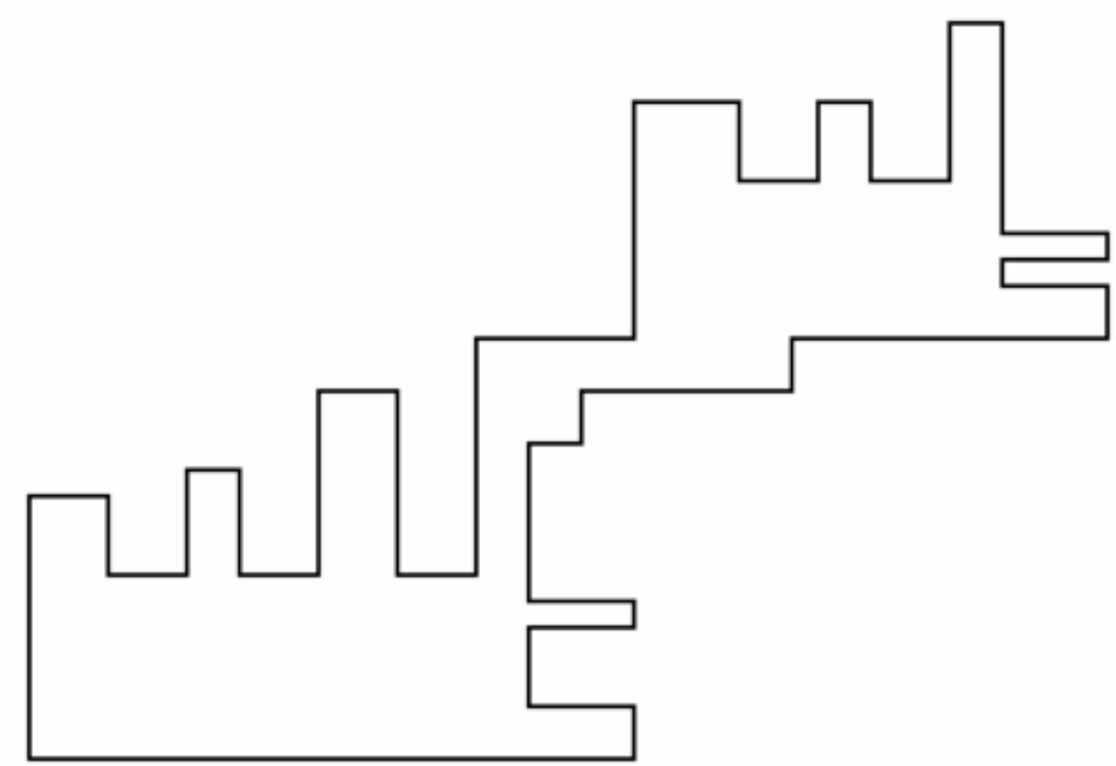

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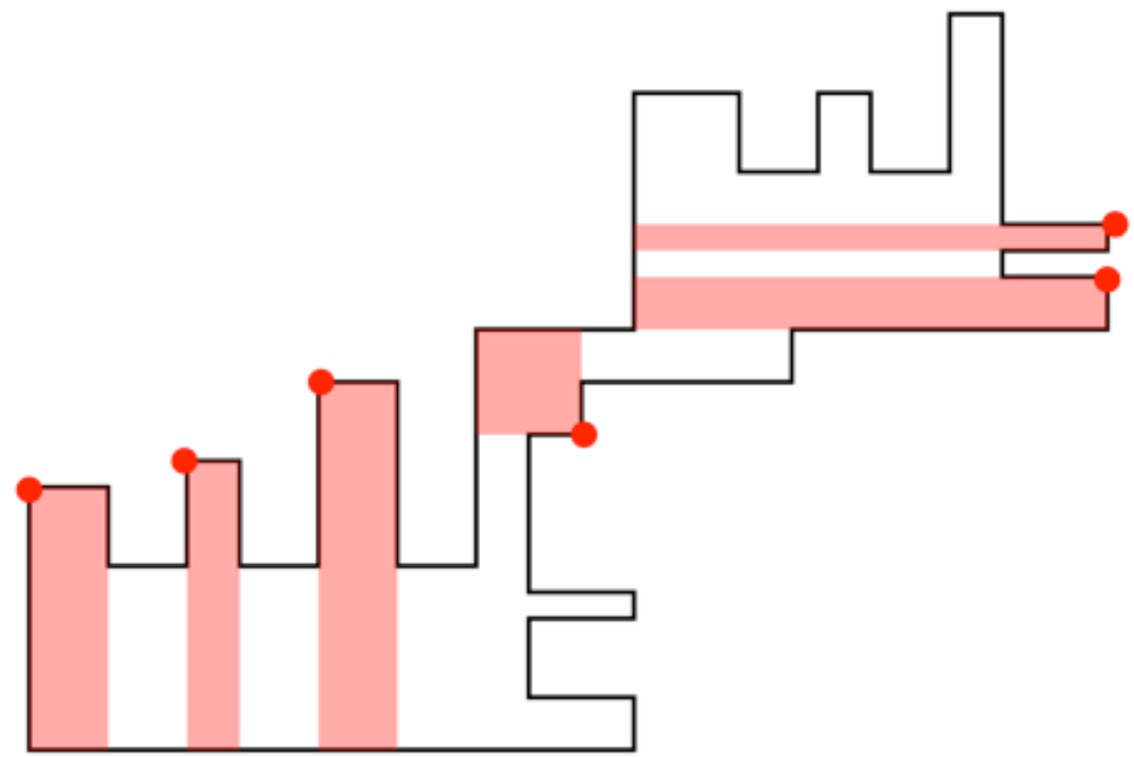

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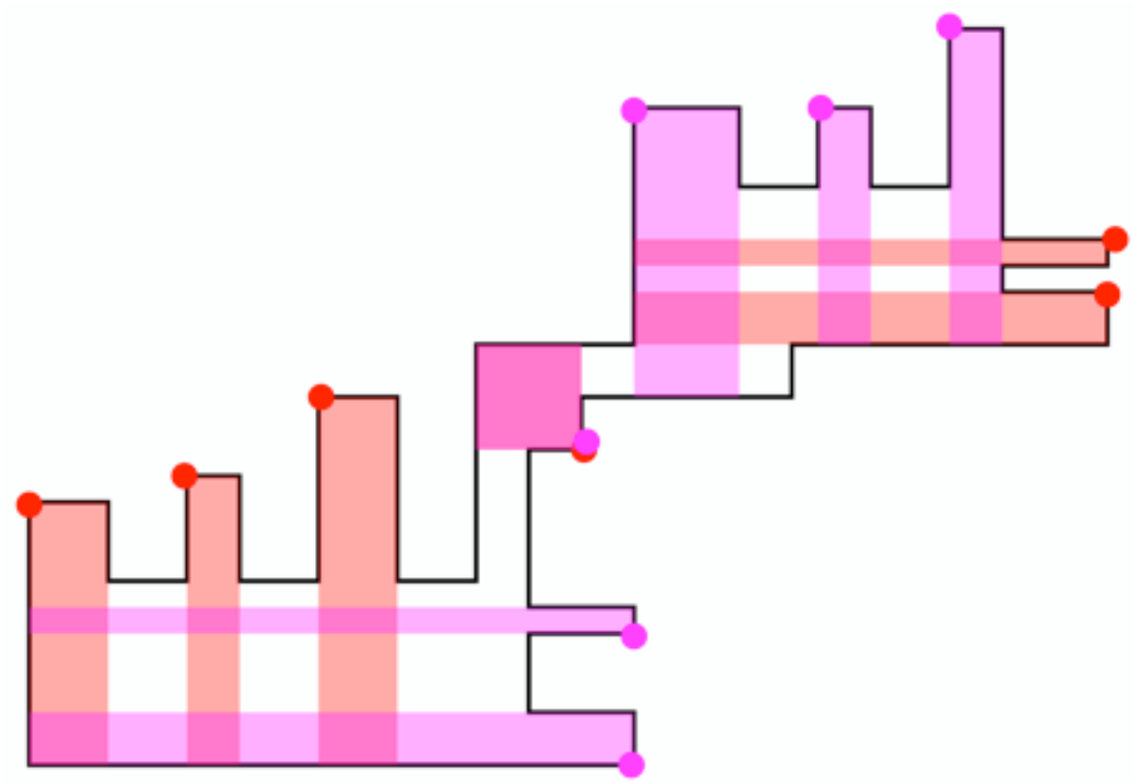

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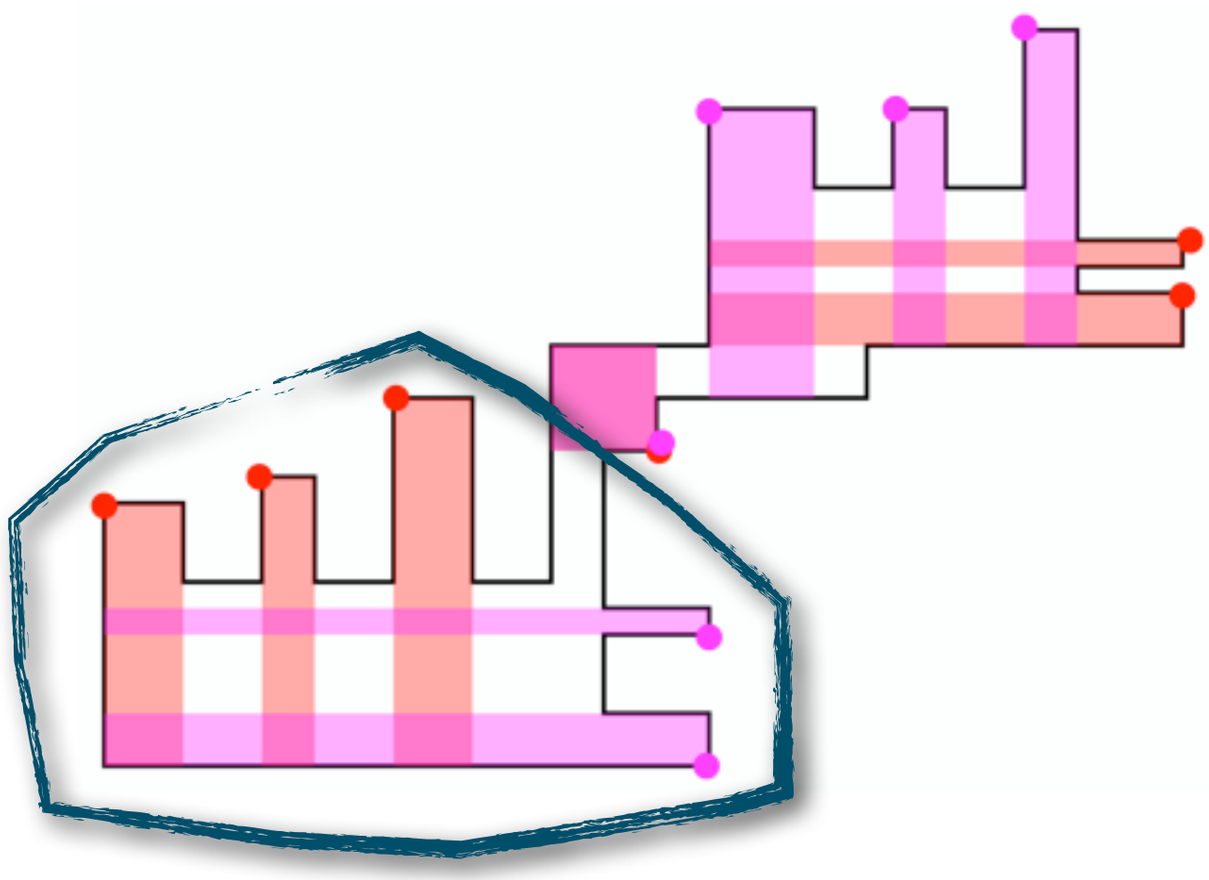

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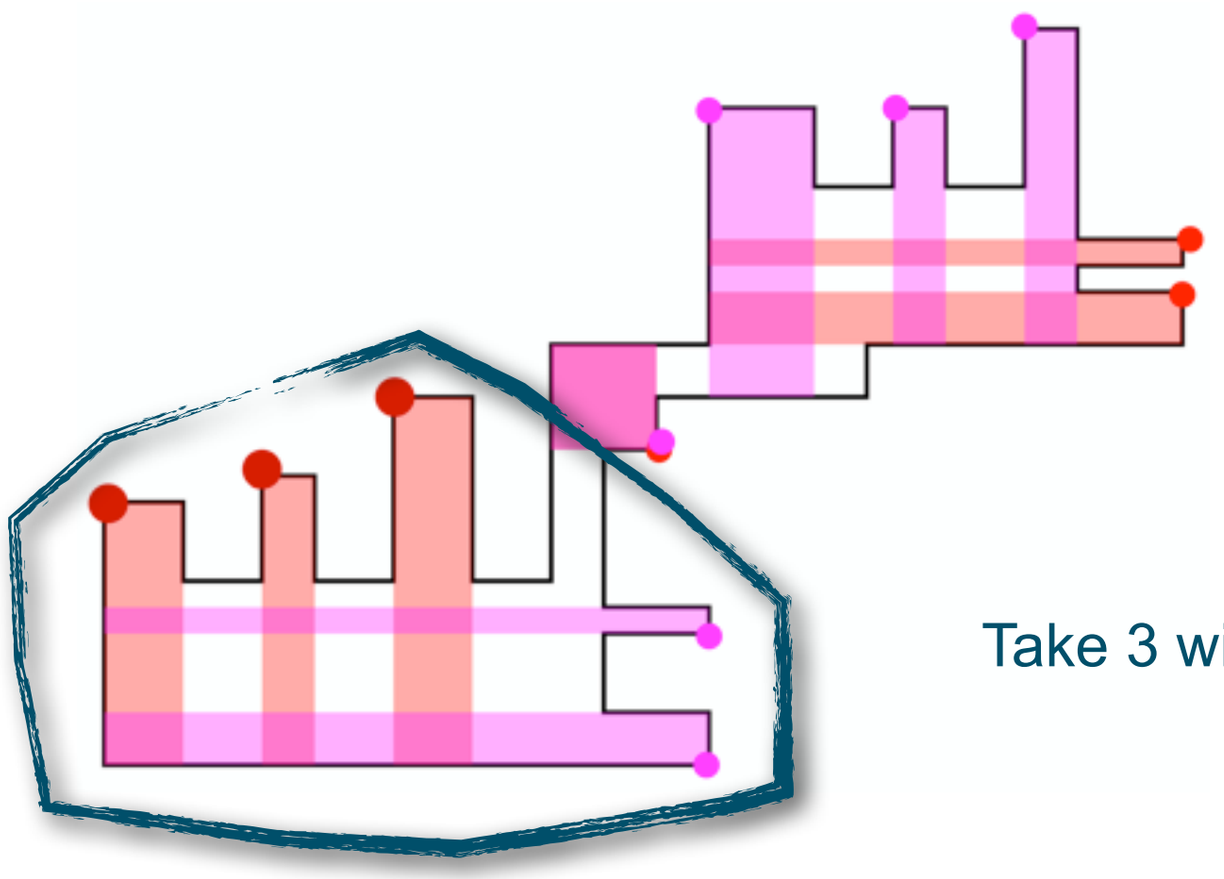

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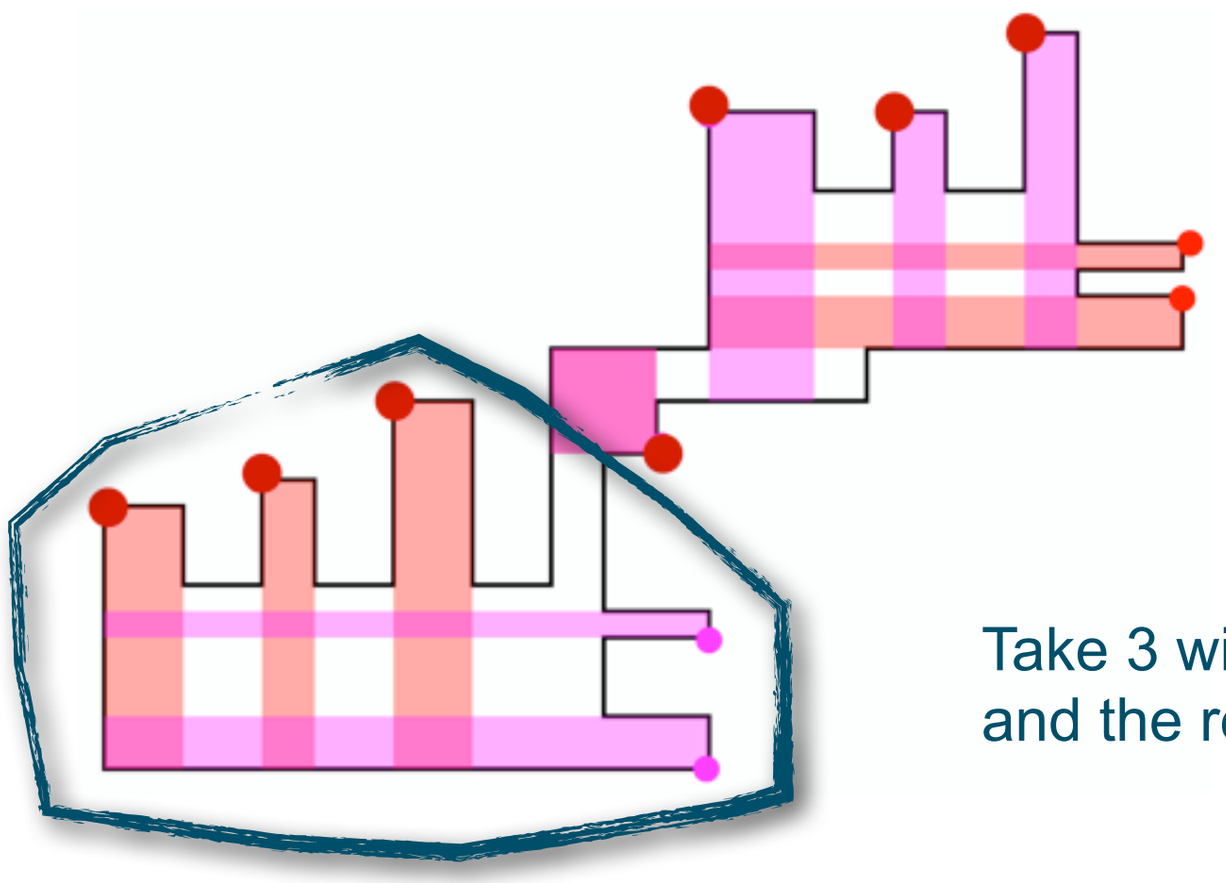

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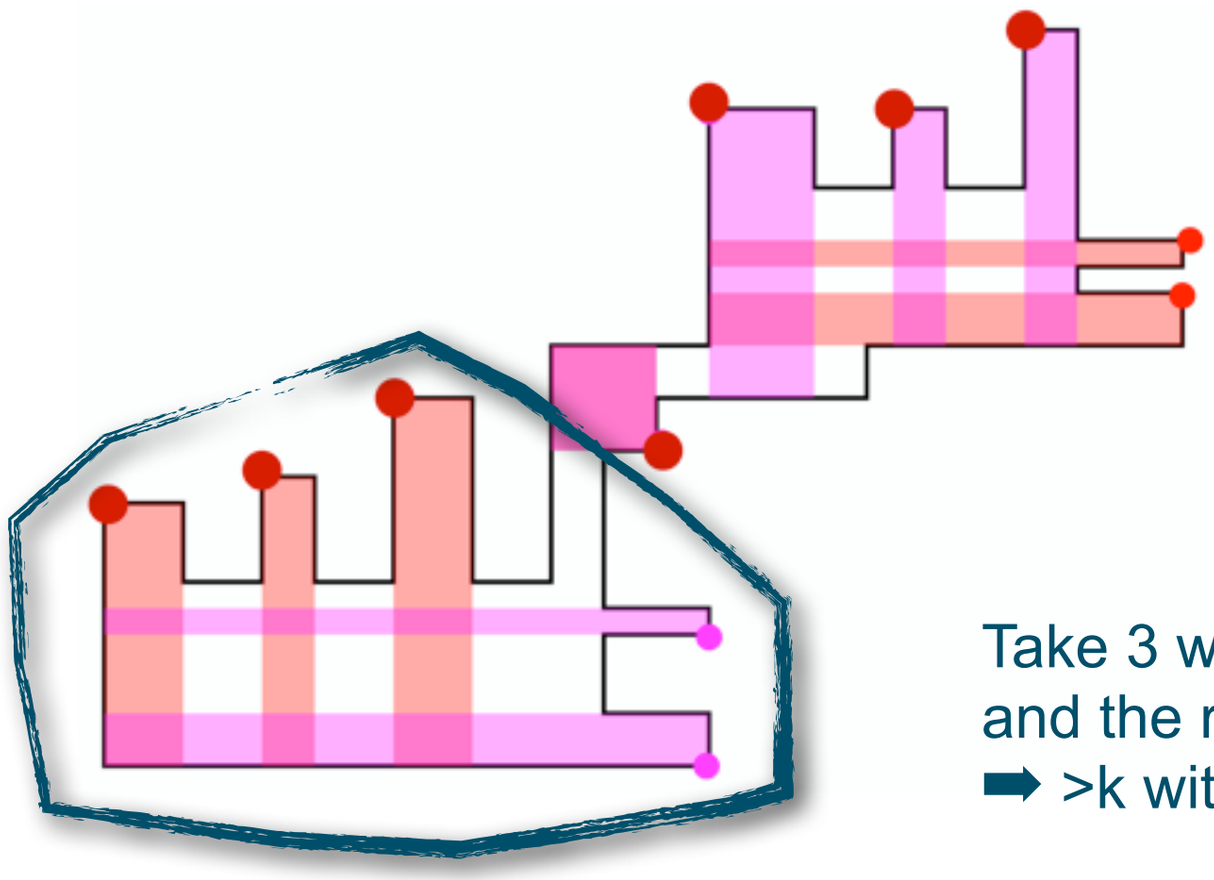

