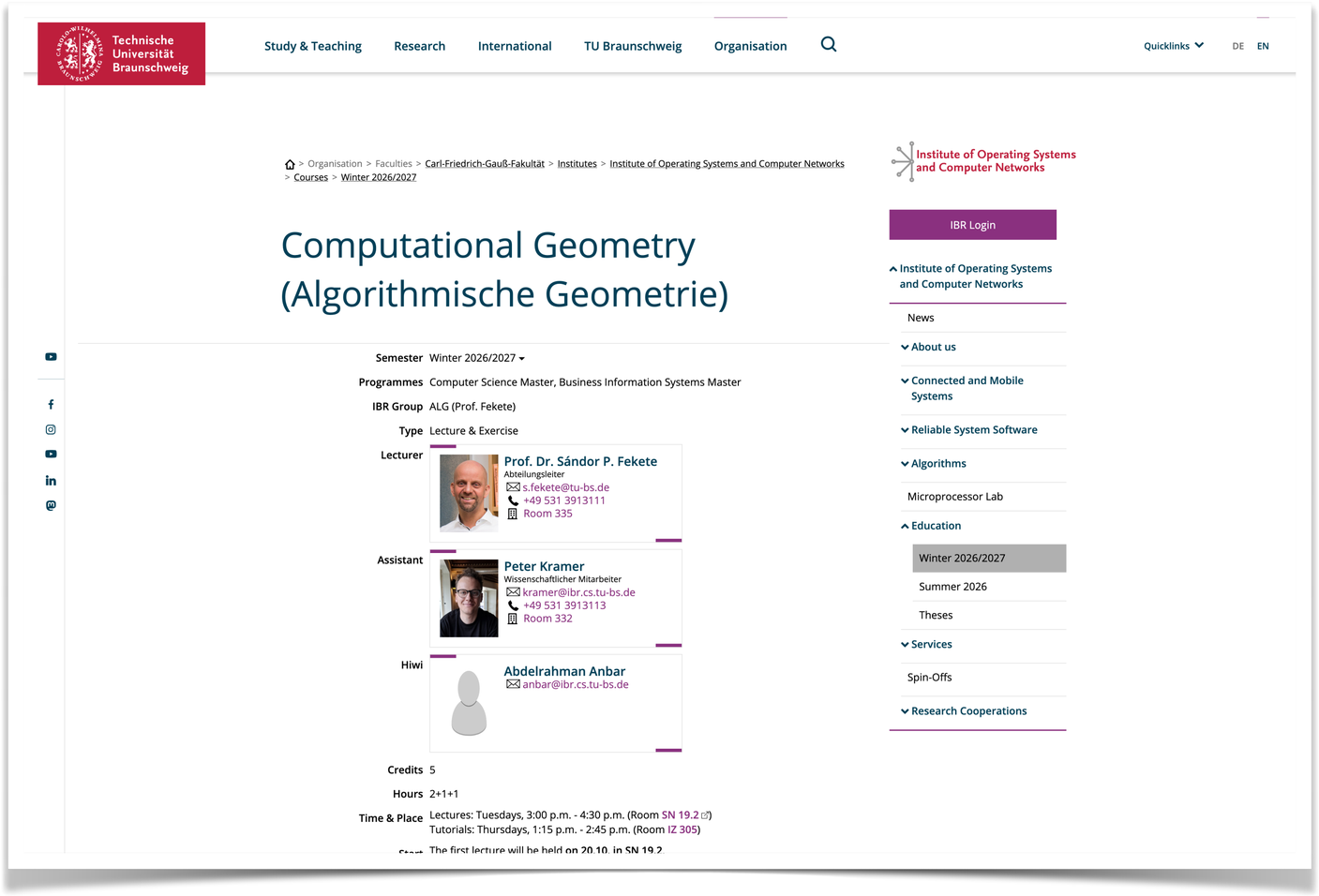
Computational Geometry

Big Tutorial #1 — Kickoff and convex hulls

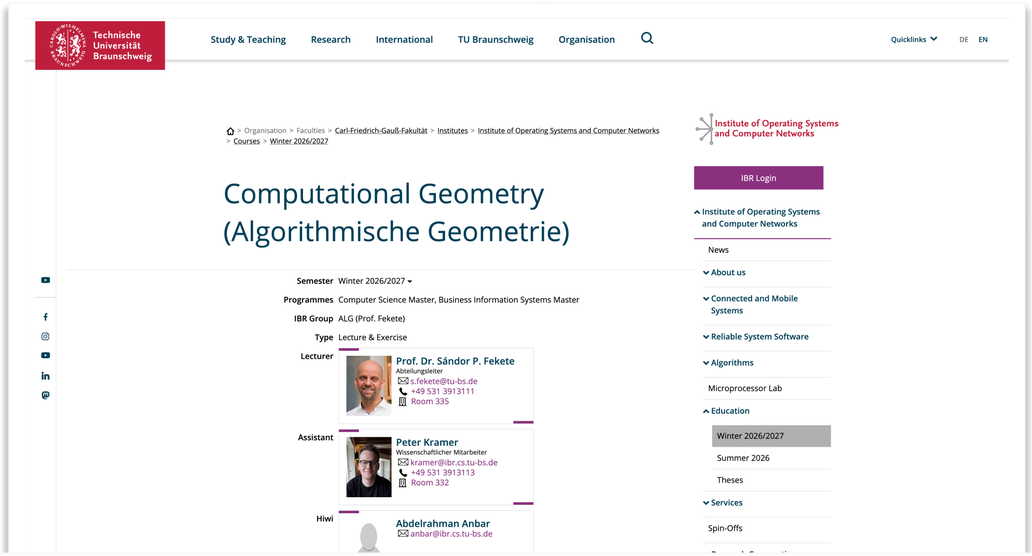
Institut für Betriebssysteme
und Rechnerverbund
Algorithmik