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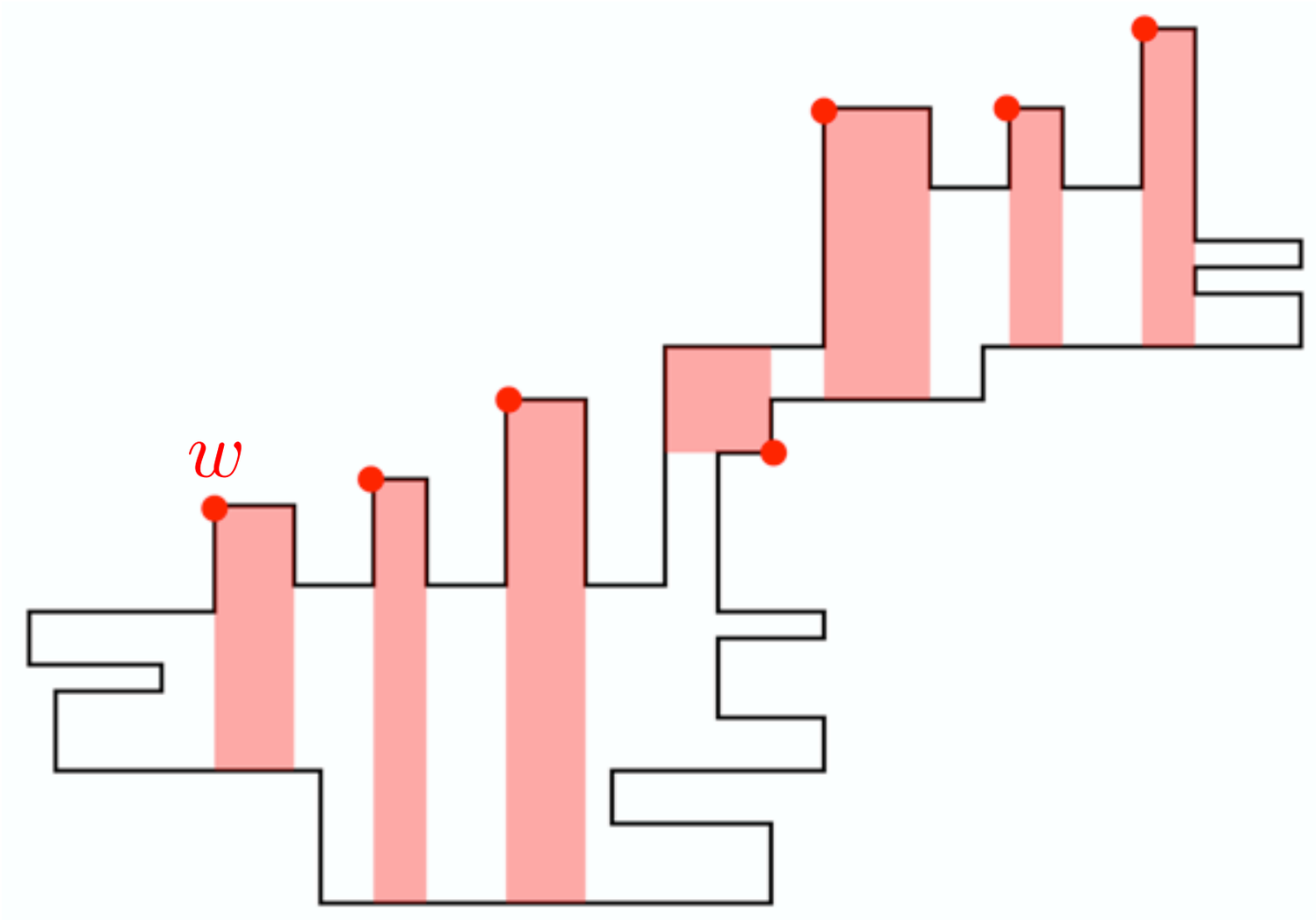

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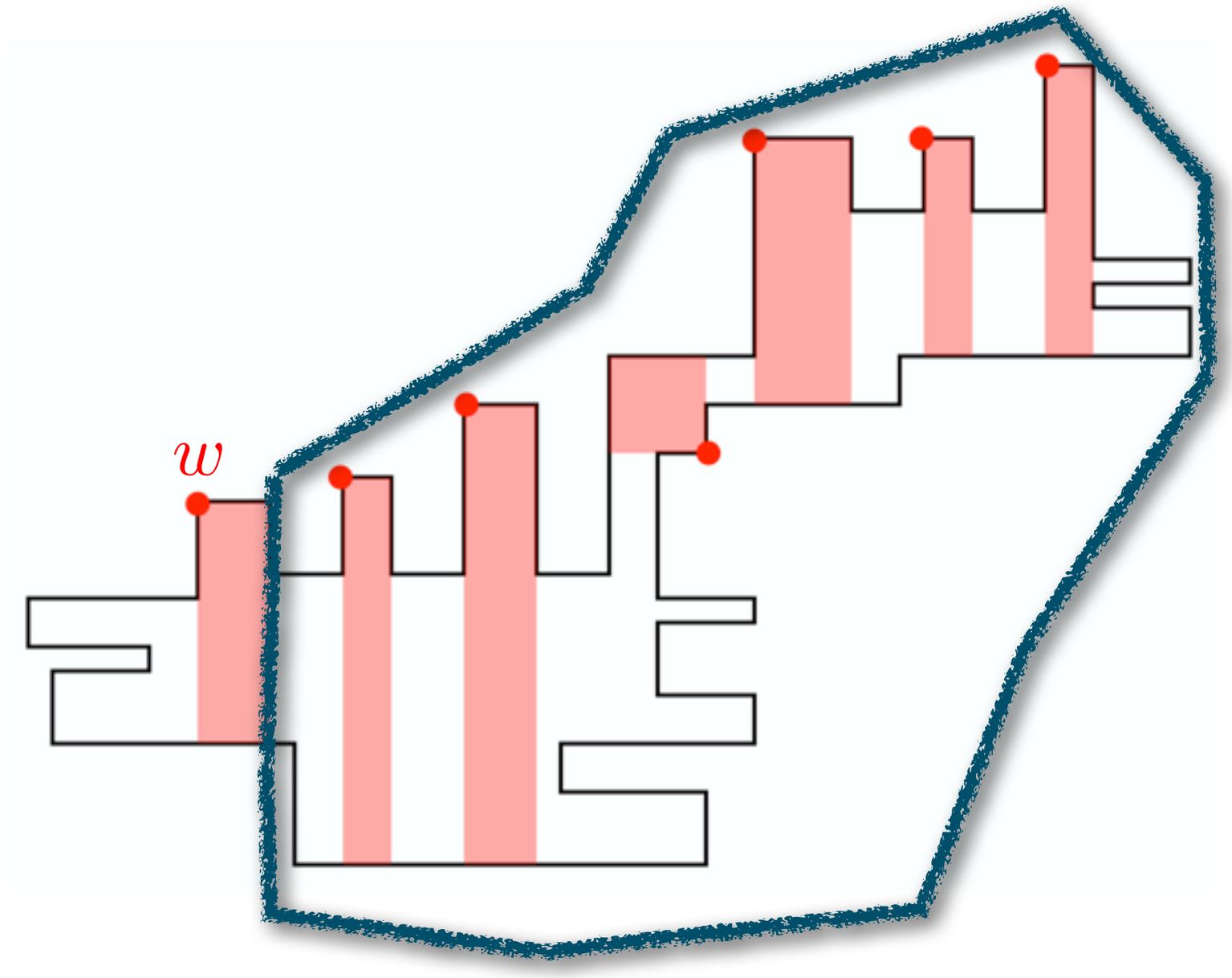

CC_k

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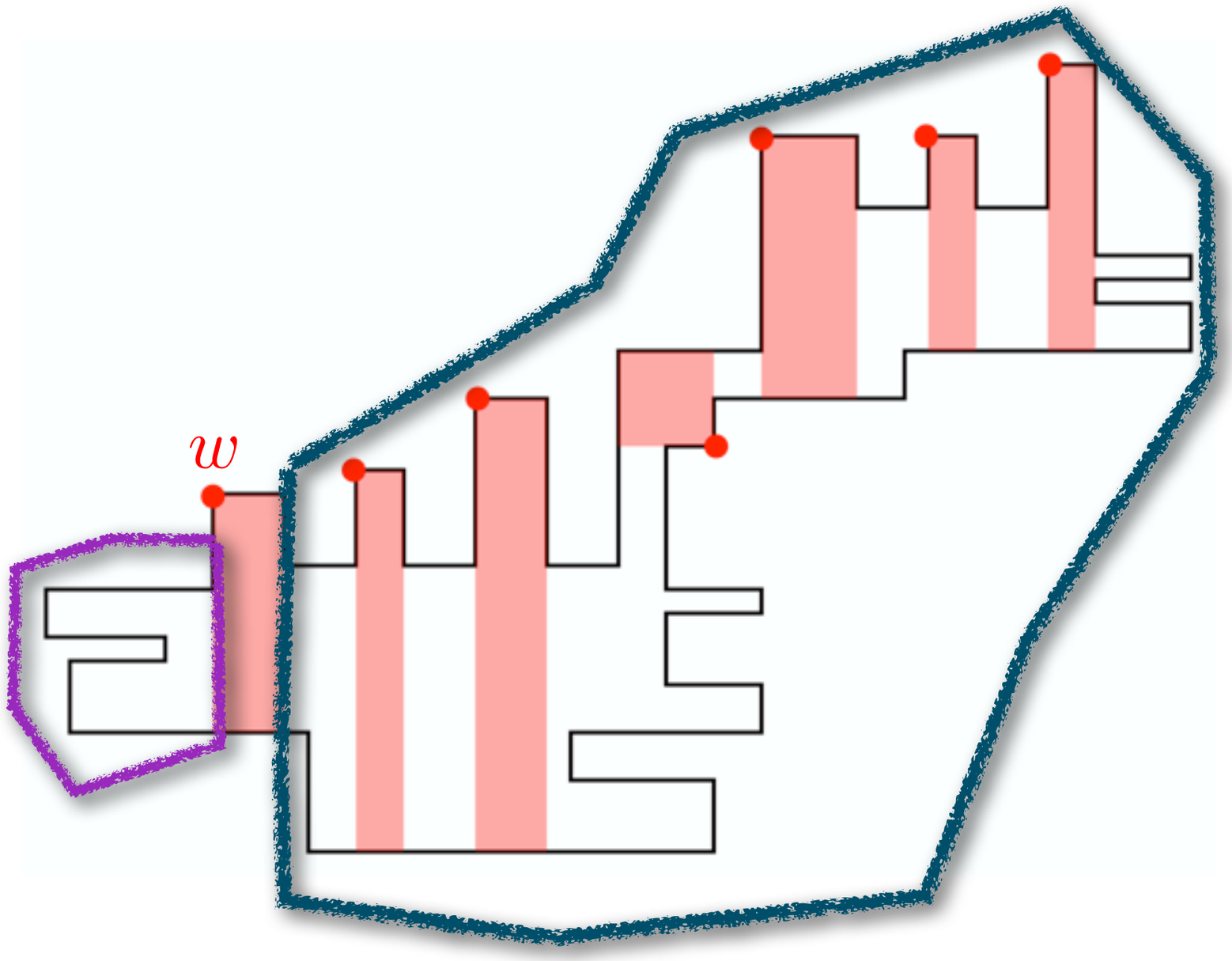
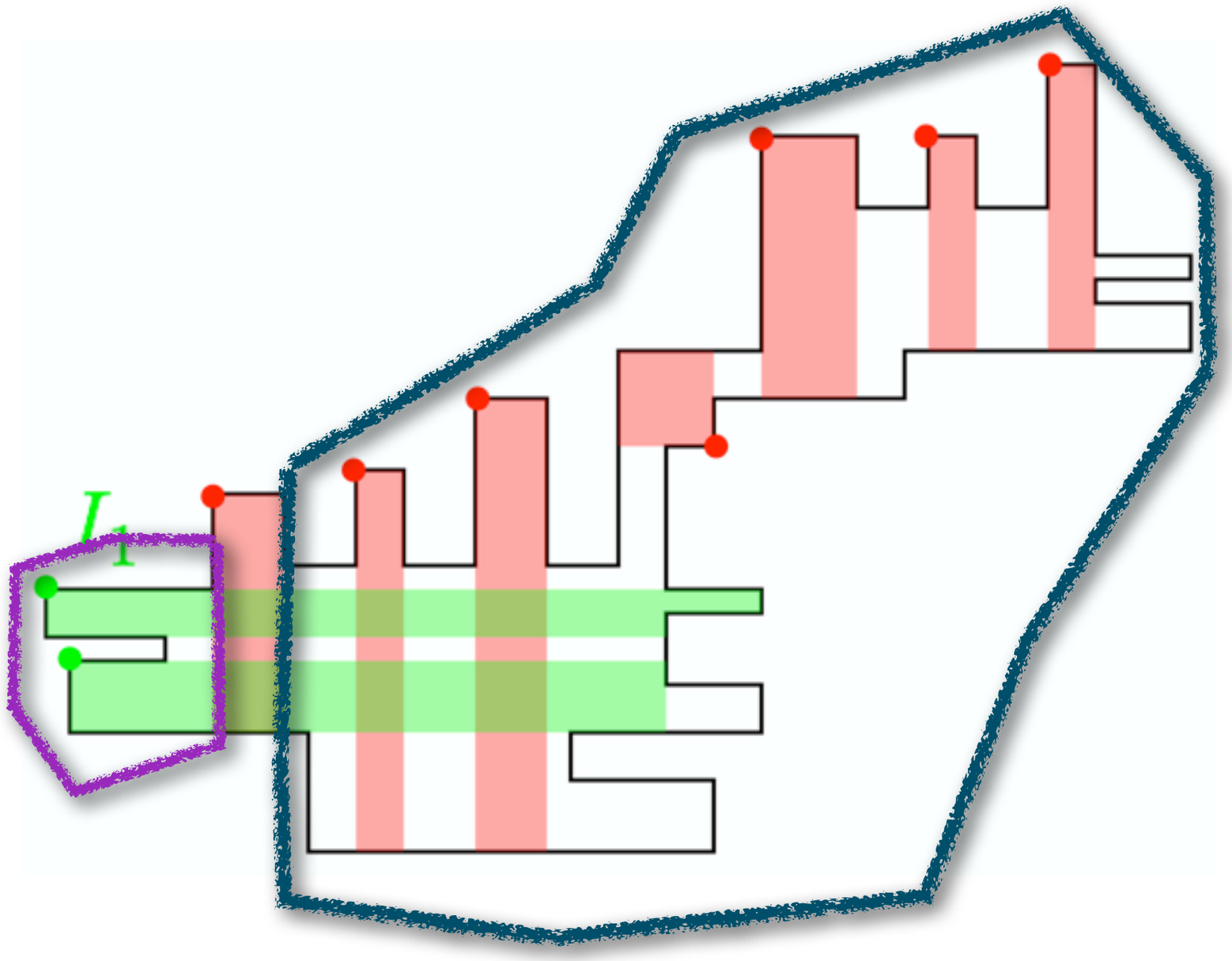


$P' \setminus \{CC_k \cup VP(w)\}$ $\qquad$ CC_k

Perfectness



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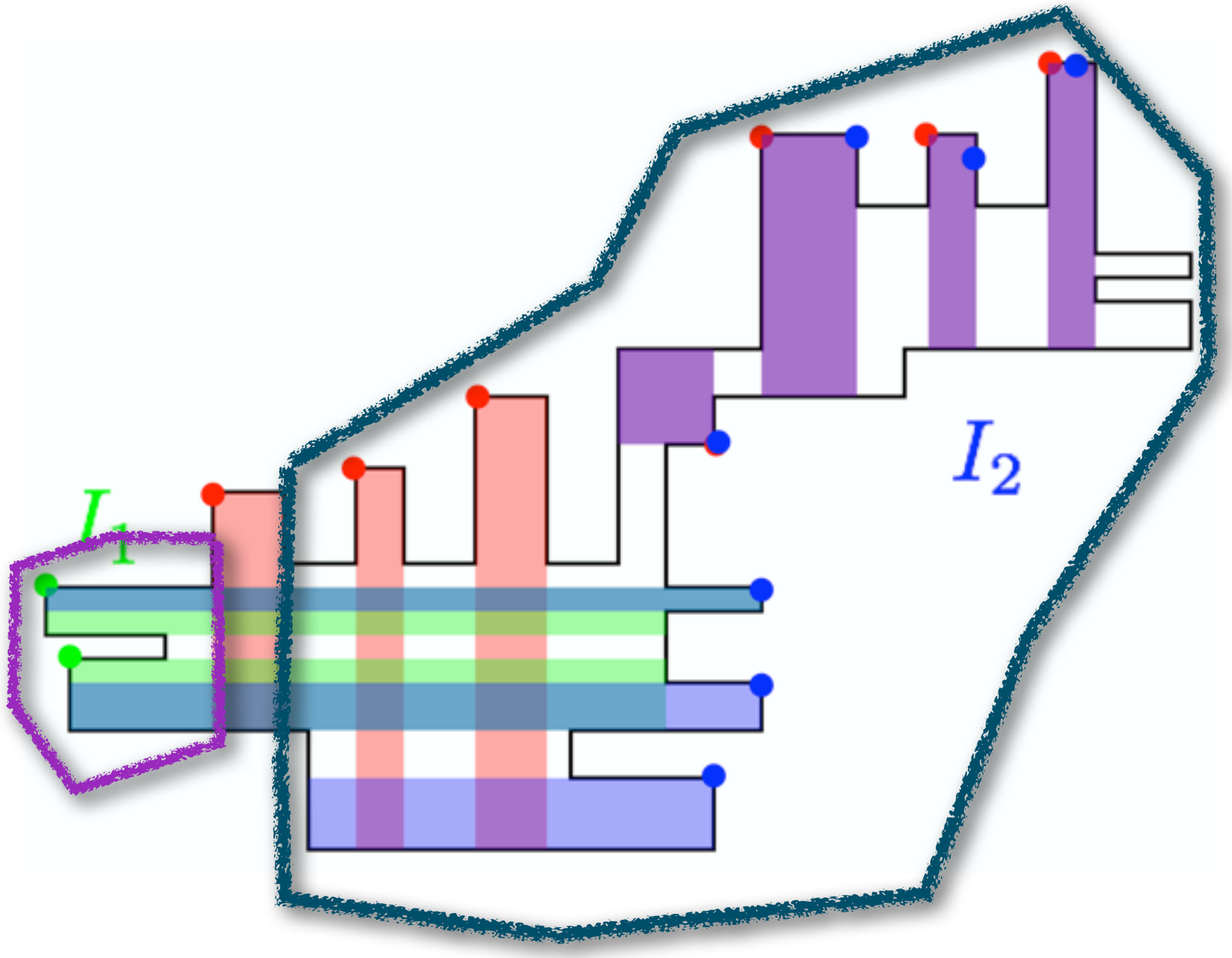

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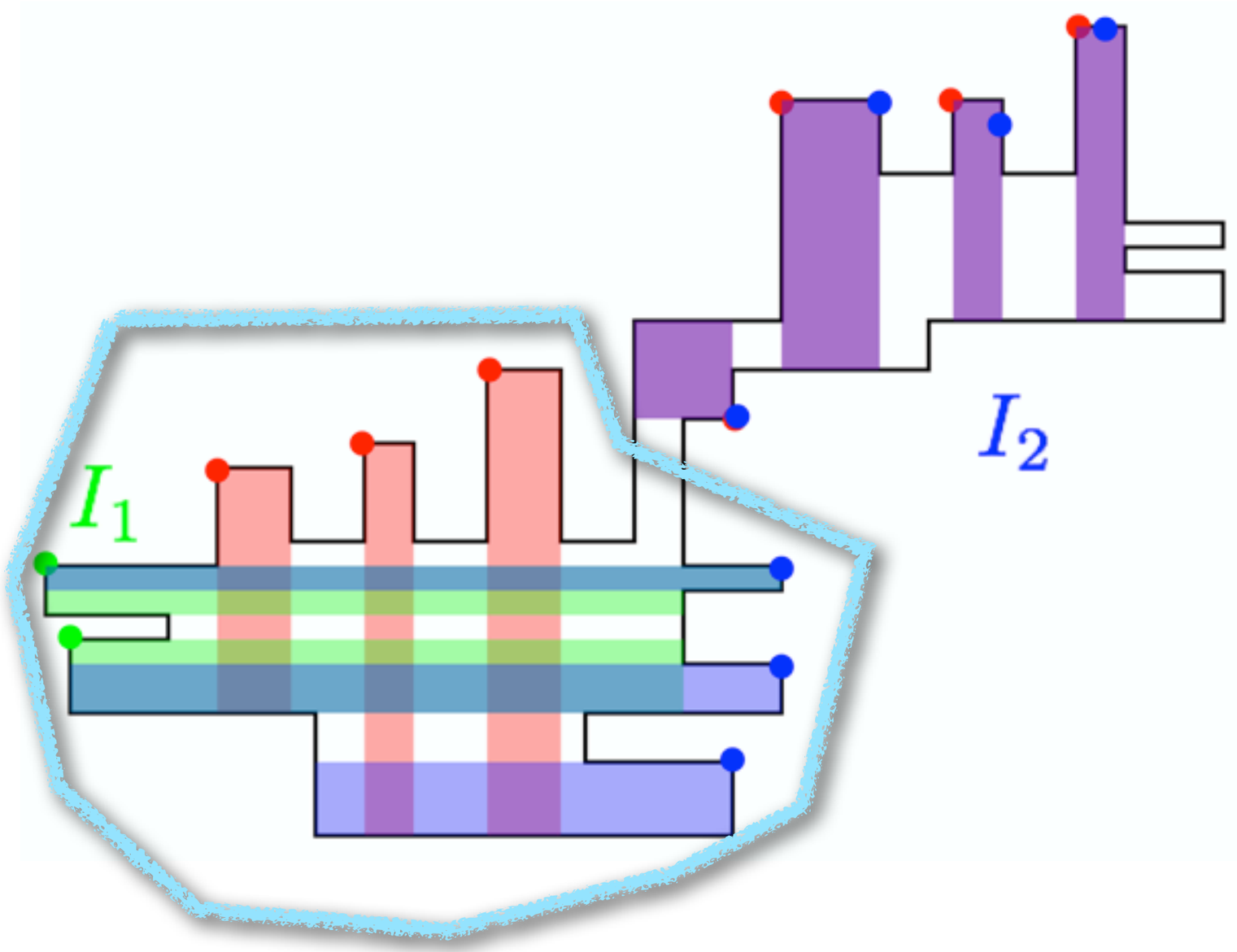