Peter Kramer | *October 22nd, 2026*

Website


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Study & Teaching
Research
International
TU Braunschweig
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Semester Winter 2026/2027
Programmes Computer Science Master, Business Information Systems Master
IBR Group ALG (Prof. Fekete)
Type Lecture & Exercise
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Room 332
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anbar@ibr.cs.tu-bs.de
Credits 5
Hours 2+1+1
Time & Place Lectures: Tuesdays, 3:00 p.m. - 4:30 p.m. (Room SN 19.2)
Tutorials: Thursdays, 1:15 p.m. - 2:45 p.m. (Room IZ 305)
The first lecture will be held on 20.10. in SN 19.2

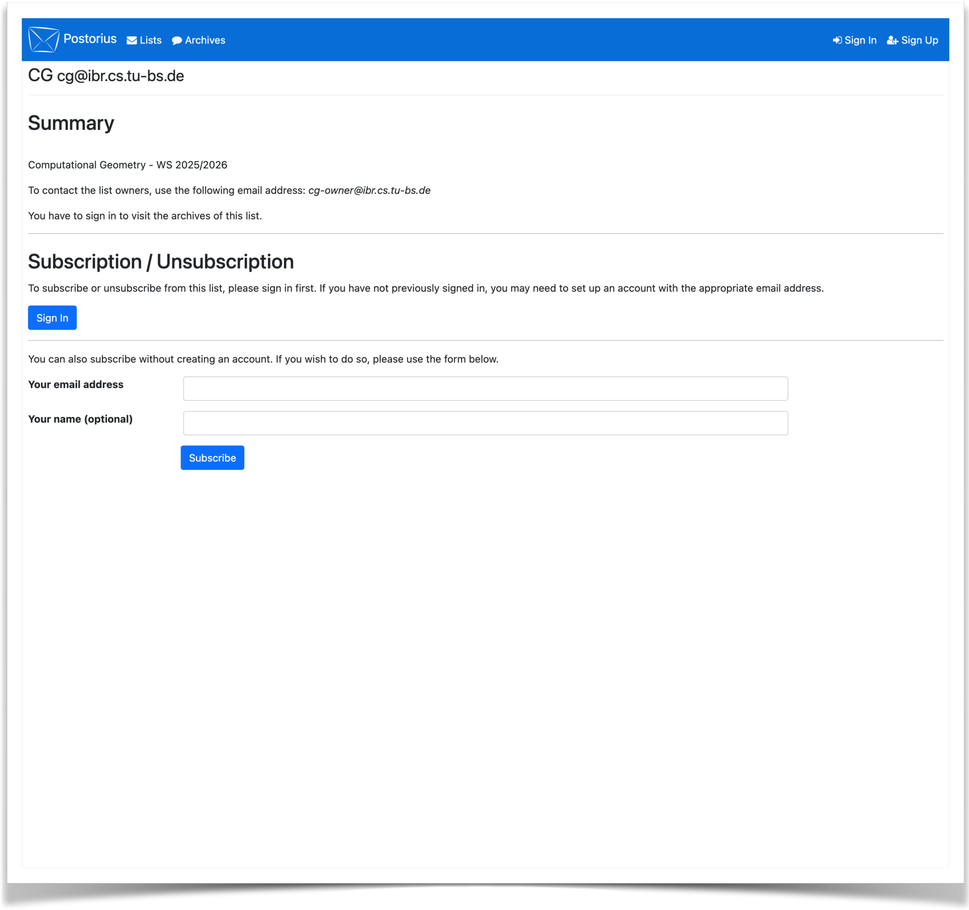
Website



Material

We provide video recordings of the lecture from 2021 below. These alone cannot replace attendance in the ongoing semester: The course structure may differ.

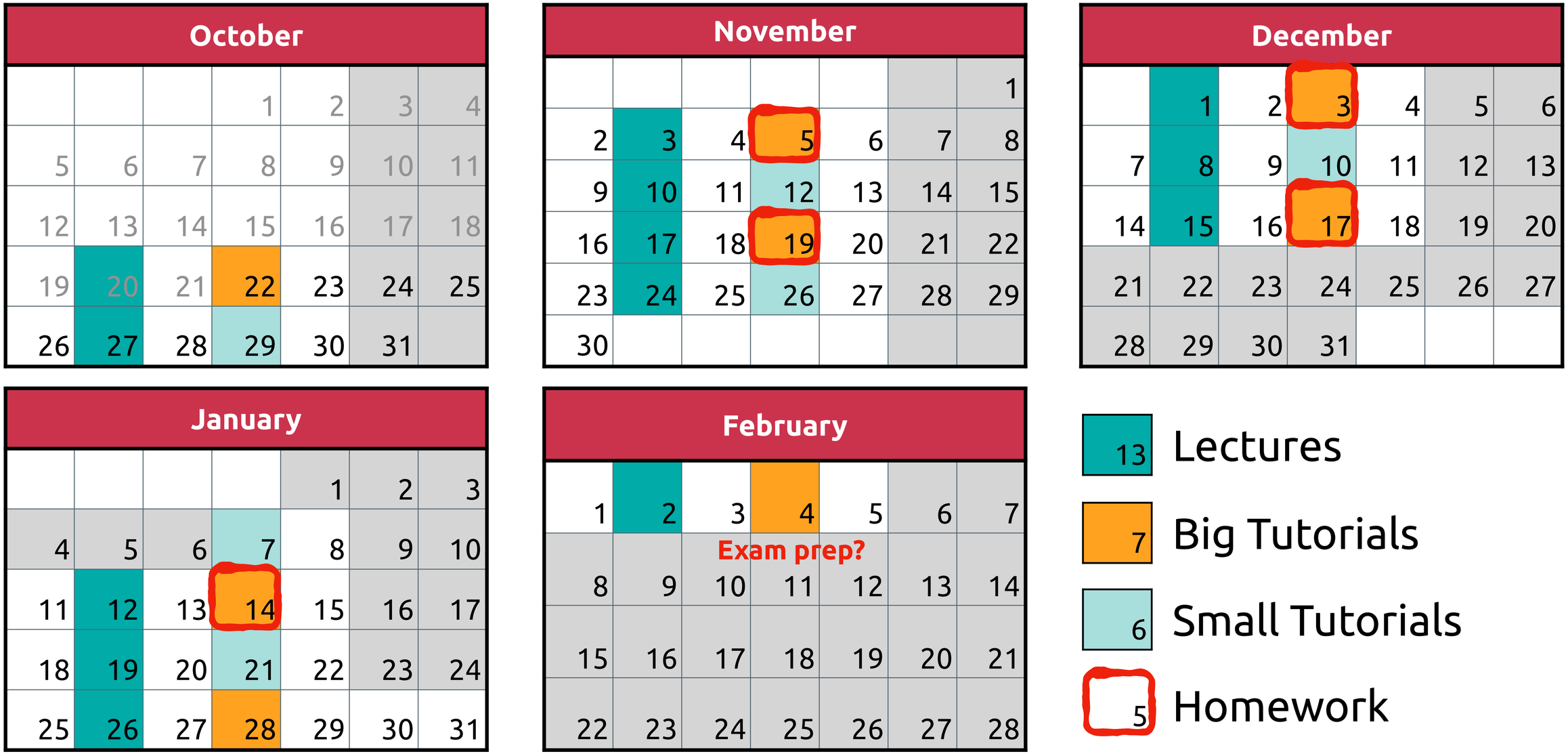
Topic	Lecture Material	References
Introduction	[PDF] [Video]	



Tutorial

- **Every Thursday at 1:15pm** in this room, either a big or a small tutorial.

- **Big tutorial:** Expand upon (and use!) concepts from the lecture.
- **Small tutorial:** Homework discussion and live exercises.


October
1 2 3 4
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January
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February
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Exam prep?
8 9 10 11 12 13 14
15 16 17 18 19 20 21
22 23 24 25 26 27 28
13 Lectures
7 Big Tutorials
6 Small Tutorials
5 Homework

Tutorial

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- **Big tutorial:** Expand upon (and use!) concepts from the lecture.
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- **Studienleistung:** Five homework sheets.
  - Biweekly assignments.
  - 50% of total possible points on the sheets; roughly 85 - 90 points.

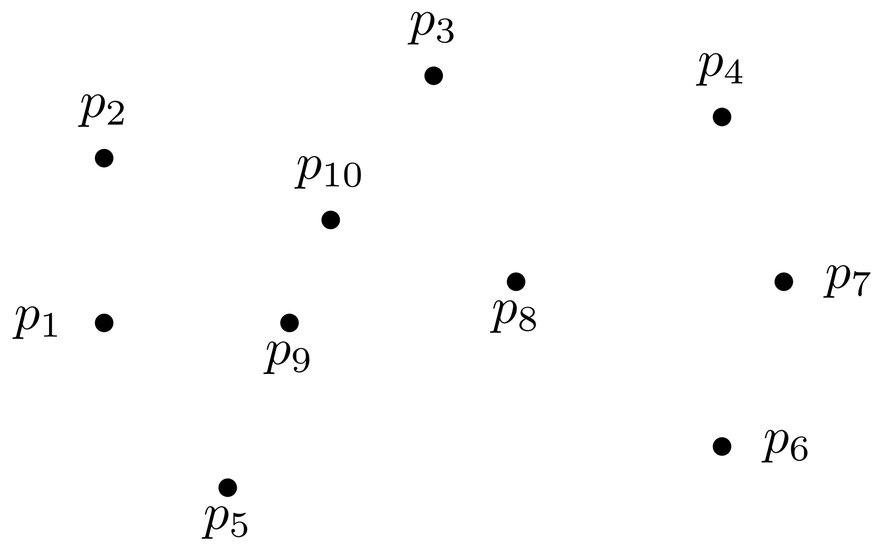
Convex Hulls

Peter Kramer | October 22nd, 2026

The convex hull

… is a point set? A convex polygon? A subset?