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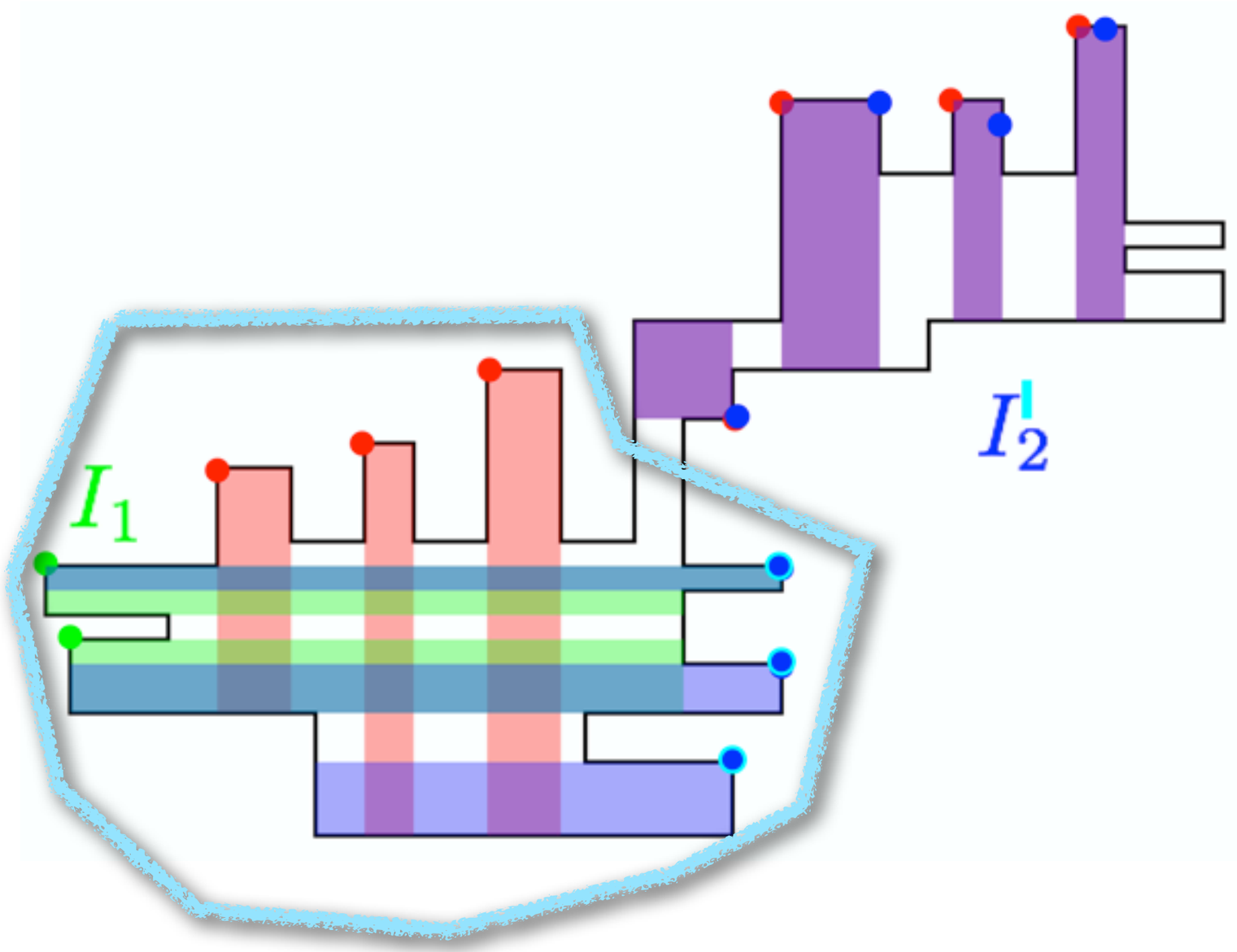

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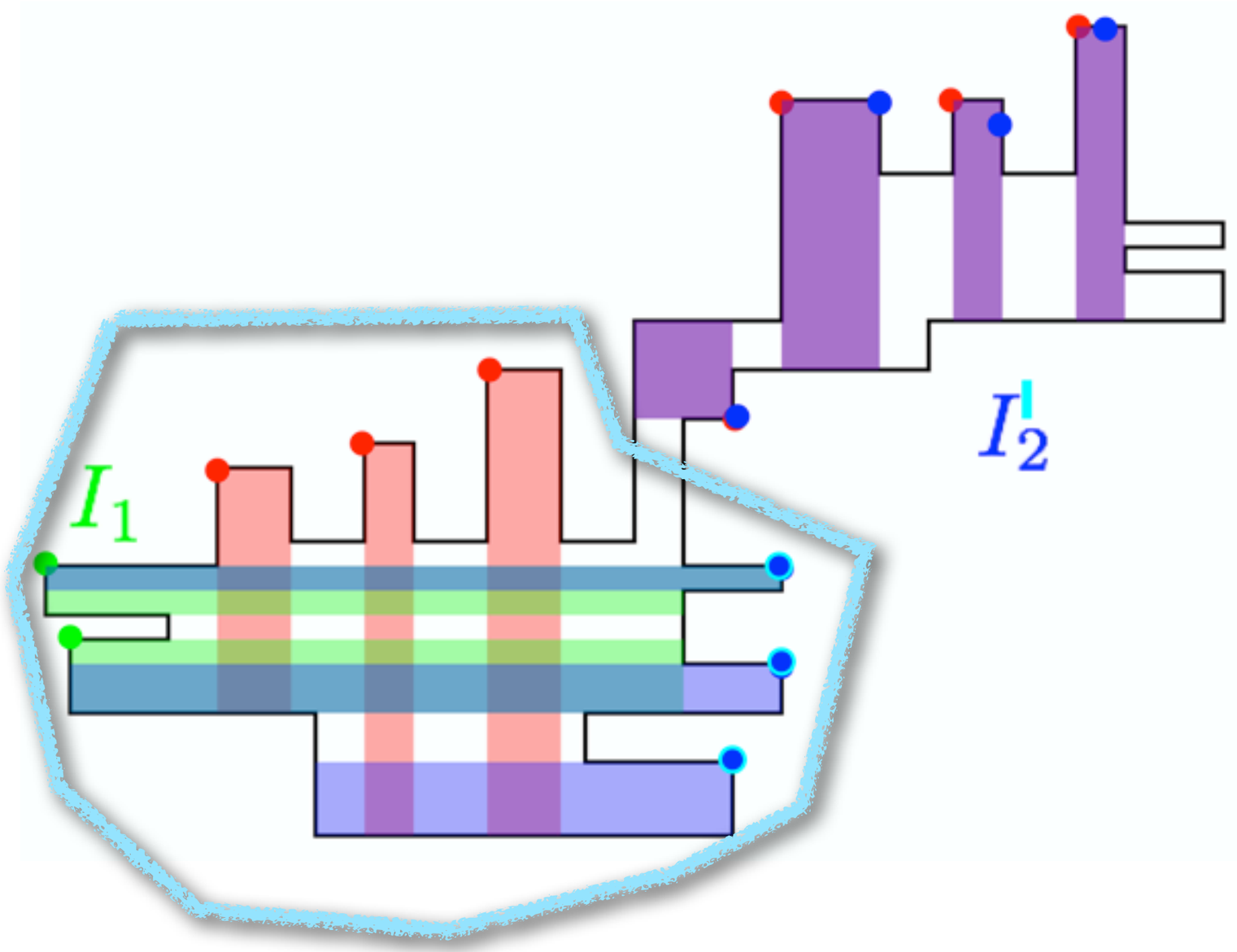
For any IS X of visibility polygons in $I_1 \cup I_2'$:



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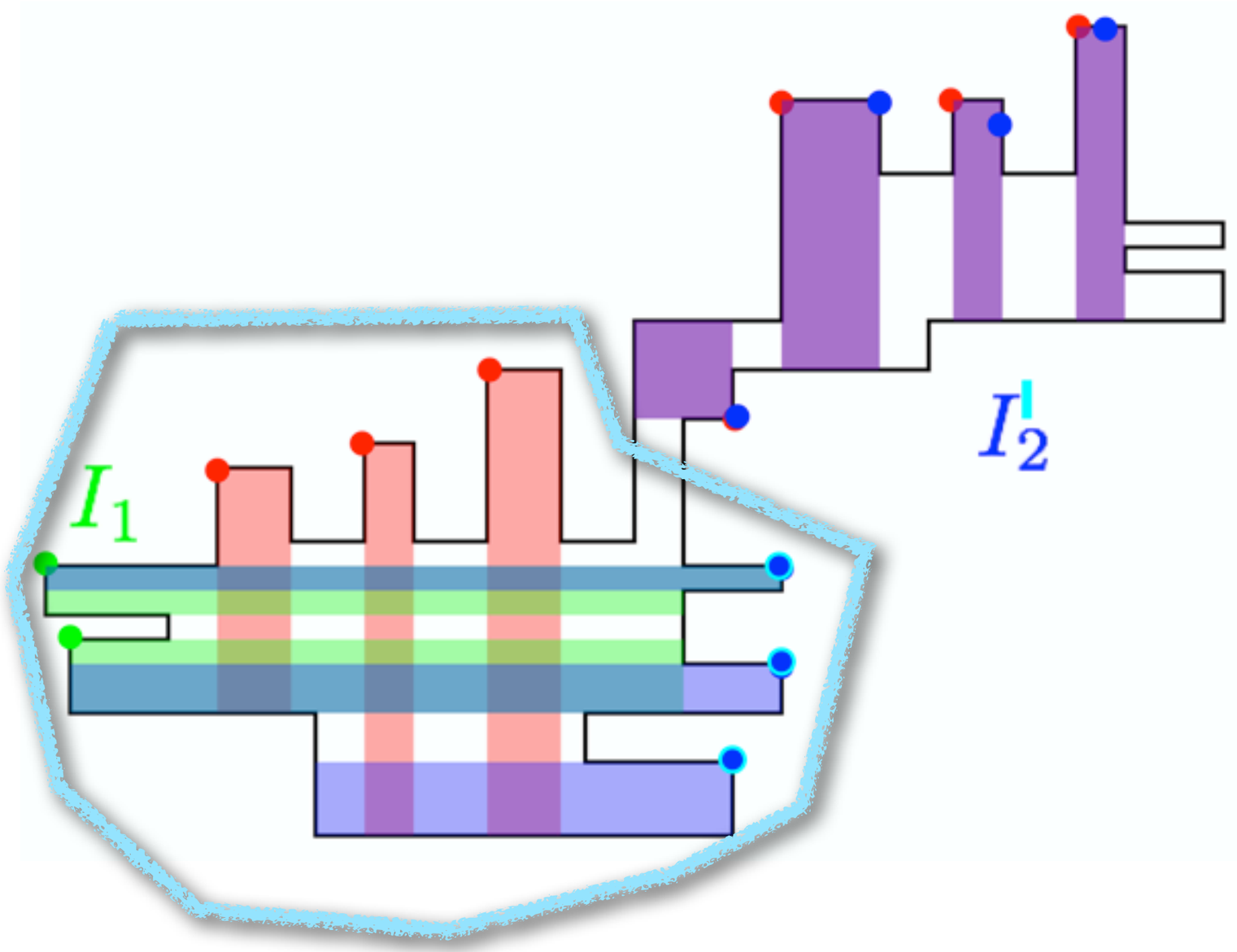

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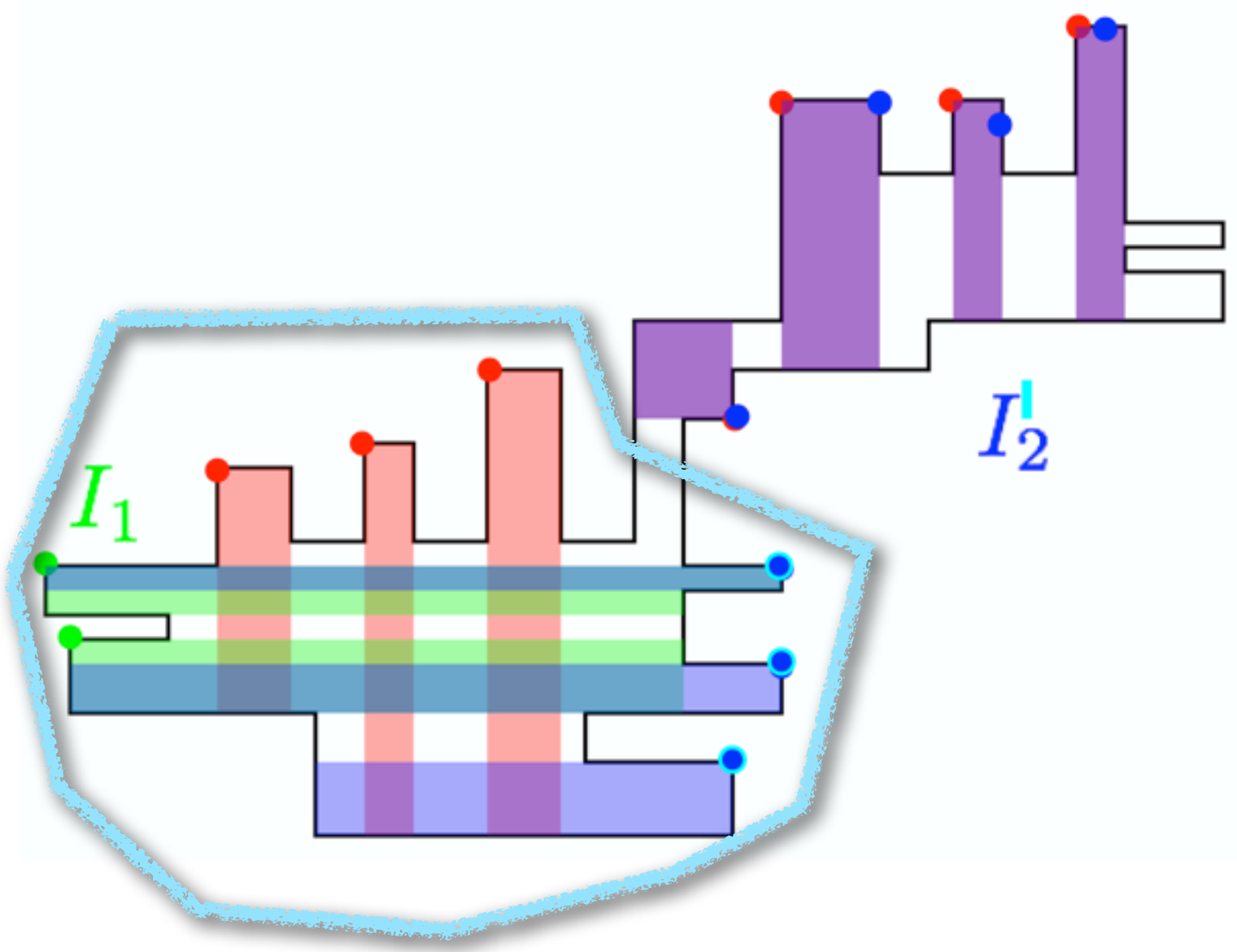

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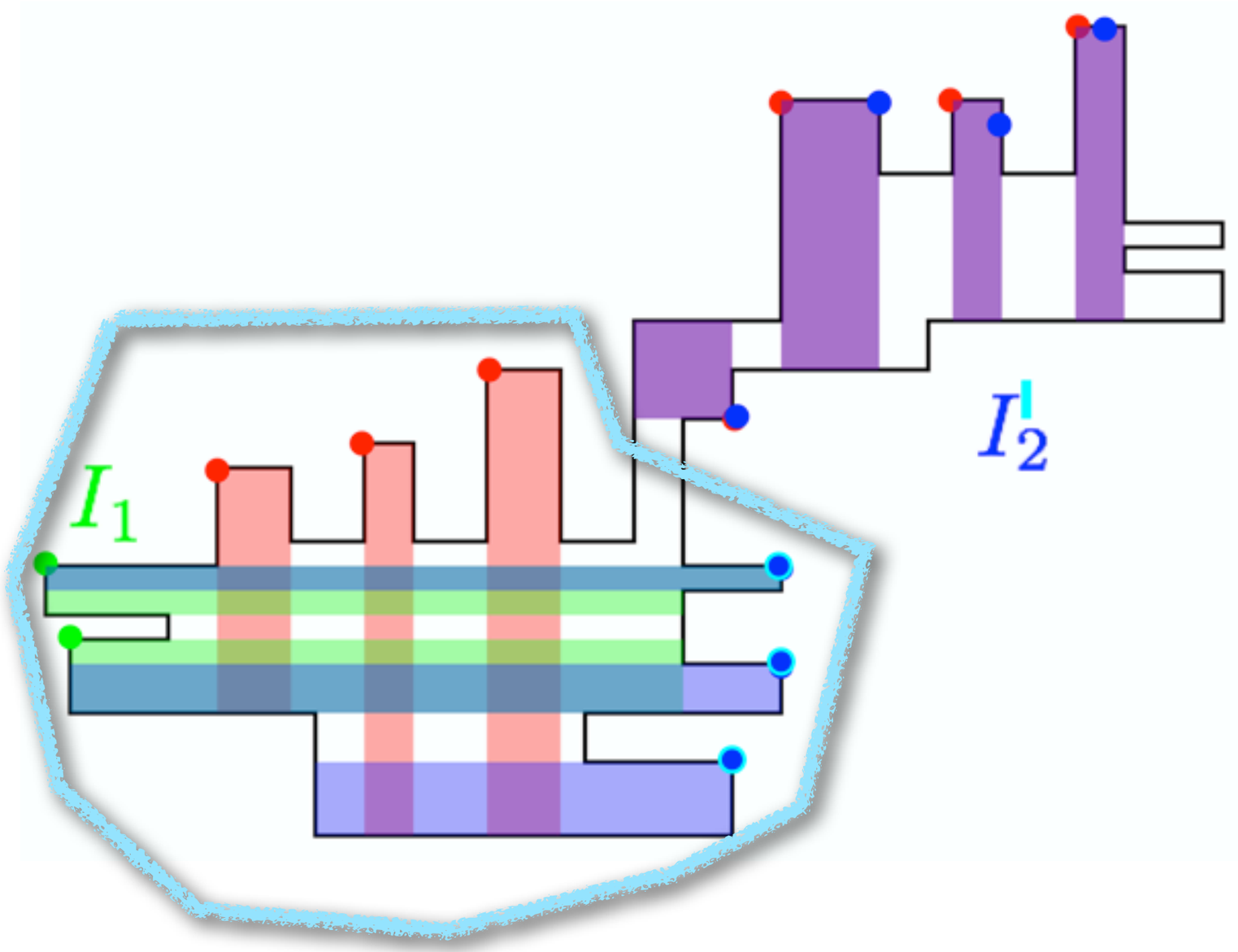