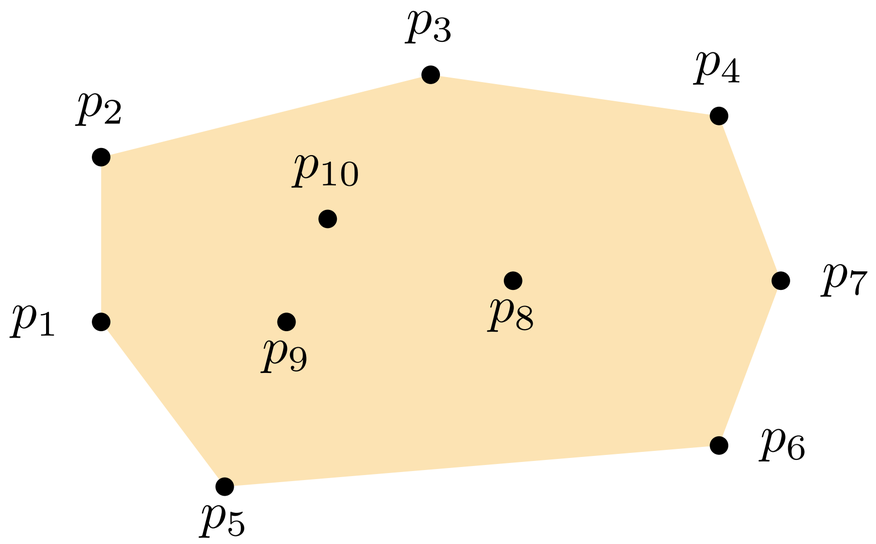
- For a finite point set $\mathscr{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$, $\mathrm{conv}(\mathscr{P}) = \ldots$?



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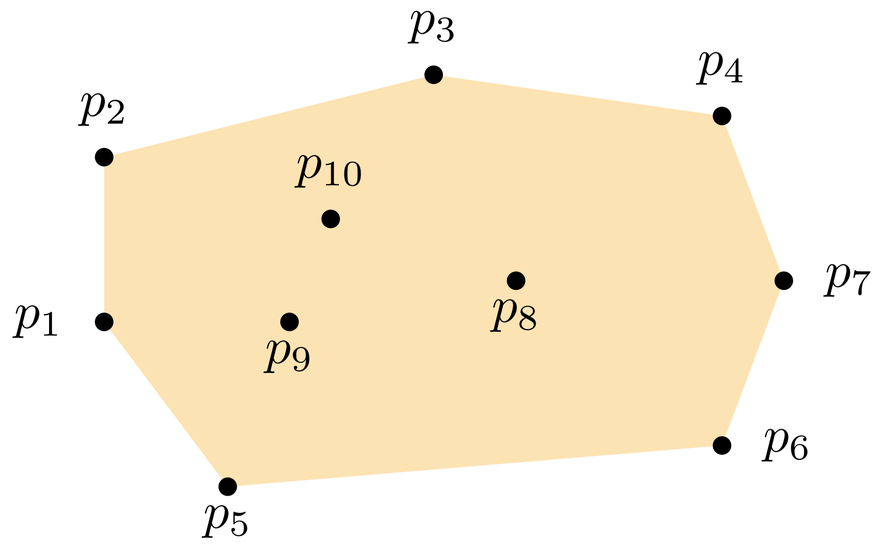
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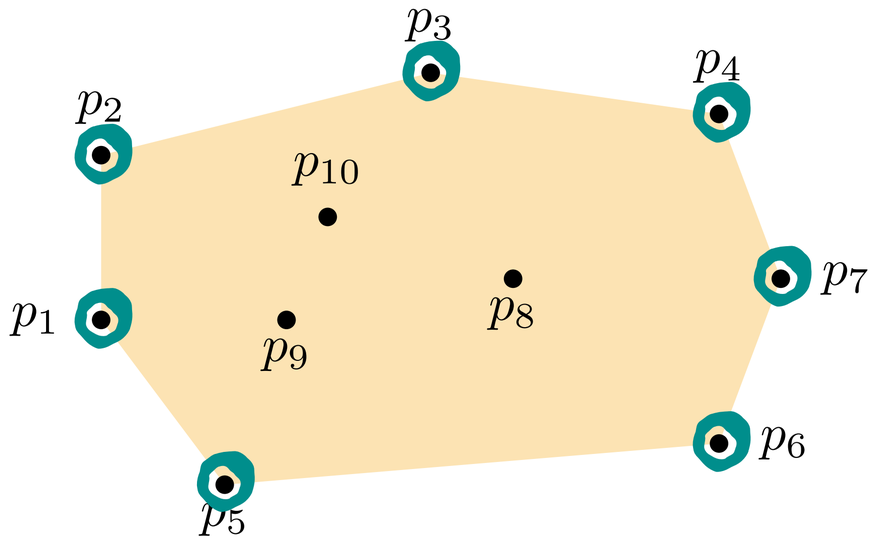
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- *This is an infinite set, but our algorithm(s) only give us points from $\mathscr{P}$!?*

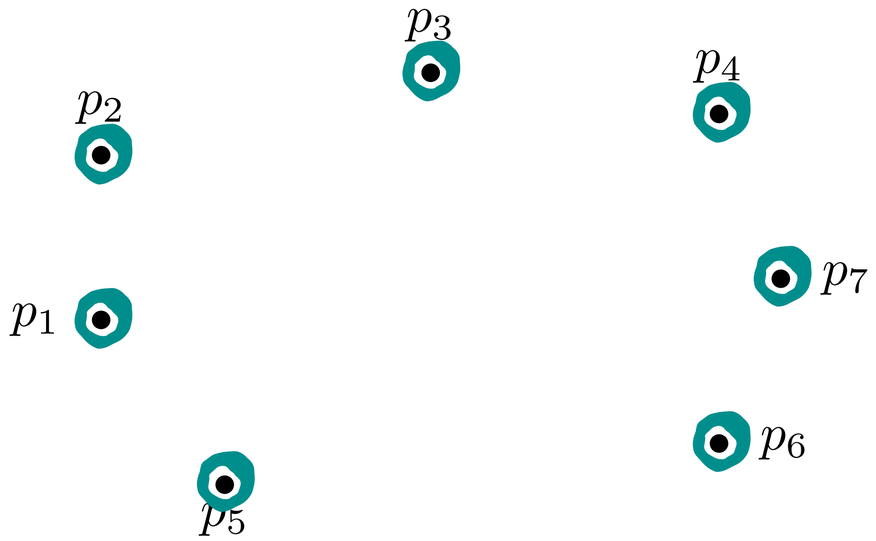


The convex hull

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- $\mathrm{conv}(\mathscr{P}) = \mathscr{P}$ *???*

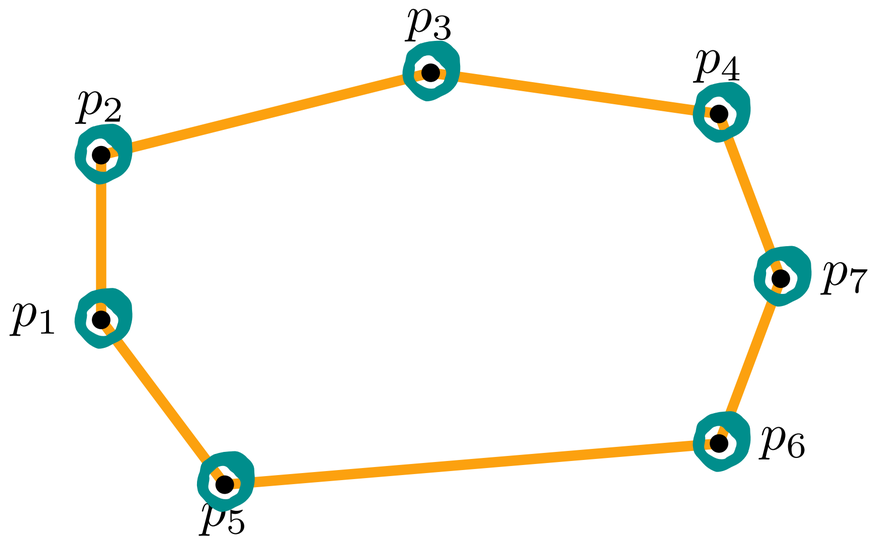
Point sets like this one are referred to as being in “convex position”.



The convex hull

… is a point set? A convex polygon? A subset?

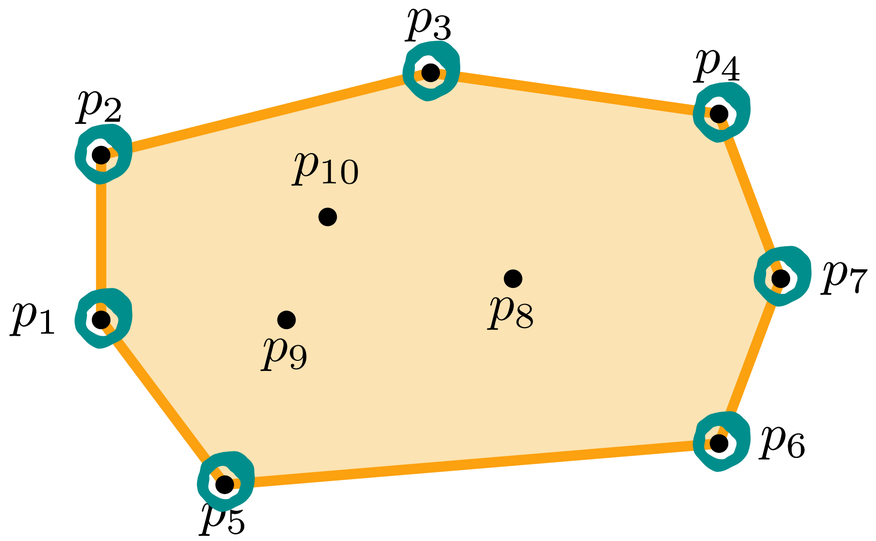
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- ~~$\mathrm{conv}(\mathscr{P}) = \mathscr{P}$~~ *???*



The convex hull

… is a (usually infinite) point set, described by a convex polygon.