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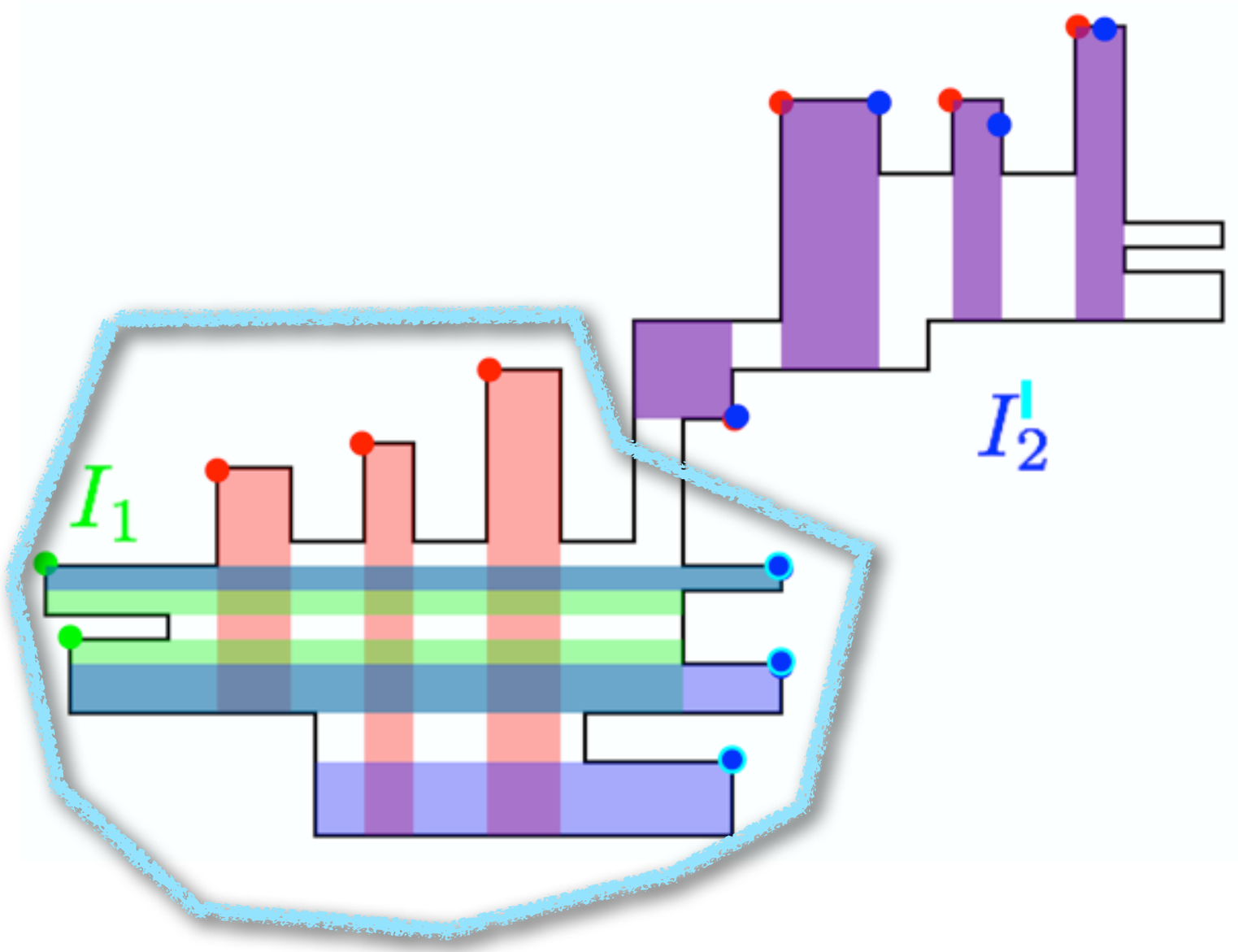

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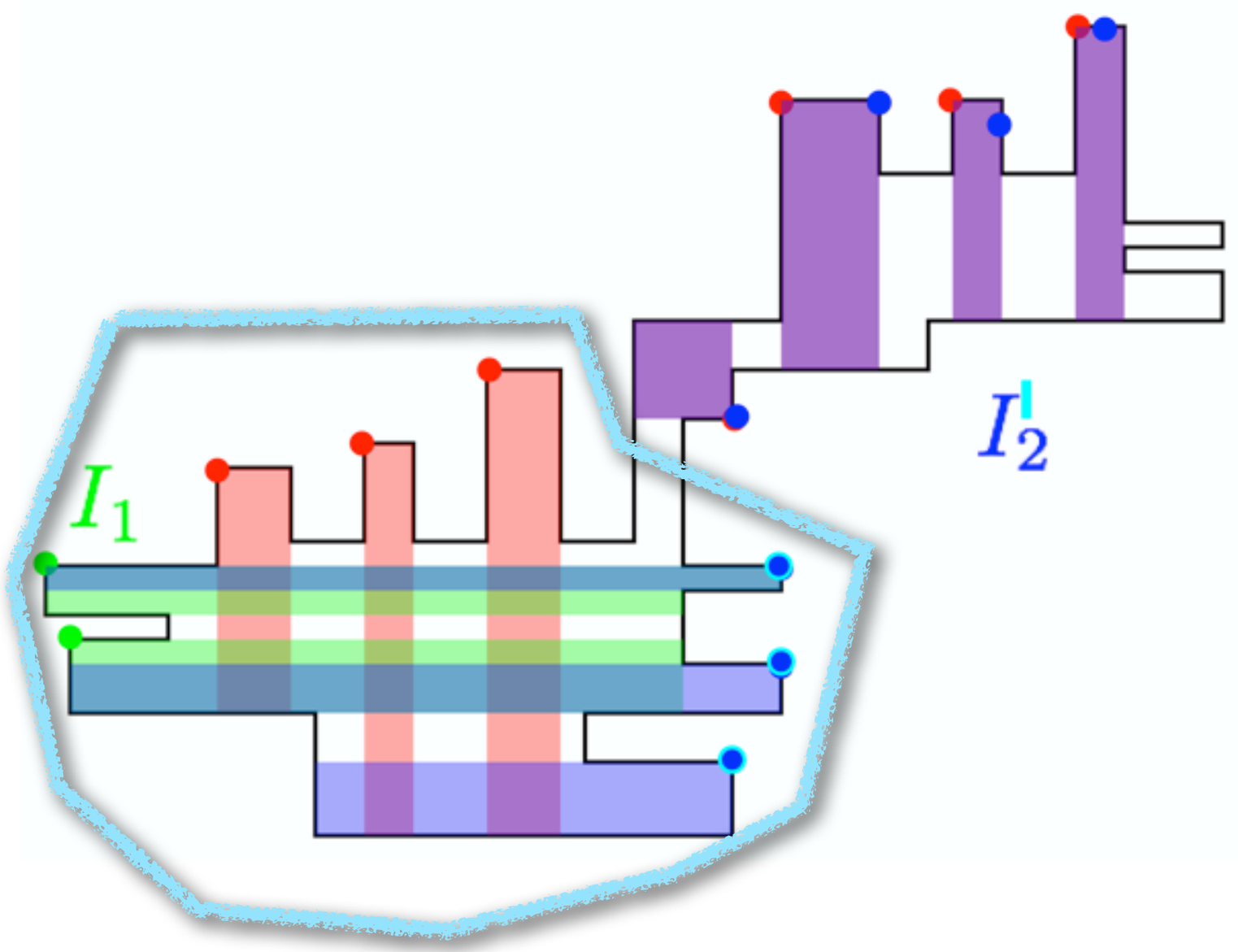
Perfectness



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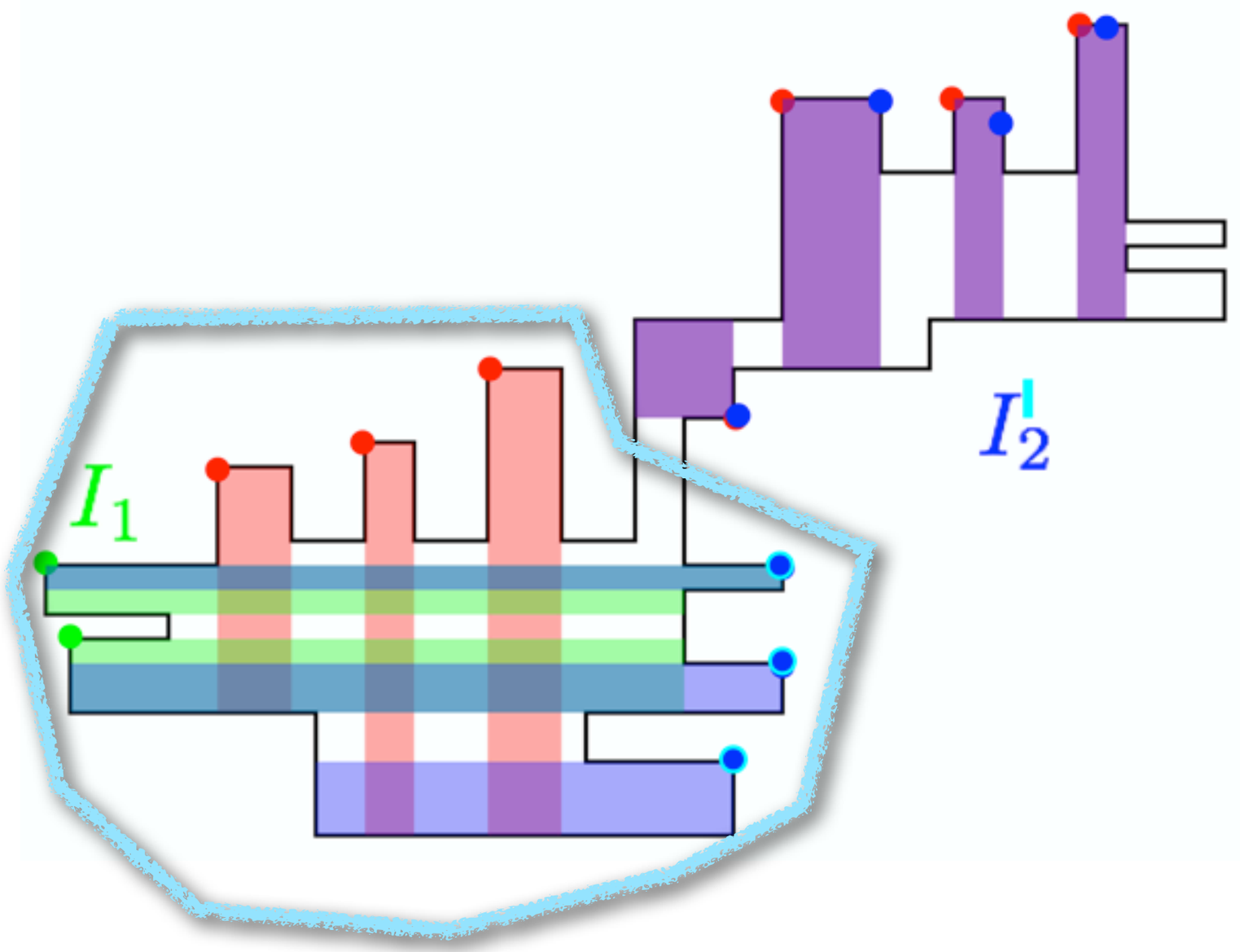
Perfectness



For any IS X of visibility polygons in $I_1 \cup I_2'$:

$$|\{v \in W : VP(v) \cap VP(x), x \in X\}| \geq |X|$$

If X is a max IS of visibility polygons in $I_1 \cup I_2'$ of size t

➡ exists a matching of size t in the intersection graph of VPs from points in W and points in X

Let the stabbing number of visibility polygons from $I_1 \cup I_2'$ be s.

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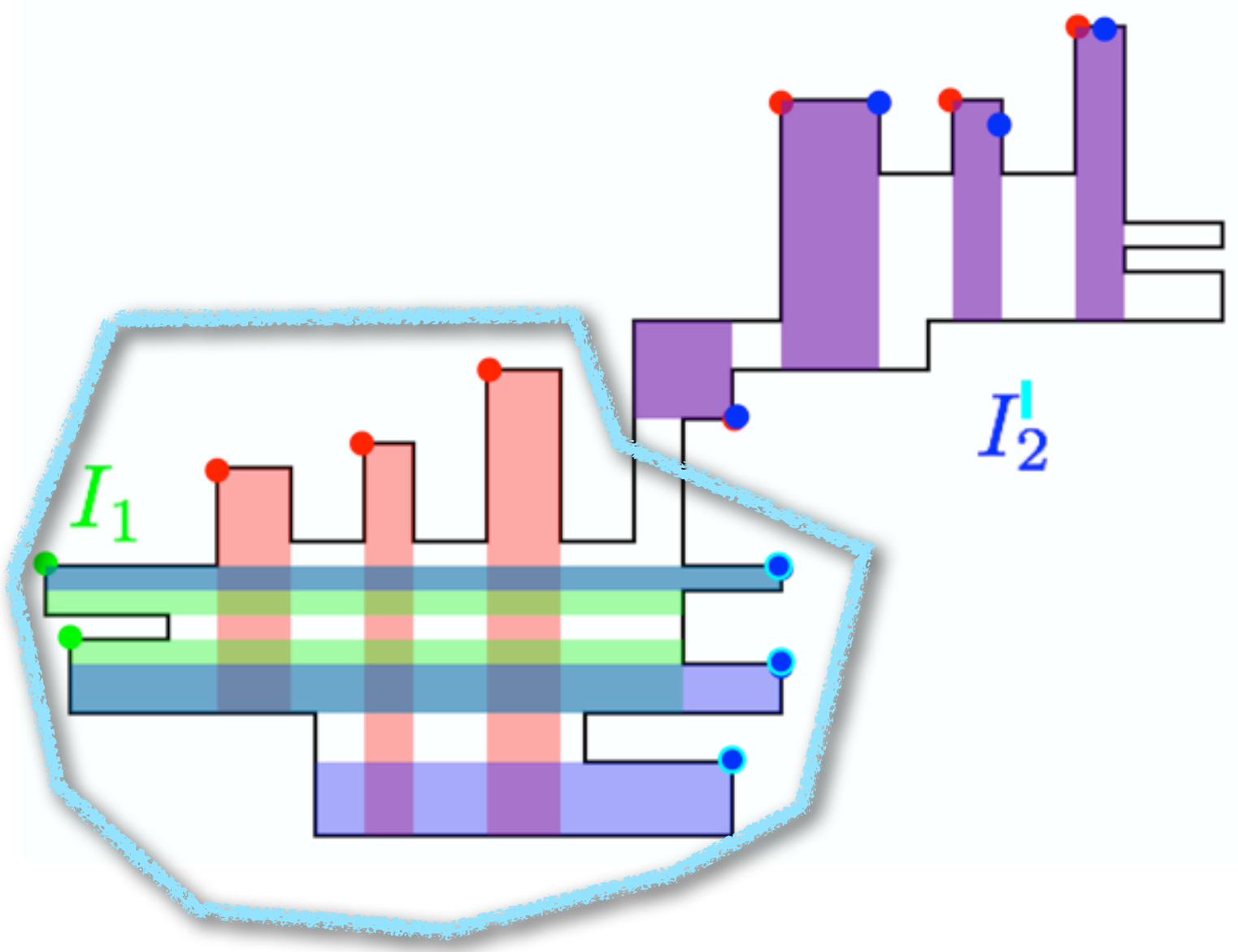
- s_1 polygons that do not interfer

Perfectness



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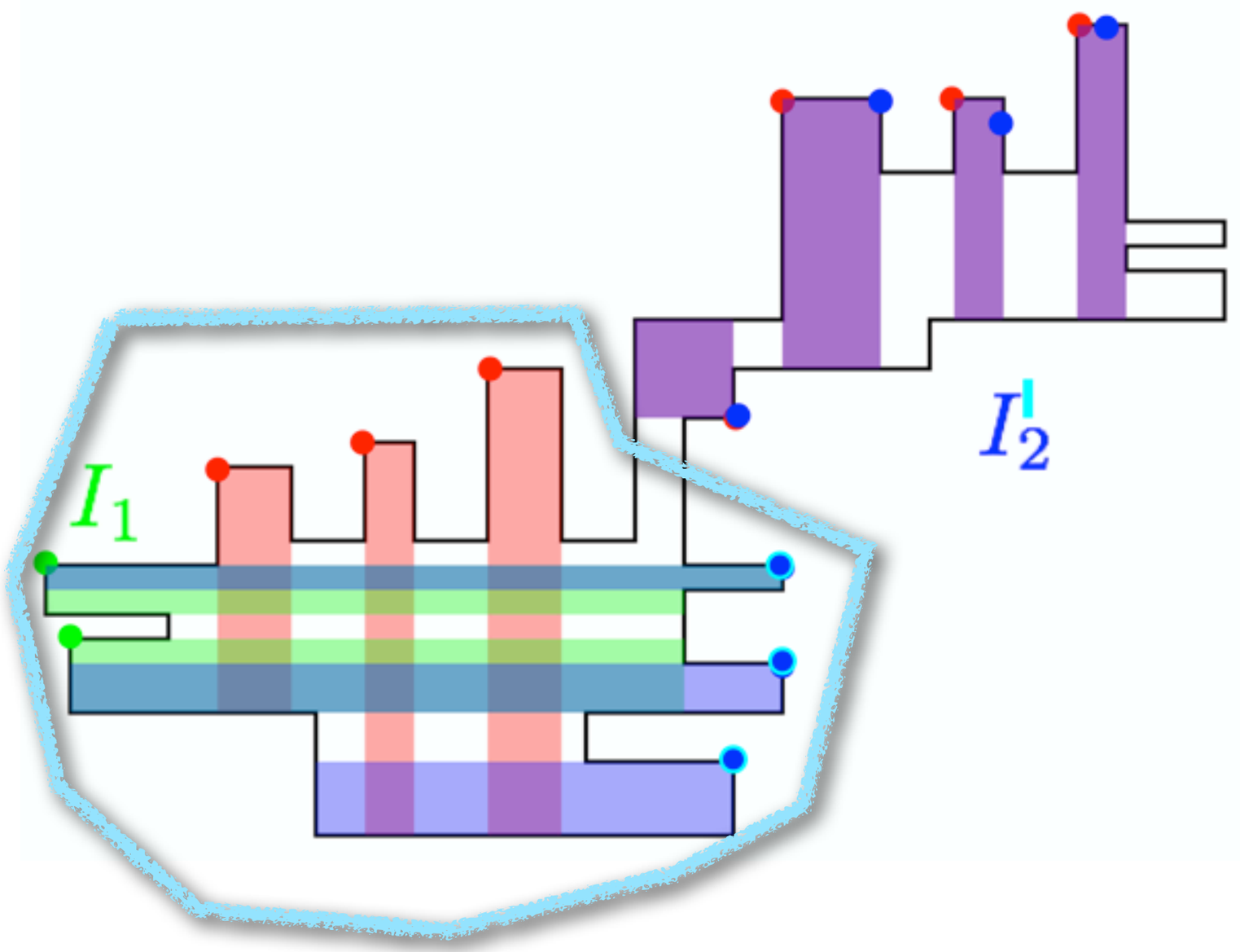
Perfectness



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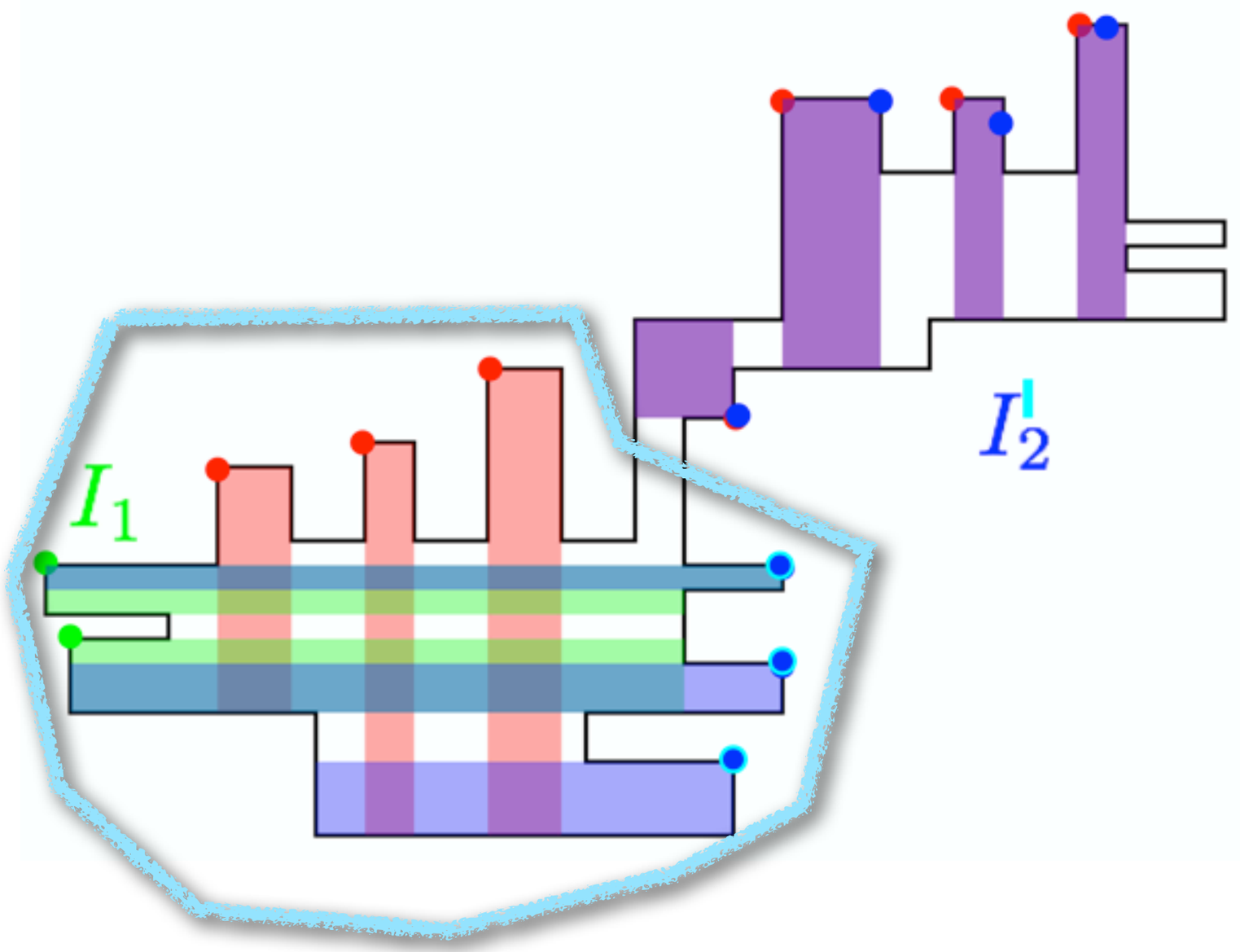