- Given a finite point set $\mathscr{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$.
- *Definition 2.3:* $\mathrm{conv}(\mathscr{P}) = \{x \in \mathbb{R}^2 \mid x \text{ is a convex combination of } \mathscr{P}\}$
- *Our algorithms compute the <u>boundary set</u>* $\partial\,\mathrm{conv}(\mathscr{P})$ *as a convex polygon!*



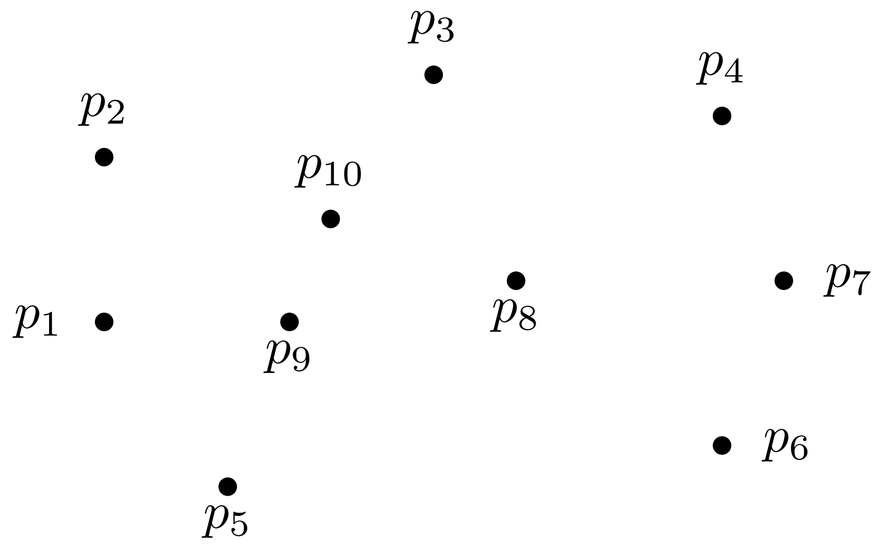
Detour: Point sets & Polygons

Peter Kramer | October 22nd, 2026

Point sets & Polygons

What's the difference?

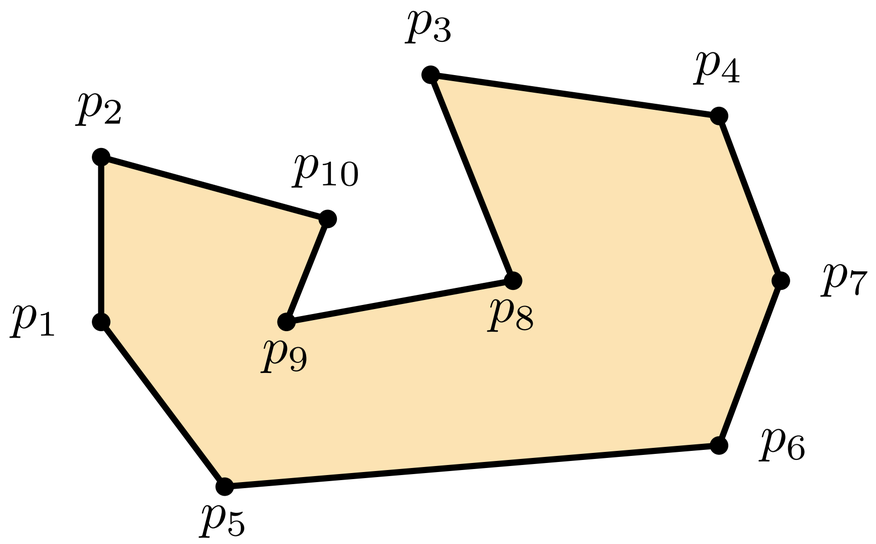
- Consider a finite point set $\mathcal{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$.
- What's a polygon P using these points?



Point sets & Polygons

What's the difference?

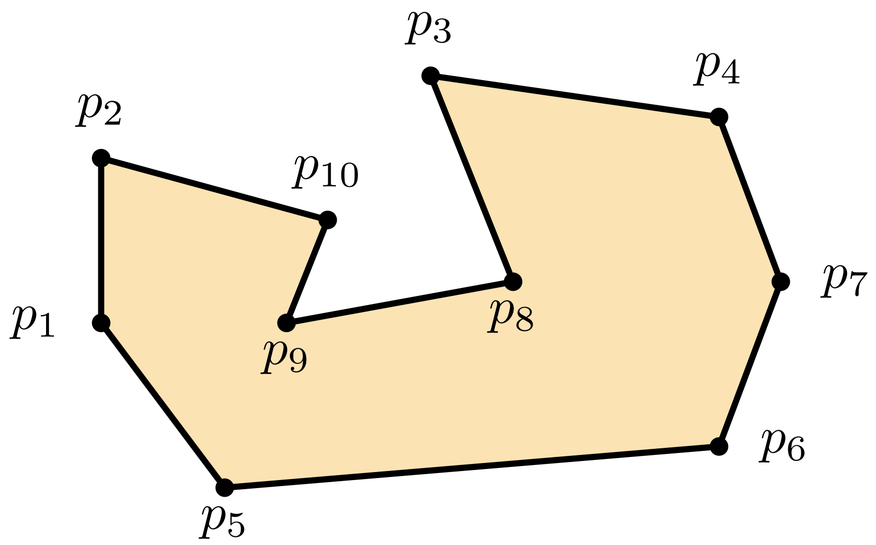
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Point sets & Polygons

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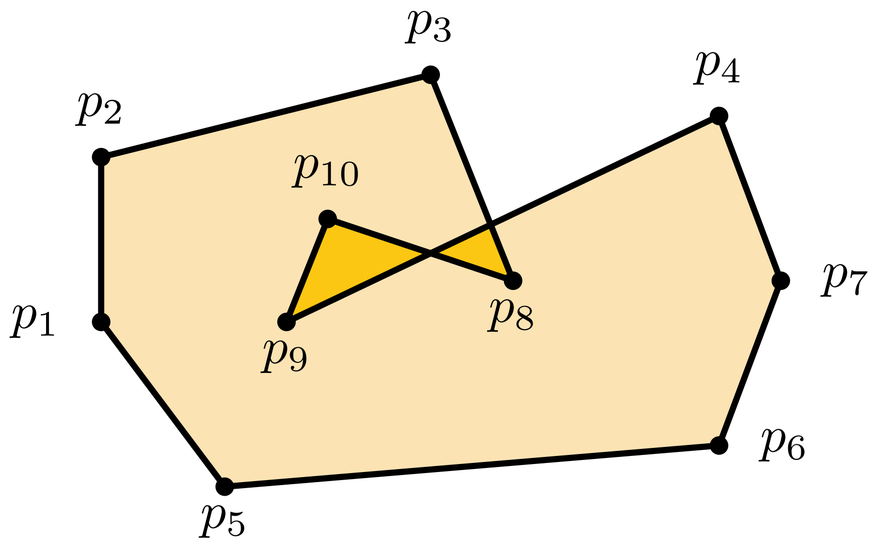
- Consider a finite point set $\mathcal{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$.
- An ordered sequence $P := p_{\pi(1)}, p_{\pi(2)}, \ldots, p_{\pi(n)} \in (\mathbb{R} \times \mathbb{R})^n$ is a polygon.



Point sets & Polygons

What's the difference?

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Point sets & Polygons

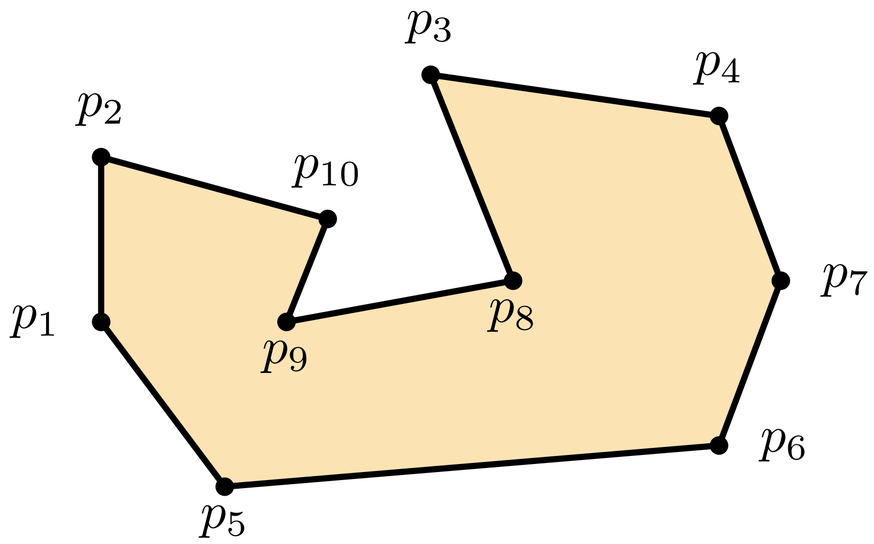
What's the difference?

- Consider a finite point set $\mathcal{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$.
- A *simple* polygon P does not self-intersect.

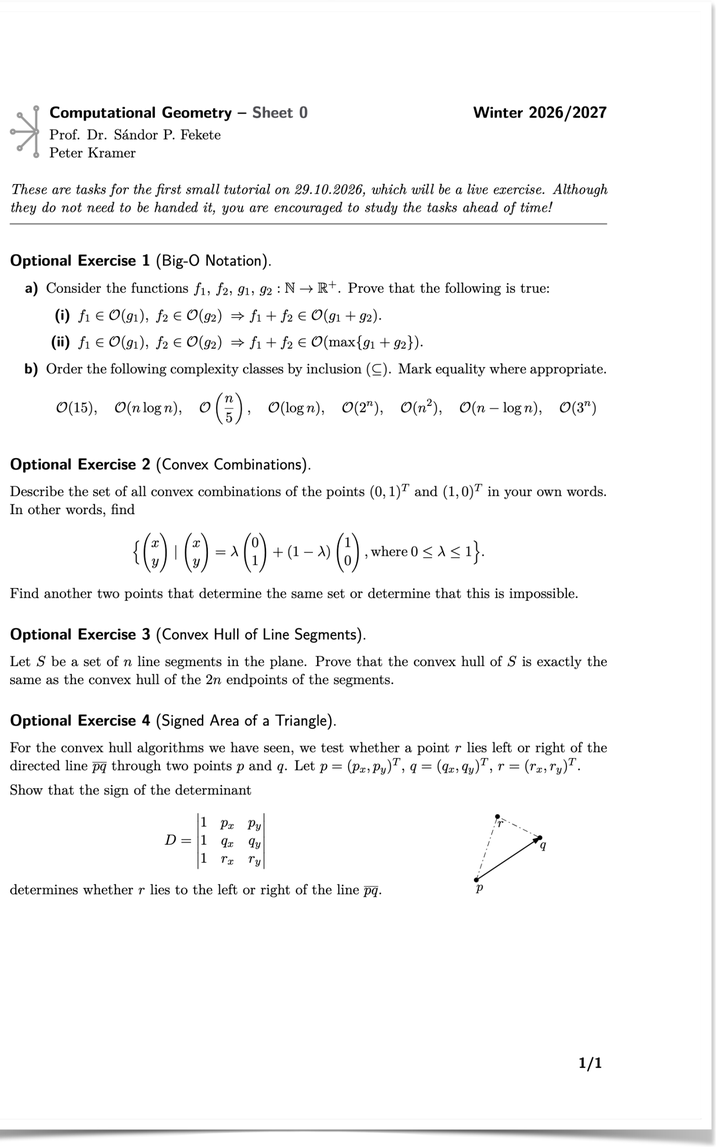
Note: We will encounter further types of polygons later on :)