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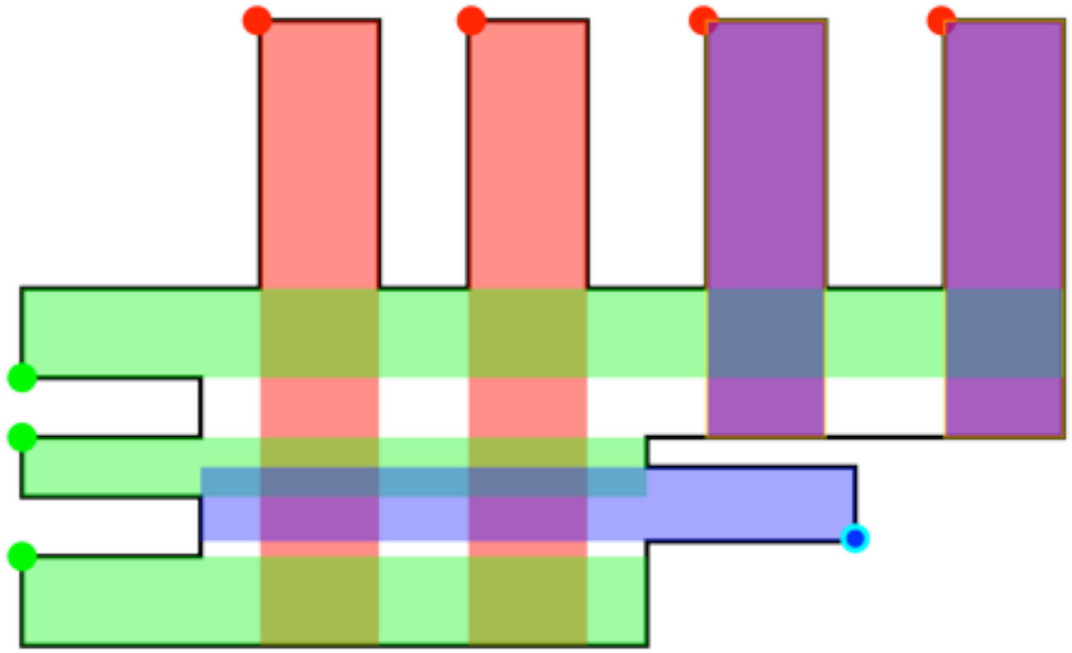

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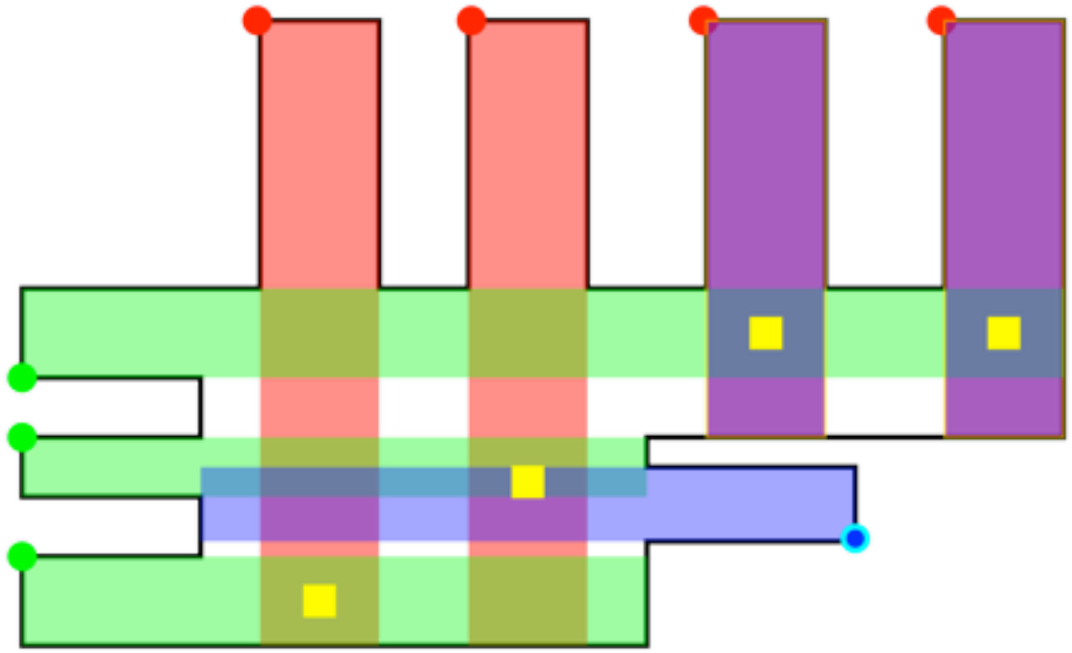

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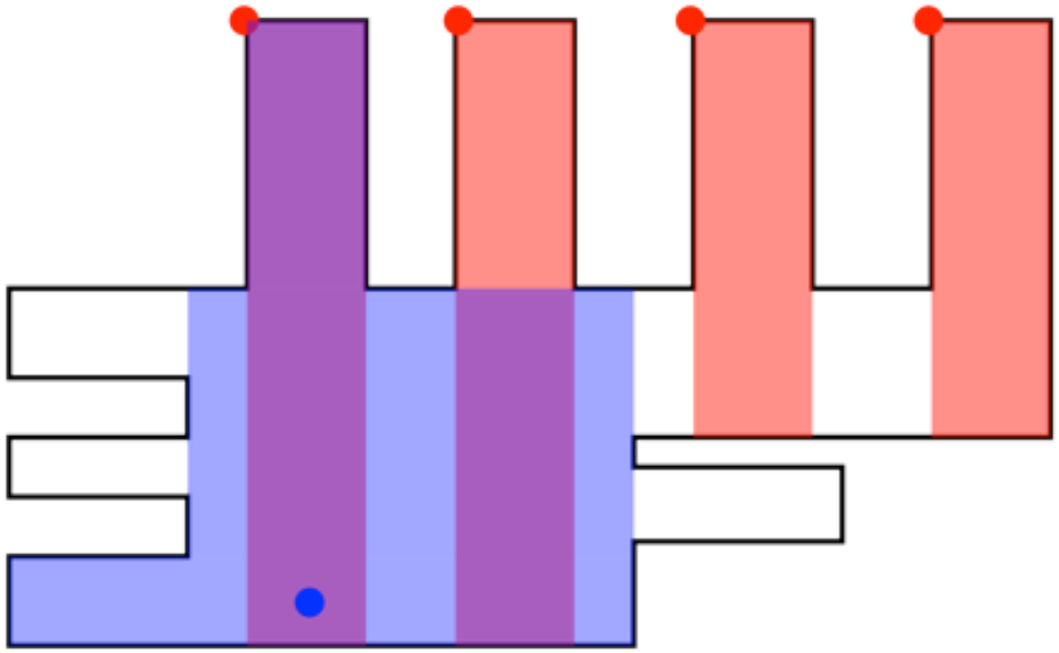
$P_1 = VP(g)$



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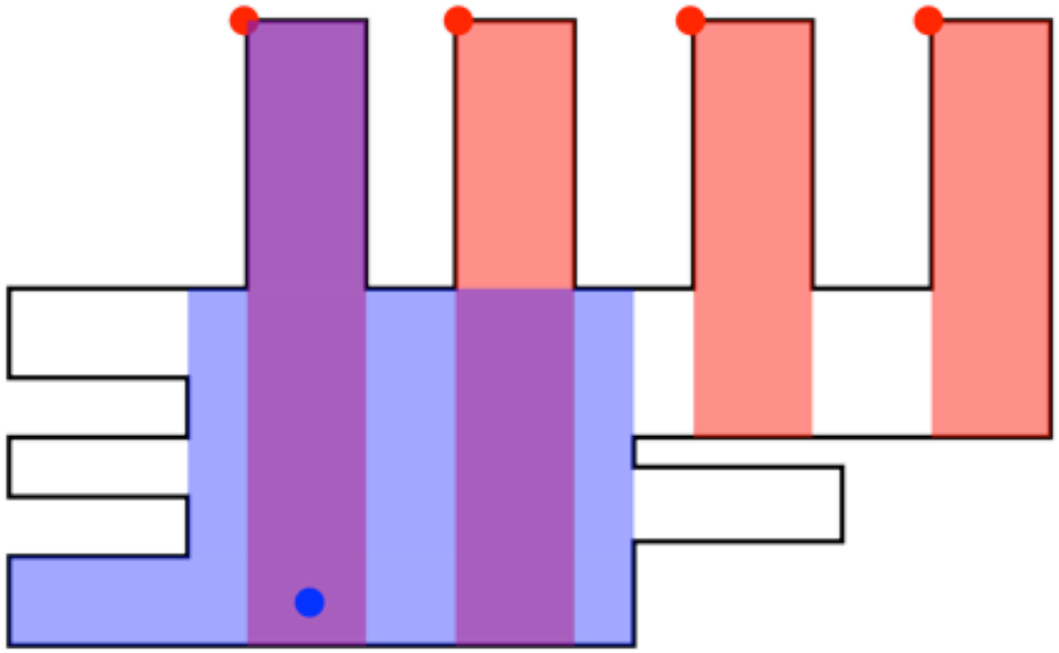

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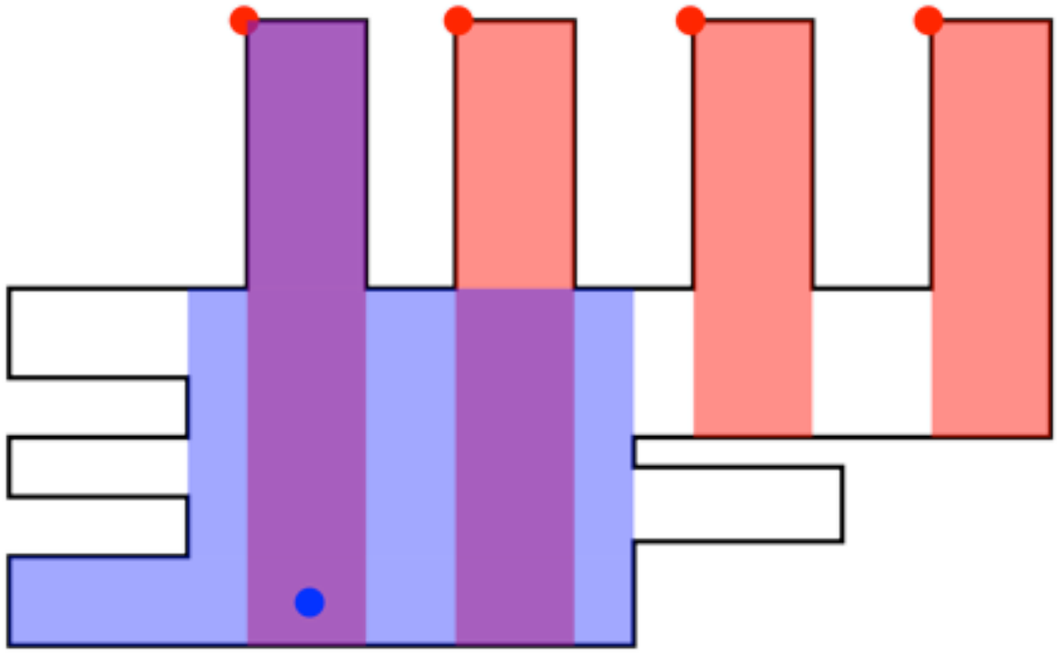

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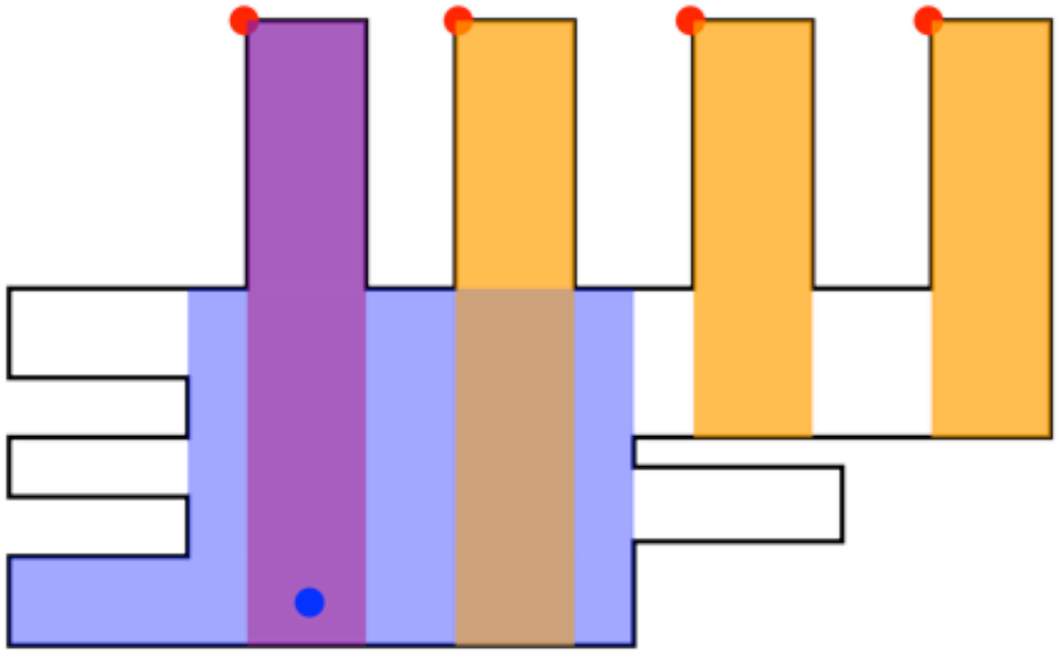

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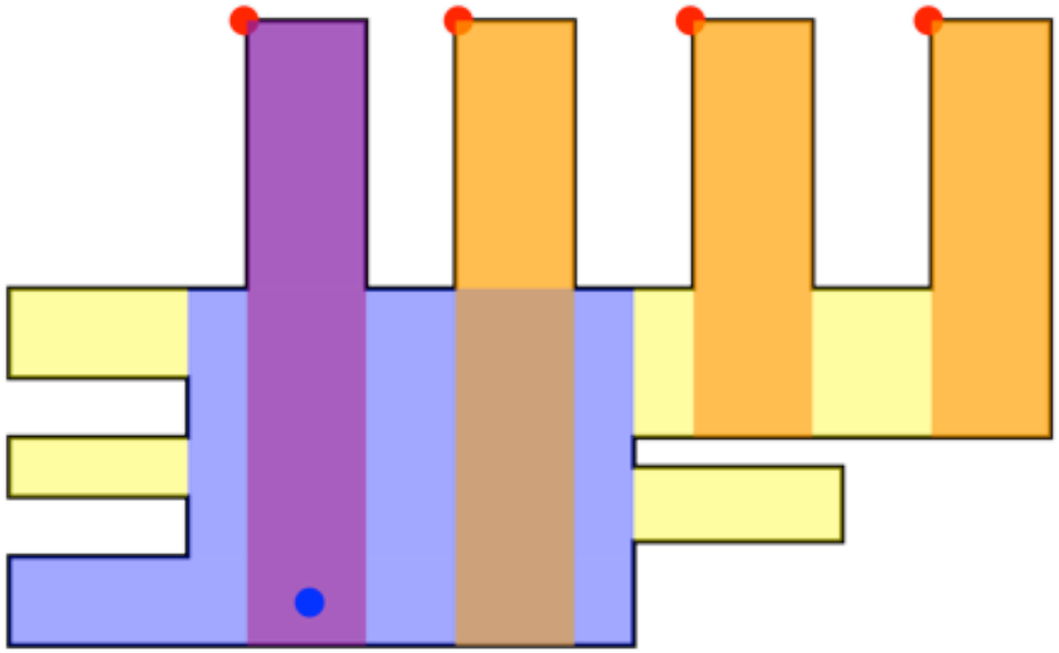

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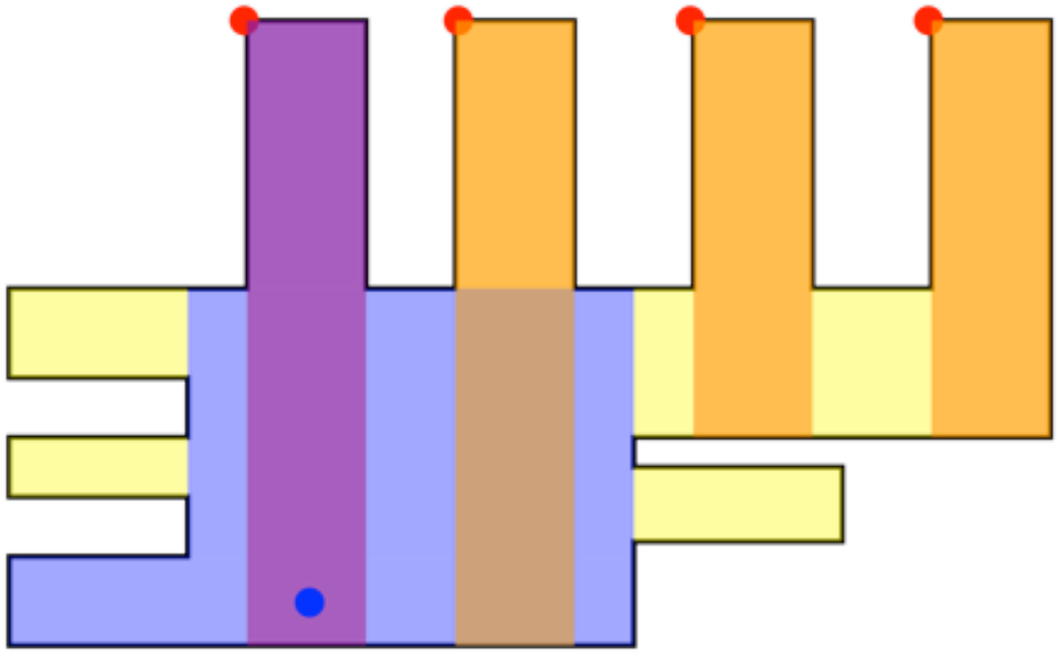

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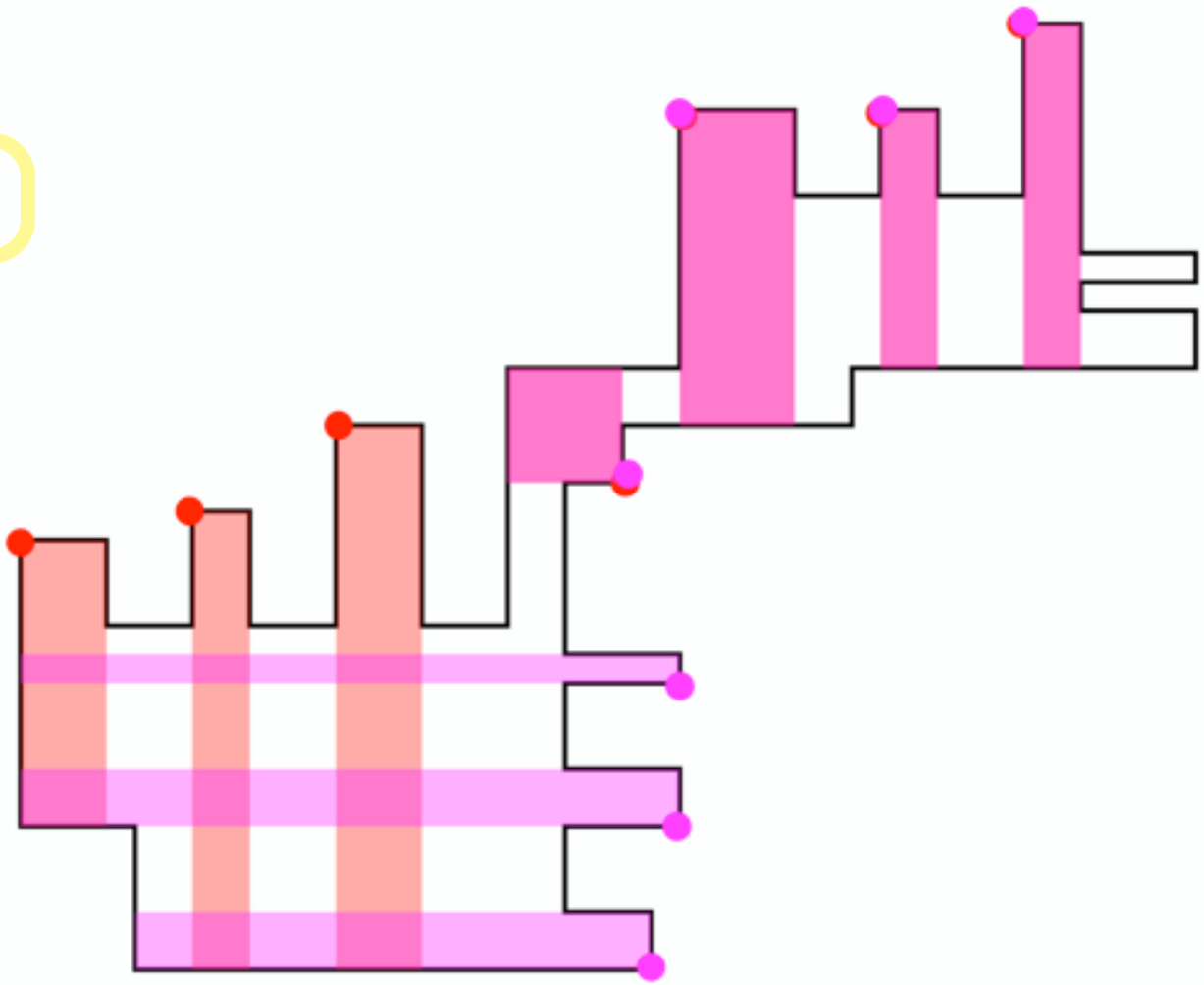

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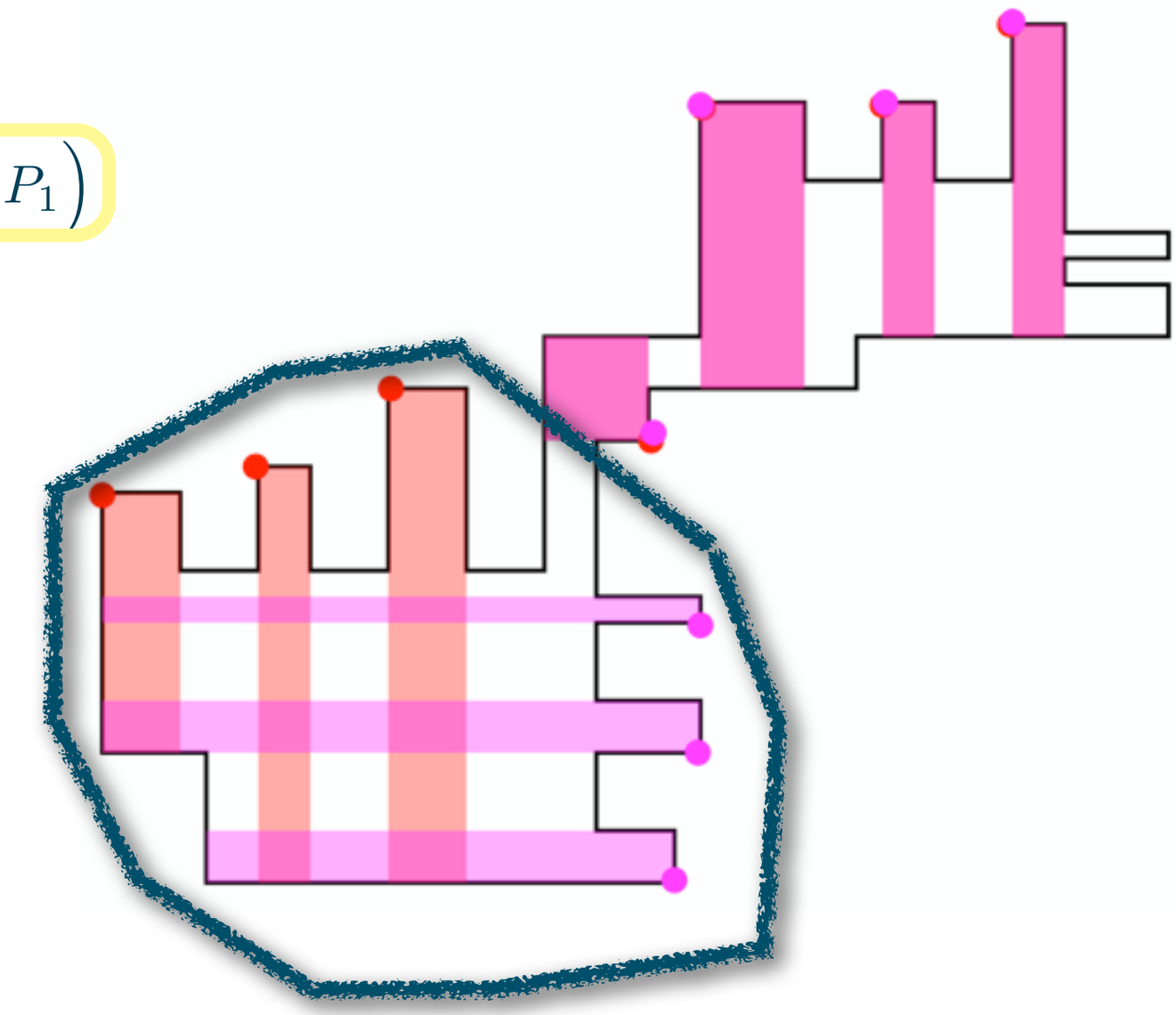

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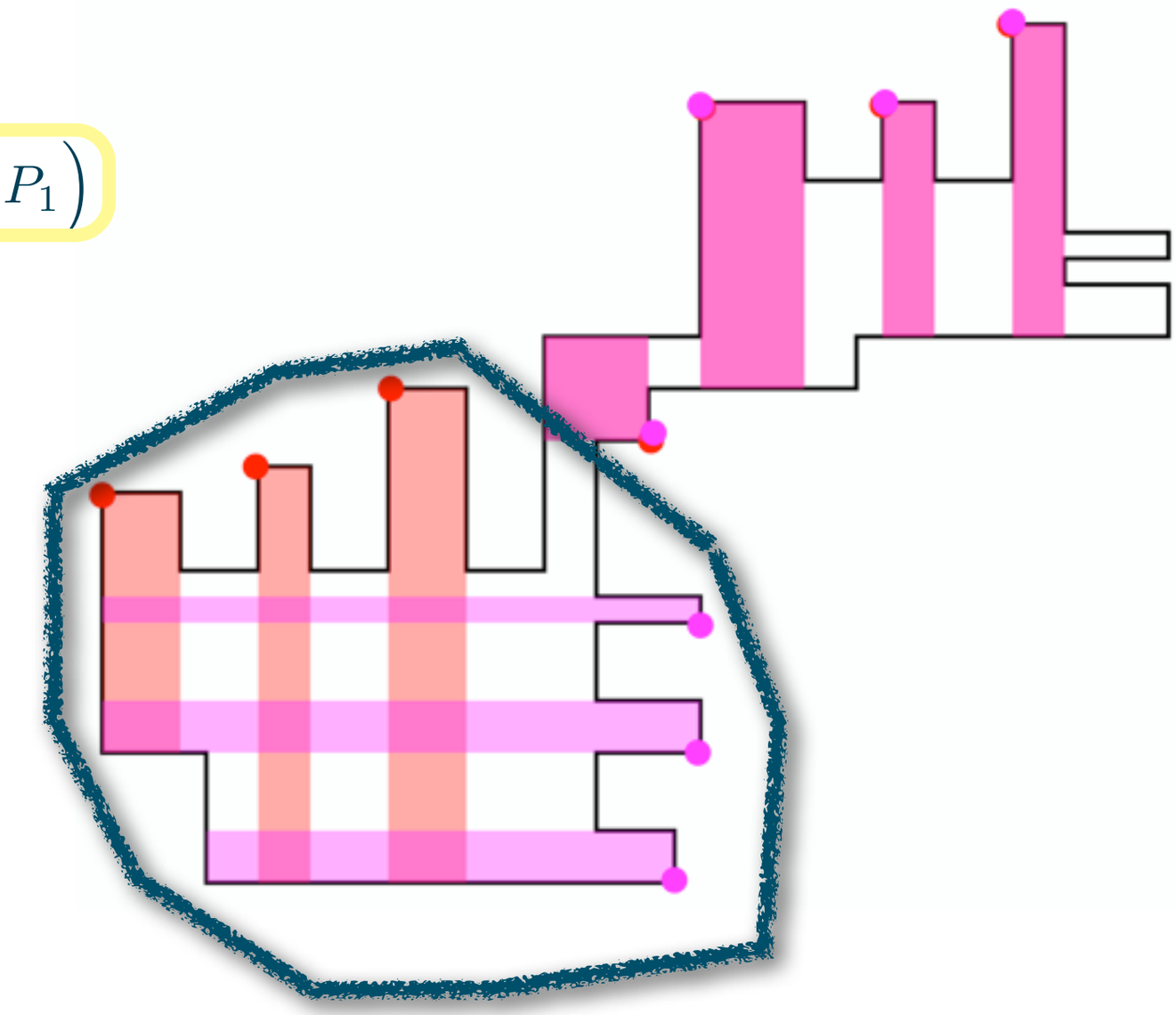

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Finally: $P' = P_1 \cup P_2$

Outlook

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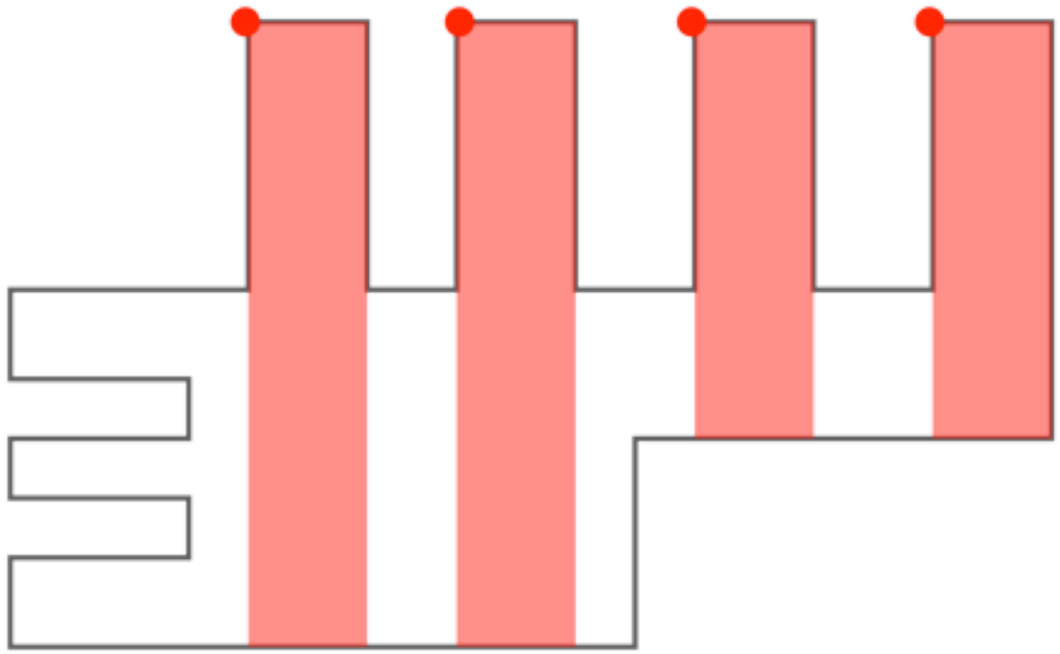

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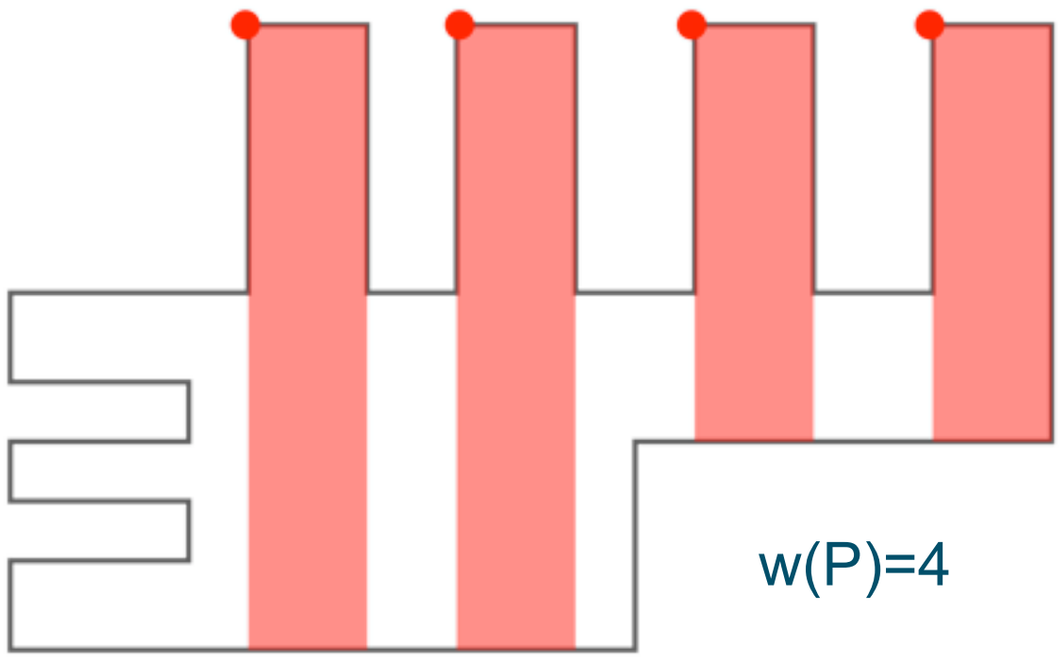
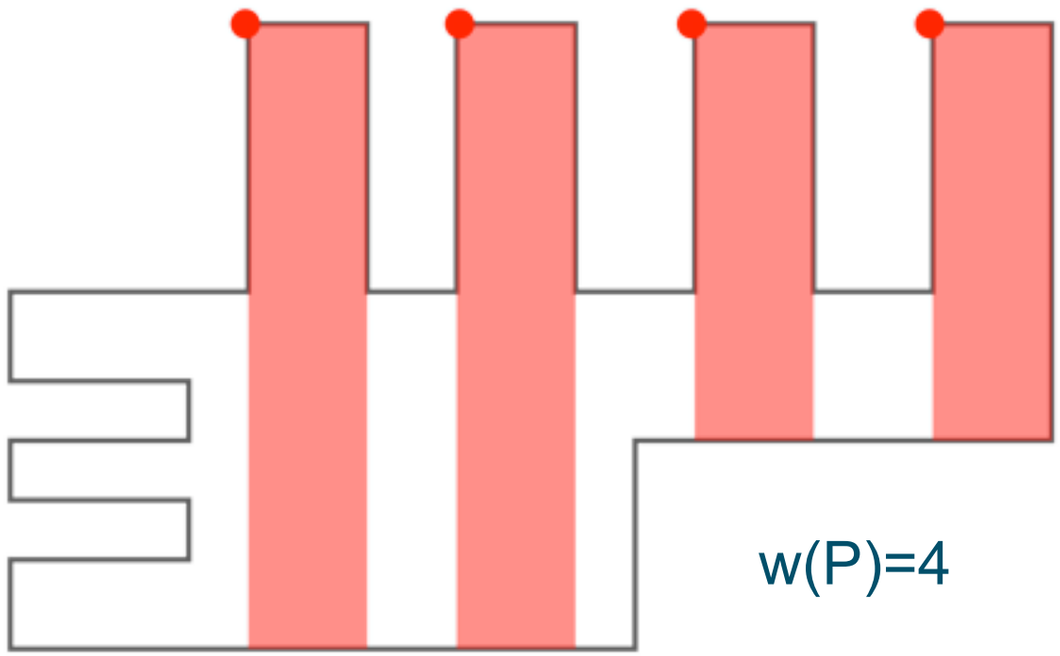


- computation of w(P) in substantiell lower running time (<$O(n^{17})$) via Dynamic Programming

Outlook



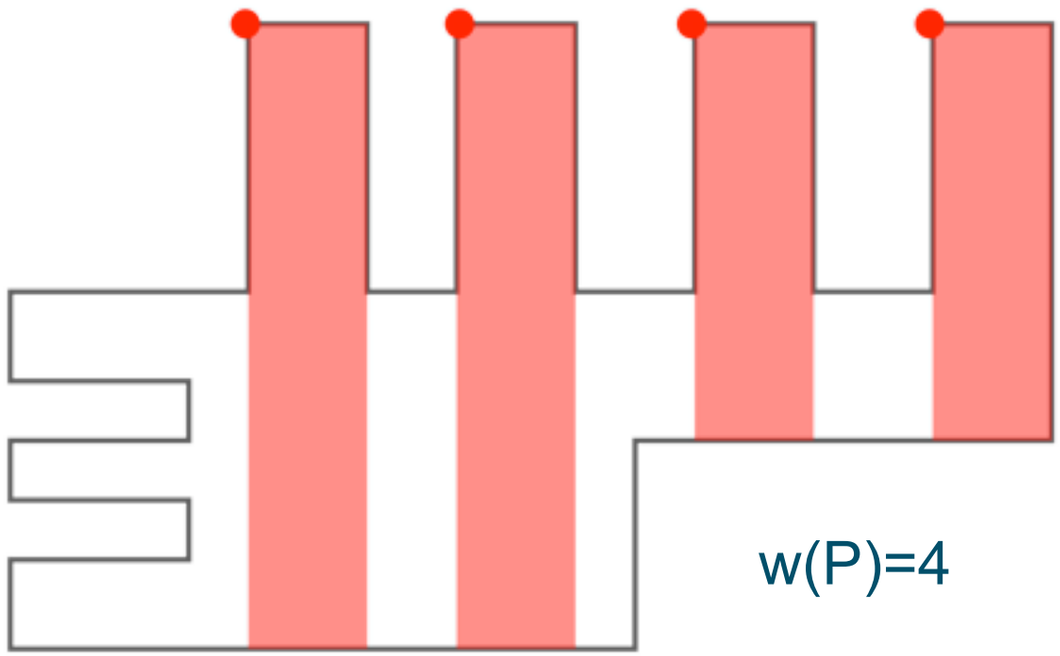
w(P)=4

g(P)=4

- computation of w(P) in substantiell lower running time (<$O(n^{17})$) via Dynamic Programming

➡ faster algorithm to compute g(P)

Outlook



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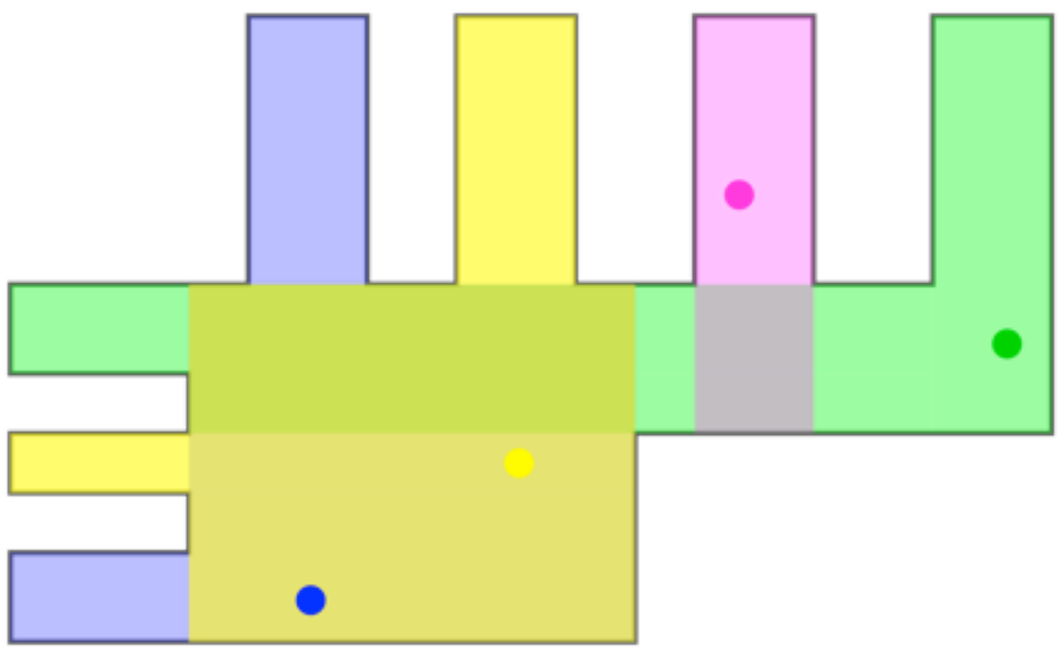

- computation of w(P) in substantiell lower running time (<$O(n^{17})$) via Dynamic Programming

➡ faster algorithm to compute g(P)

➡as the above is constructive: also computation of the location of guards possible?

Outlook

- computation of w(P) in substantiell lower running time ($<O(n^{17})$) via Dynamic Programming

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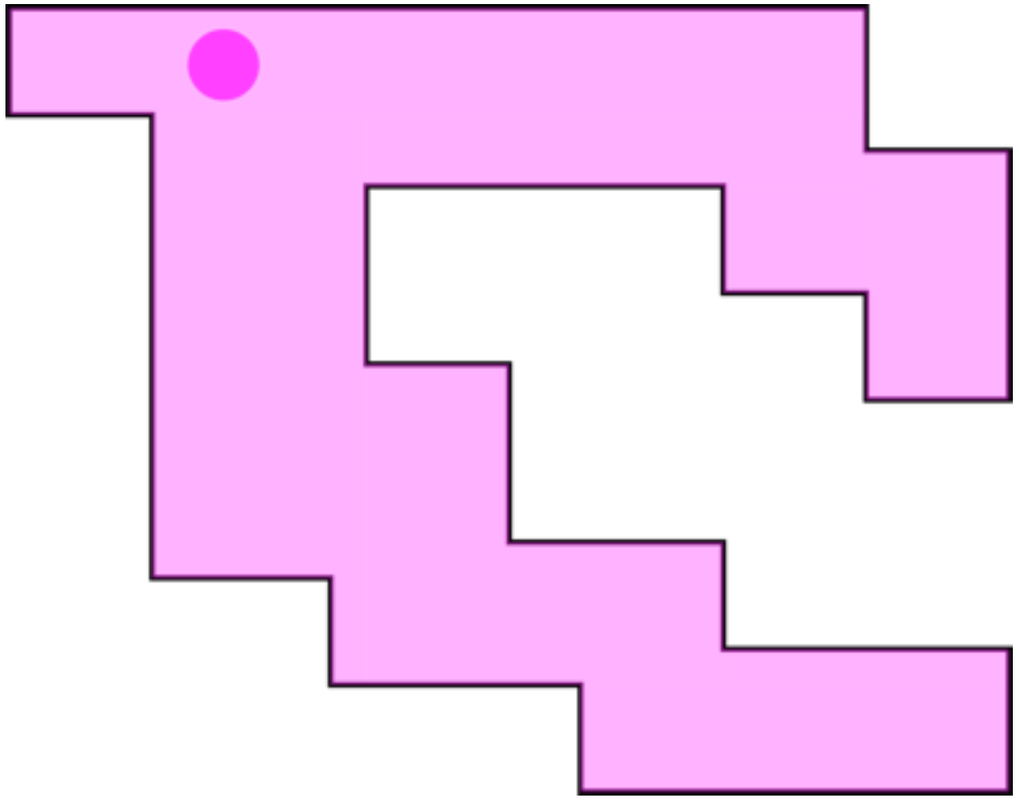
➡as the above is constructive: also computation of the location of guards possible?

- other types of vision?

Outlook



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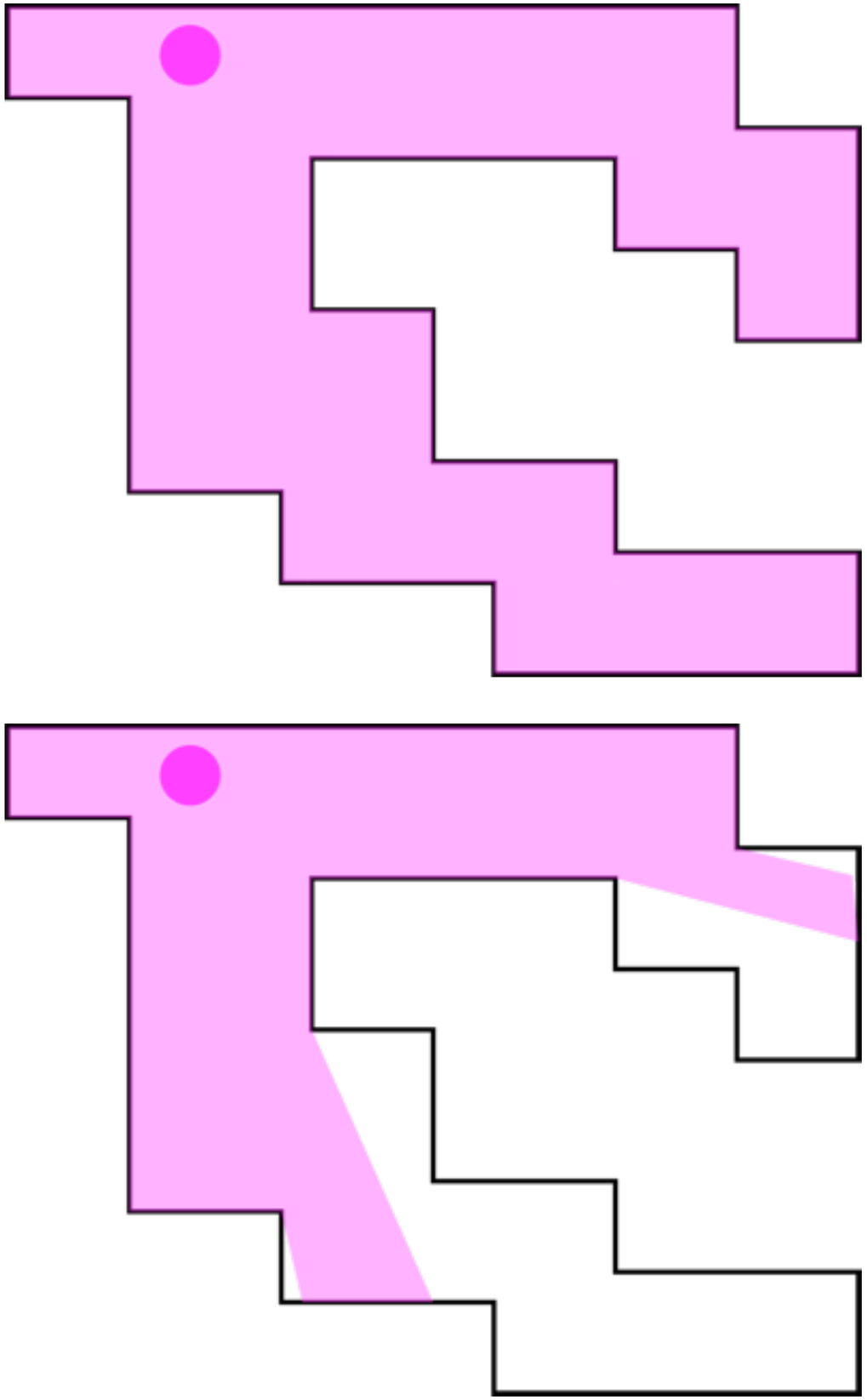
- other types of vision?
  - „staircase“ visibility

Outlook



- computation of w(P) in substantiell lower running time (<$O(n^{17})$) via Dynamic Programming

➡ faster algorithm to compute g(P)

➡as the above is constructive: also computation of the location of guards possible?

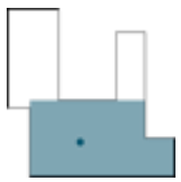
- other types of vision?
  - „staircase“ visibility
  - ordinary visibility

Thanks.

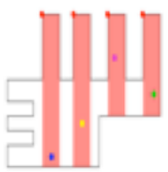
Thanks.

Computation of w(P)

The Art Gallery Problem, Guards and Witnesses



Rectangular Vision

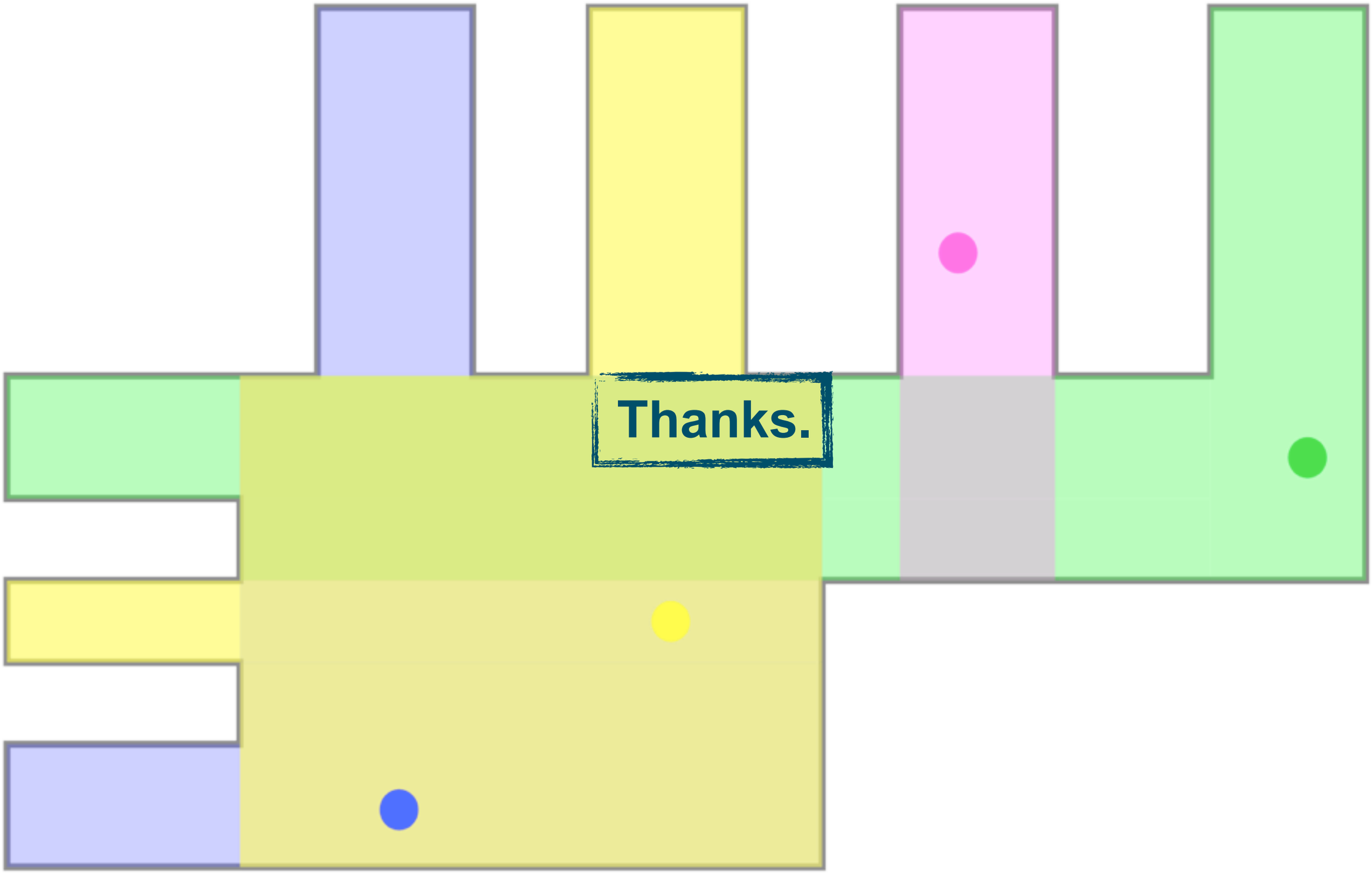


Perfectness

Computation of w(P)

Outlook

Thanks.



Thanks.

Rectangular Vision

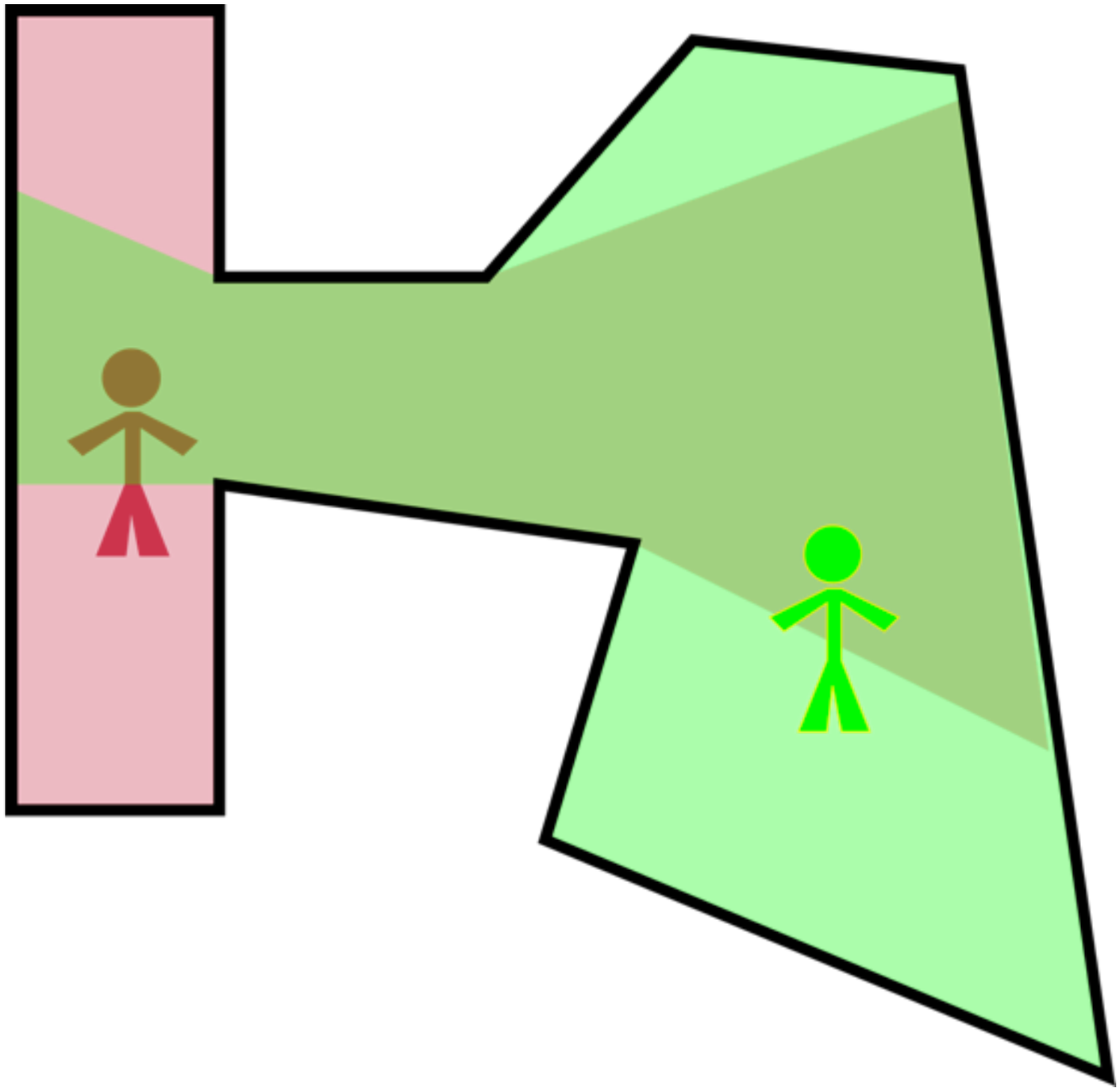
Theorem by Breen [1996]:

Let $\mathcal{P}$ be a family of simple polygons in the plane.

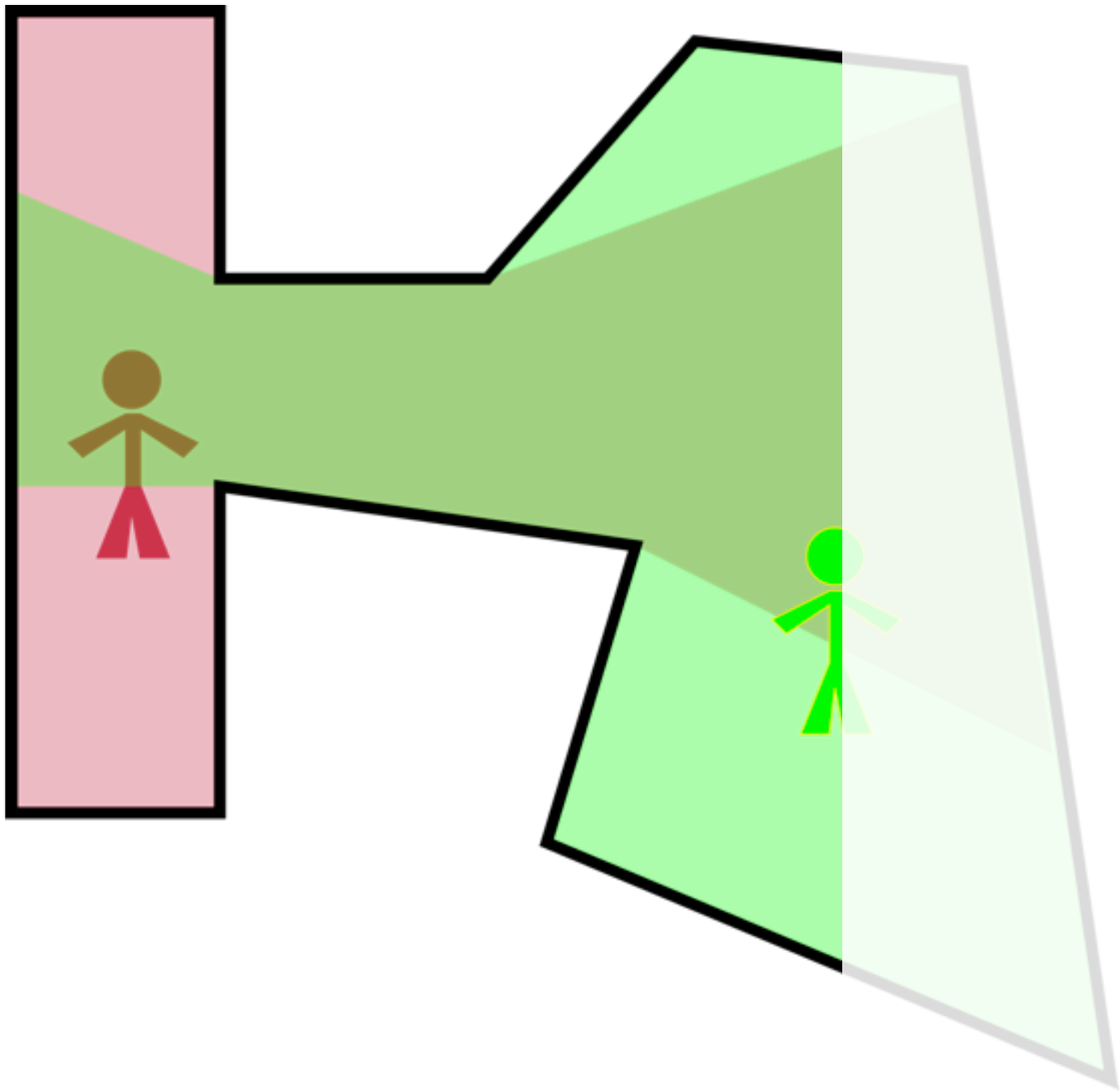
- If every three members of $\mathcal{P}$ have a simply connected union
- and every two have a nonempty intersection

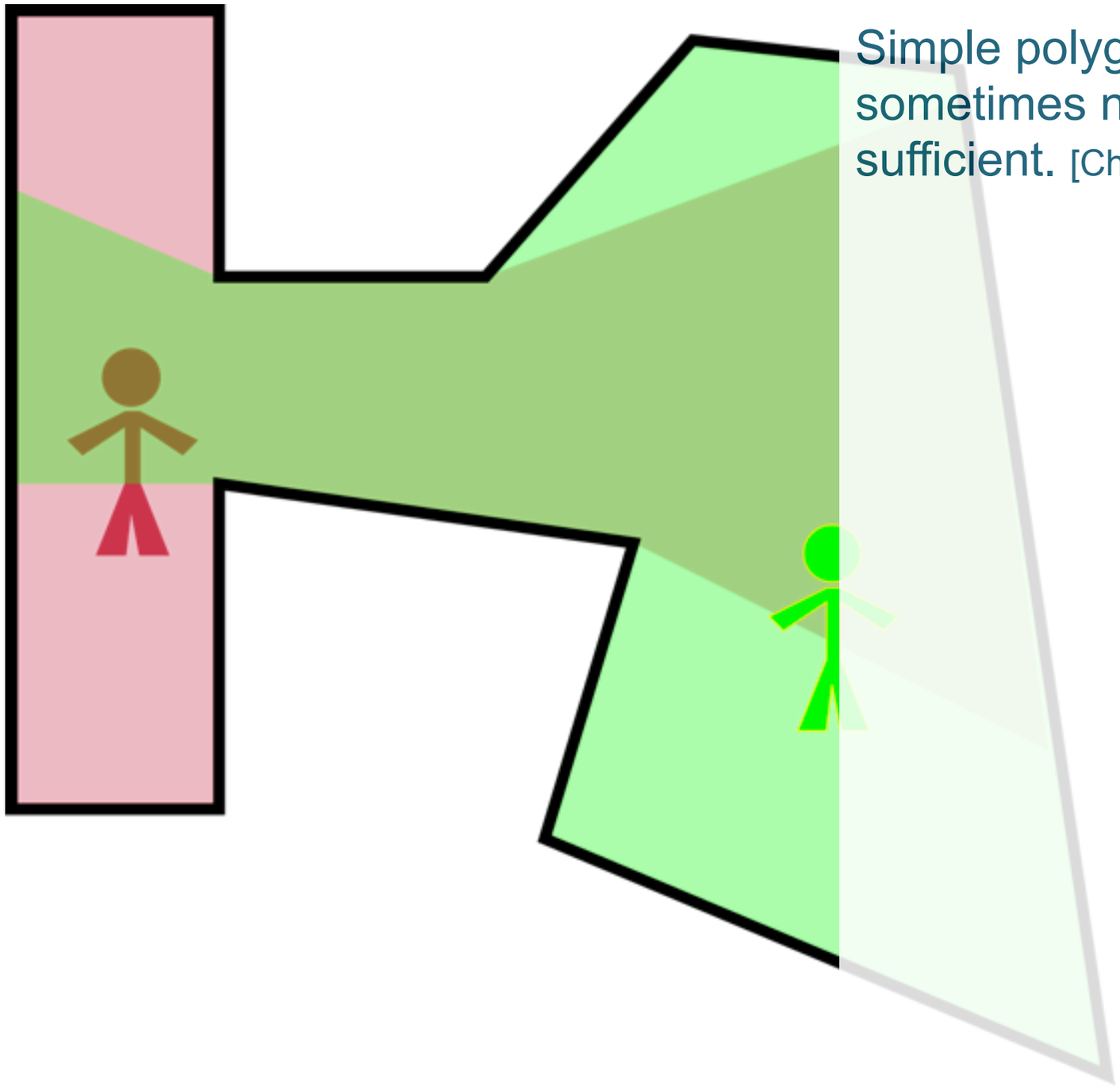
➡ $\cap\{P : P \in \mathcal{P}\} \neq \emptyset$

- for a family of orthogonally convex polygons the intersection is again an orthogonally convex polygon.



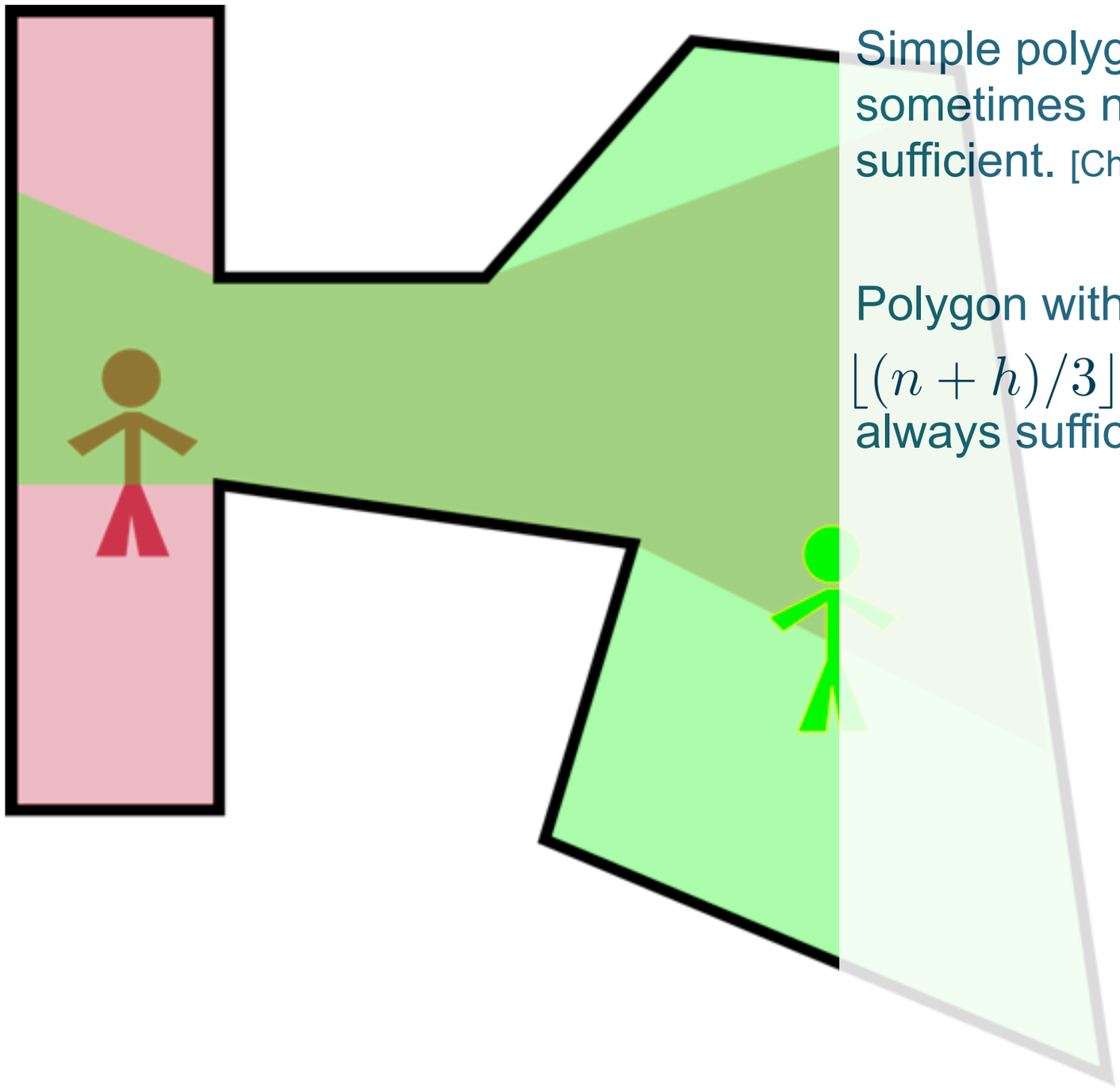
Known:





Known:

Simple polygon with n vertices: $\lfloor n/3 \rfloor$ are sometimes necessary and always sufficient. [Chvátal '75]



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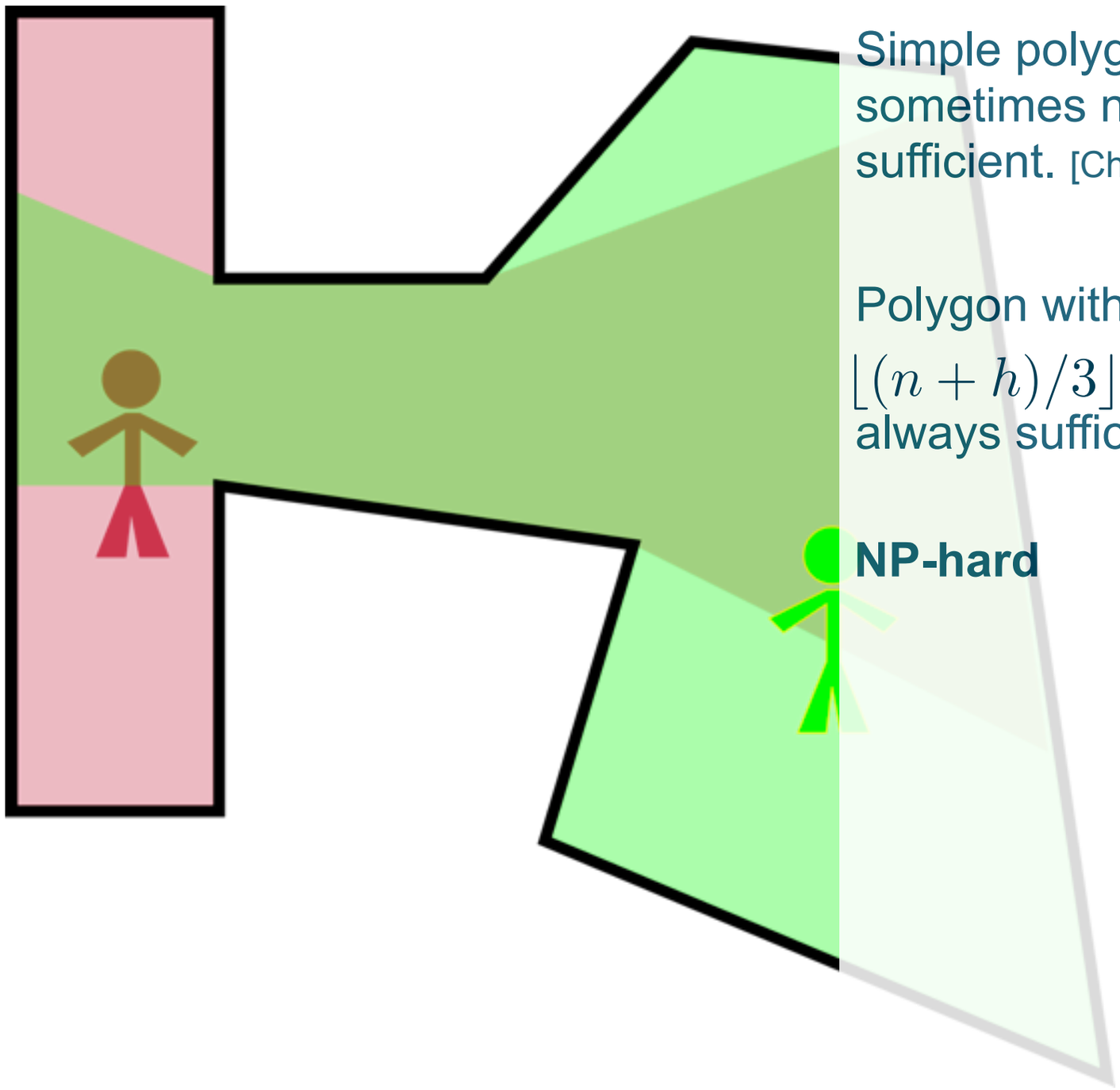
Polygon with n vertices and h holes: $\lfloor (n+h)/3 \rfloor$ are sometimes necessary and always sufficient. [Bjorling-Sachs, Souvaine '95]



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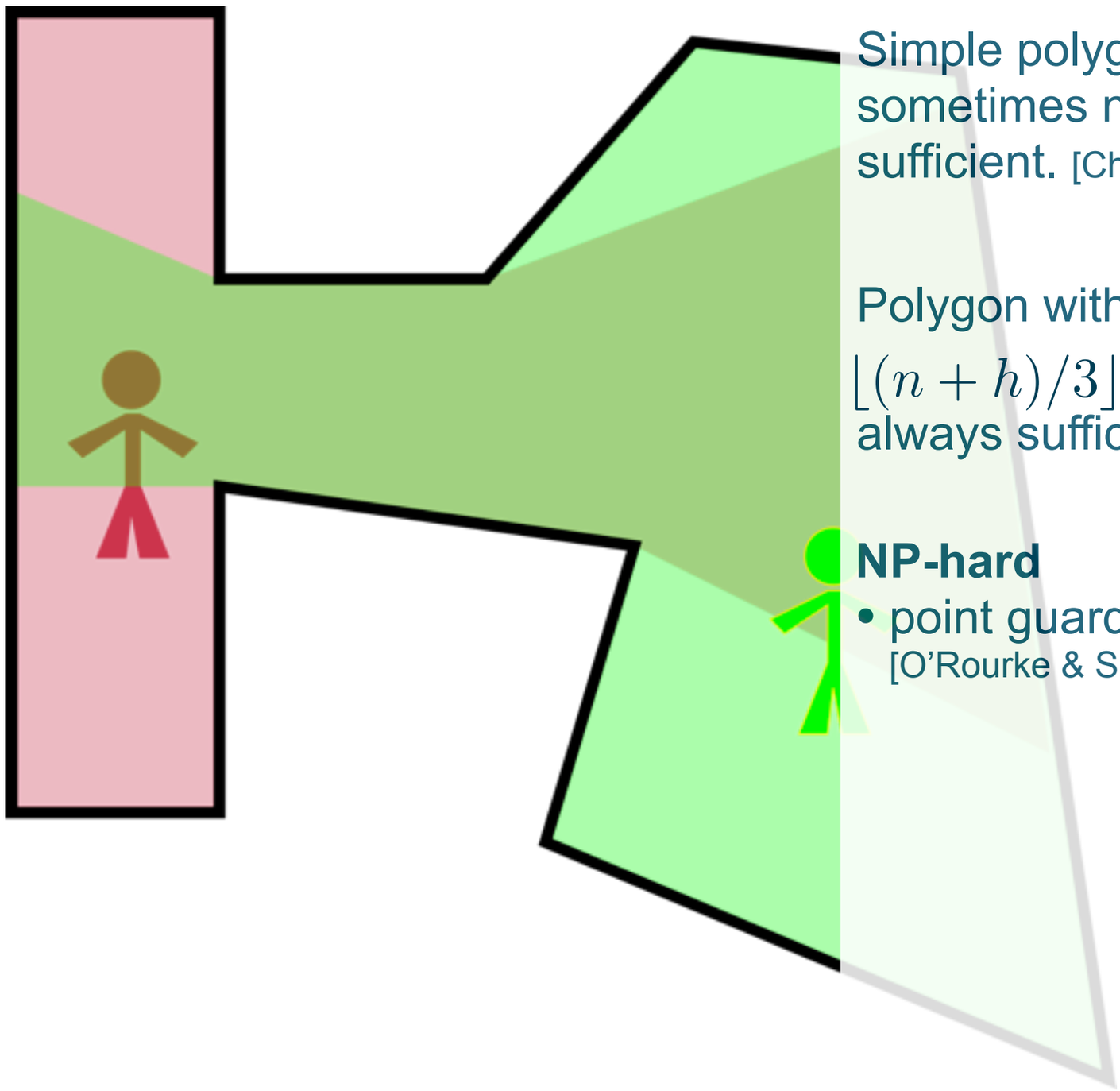
NP-hard



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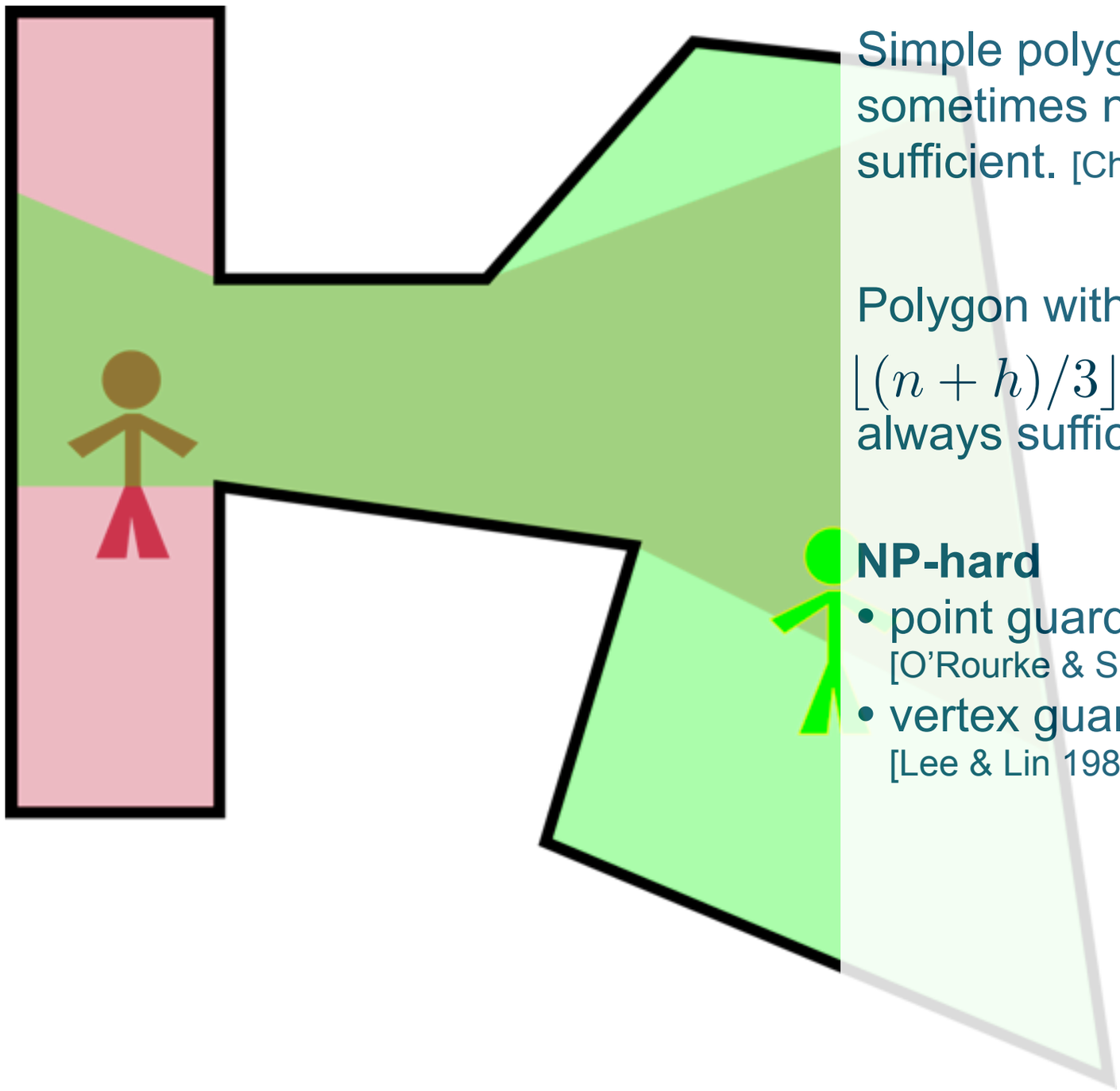
NP-hard

- point guards with holes [O'Rourke & Supowit 1983]



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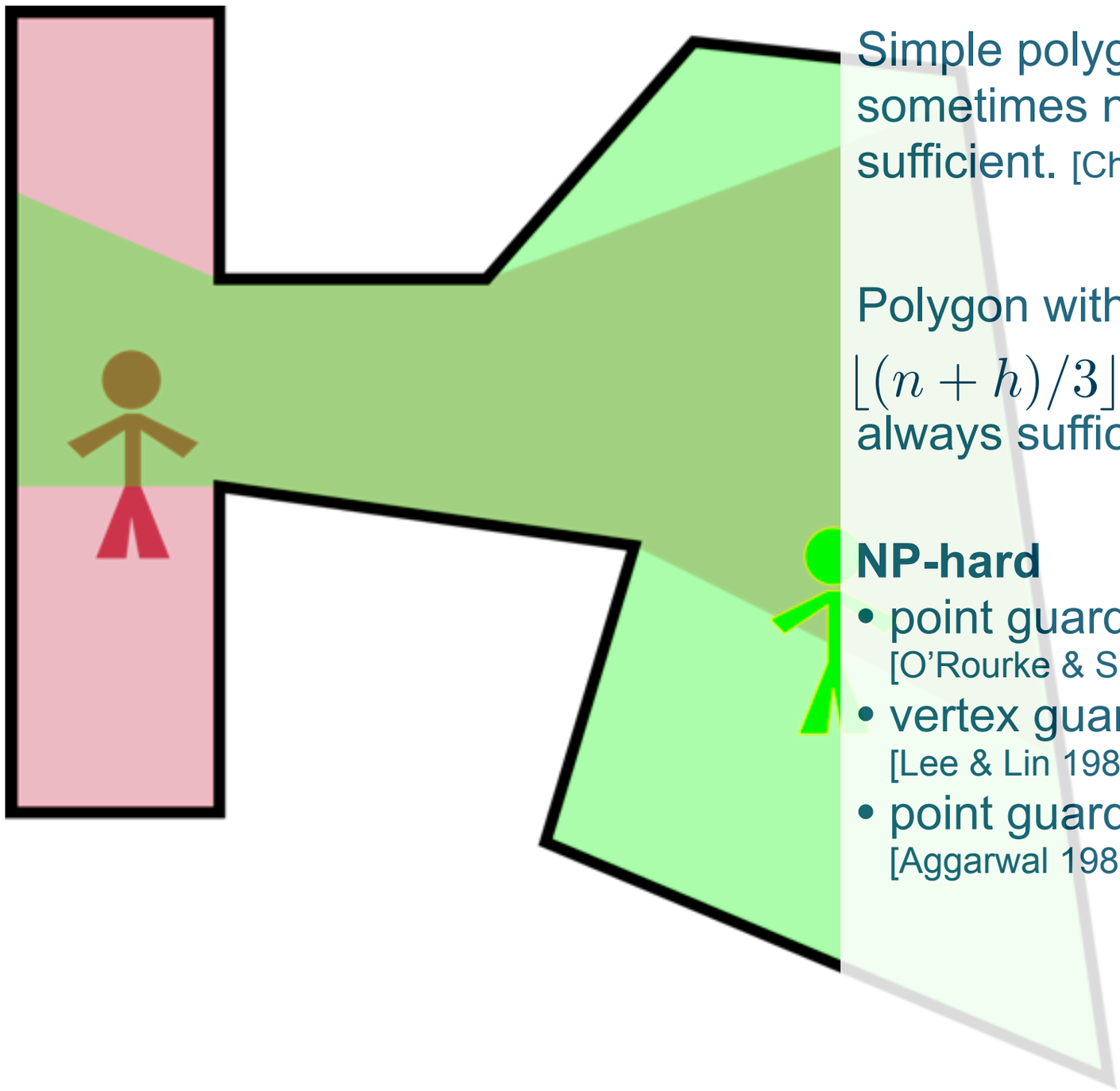
- vertex guards without holes [Lee & Lin 1986]



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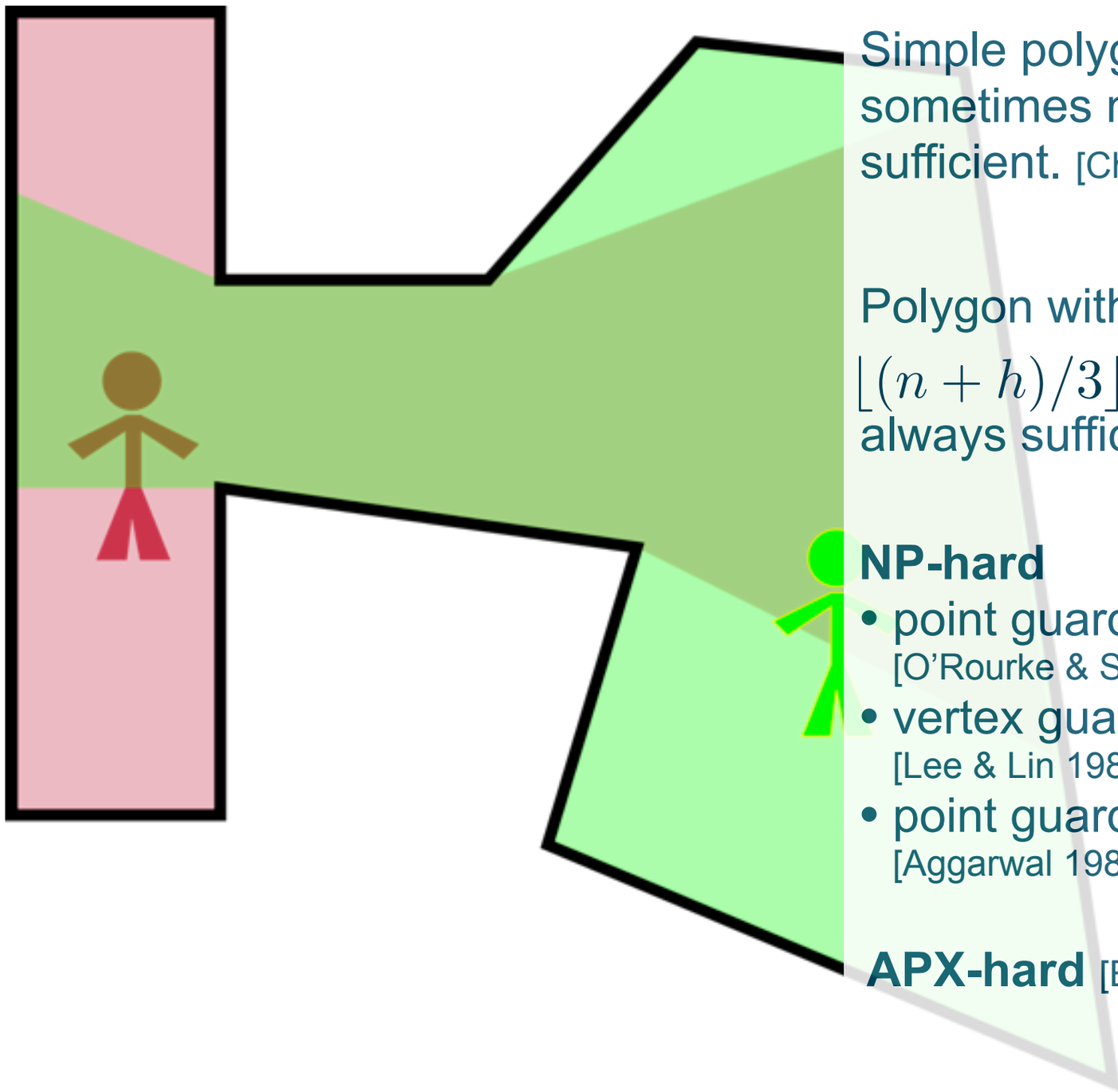

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NP-hard

- point guards with holes [O'Rourke & Supowit 1983]
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- point guards without holes [Aggarwal 1986]

APX-hard [Eidenbenz, Stamm & Widmayer 2001]