Typically, we describe polygons in CCW order:

$(p_1, p_5, p_6, p_7, p_4, p_3, p_8, p_9, p_1 0, p_2)$



Live Exercise Sheet



Computational Geometry – Sheet 0 **Winter 2026/2027**

Prof. Dr. Sándor P. Fekete
Peter Kramer

These are tasks for the first small tutorial on 29.10.2026, which will be a live exercise. Although they do not need to be handed it, you are encouraged to study the tasks ahead of time!

Optional Exercise 1 (Big-O Notation).

a) Consider the functions f_1, f_2, g_1, $g_2 : \mathbb{N} \to \mathbb{R}^+$. Prove that the following is true:

(i) $f_1 \in \mathcal{O}(g_1),\ f_2 \in \mathcal{O}(g_2) \Rightarrow f_1 + f_2 \in \mathcal{O}(g_1 + g_2)$.

(ii) $f_1 \in \mathcal{O}(g_1),\ f_2 \in \mathcal{O}(g_2) \Rightarrow f_1 + f_2 \in \mathcal{O}(\max\{g_1 + g_2\})$.

b) Order the following complexity classes by inclusion ($\subseteq$). Mark equality where appropriate.

$$\mathcal{O}(15), \quad \mathcal{O}(n \log n), \quad \mathcal{O}\left(\frac{n}{5}\right), \quad \mathcal{O}(\log n), \quad \mathcal{O}(2^n), \quad \mathcal{O}(n^2), \quad \mathcal{O}(n - \log n), \quad \mathcal{O}(3^n)$$

Optional Exercise 2 (Convex Combinations).

Describe the set of all convex combinations of the points $(0,1)^T$ and $(1,0)^T$ in your own words. In other words, find

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{where } 0 \leq \lambda \leq 1 \right\}.$$

Find another two points that determine the same set or determine that this is impossible.

Optional Exercise 3 (Convex Hull of Line Segments).

Let S be a set of n line segments in the plane. Prove that the convex hull of S is exactly the same as the convex hull of the $2n$ endpoints of the segments.

Optional Exercise 4 (Signed Area of a Triangle).

For the convex hull algorithms we have seen, we test whether a point r lies left or right of the directed line $\overline{pq}$ through two points p and q. Let $p = (p_x, p_y)^T$, $q = (q_x, q_y)^T$, $r = (r_x, r_y)^T$.

Show that the sign of the determinant

$$D = \begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$$

determines whether r lies to the left or right of the line $\overline{pq}$.

1/1

Peter Kramer | October 22nd, 2026

Homework Sheet #1

Peter Kramer | October 22nd, 2026

Computational Geometry – Sheet 1 **Winter 2026/2027**

Prof. Dr. Sándor P. Fekete — **Due** 05.11.2026

Peter Kramer — **Discussion** 12.11.2026

Please submit your individual handwritten answers, using the box in front of IZ 338 before the exercise timeslot on the due date above. Make sure to include your full name, matriculation number, *and the* programme *that you are enrolled in, e.g., computer science (Master).*

You may submit your solutions in English or German. We encourage you to avoid using AI tools such as LLMs to solve these exercises.

Exercise 1. **(5 points)**

Refer to *Definitions 2.2 and 2.3* from the lecture. Prove or disprove: The intersection of two convex hulls according to *Definition 2.3* is itself convex for all finite sets $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}^2$, i.e.,

$$\text{conv}(\text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q})) = \text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q}).$$

(Hint: Review the properties of convex hulls and combinations carefully.)

Definition E1 (General position). When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edges cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

Exercise 2 (Point in Convex Polygon Problem). **(15 points)**

Using only the known predicates, design an $\mathcal{O}(\log n)$ algorithm for the following problem. Given a convex polygon $P = (p_1, p_2, \ldots, p_n)$ with $p_i \in \mathbb{R}^2$ and an additional point $q \in \mathbb{R}^2$, does P contain q? You may assume that the point set $\{p_1, p_2, \ldots, p_n, q\}$ obeys general position.

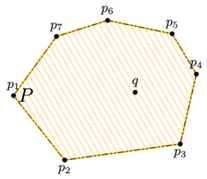


Figure 1: Example: the point q is located inside the convex polygon P.

(Hint: Let $1 < i < n$. When you use an orientation test on p_1, p_i, and q, what does this tell you? For instance, if you test $rightTurn(p_1, p_4, q)$ and obtain `true`, what can you say about q with respect to P?)

1/2

Two weeks per sheet

Computational Geometry – Sheet 1

Prof. Dr. Sándor P. Fekete

Peter Kramer

Winter 2026/2027

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These sheets are here to help you **understand** the material.

I recommend that you avoid using LLMs to solve them.

Submit individually, but of course please talk to each other! :)

Computational Geometry – Sheet 1 **Winter 2026/2027**

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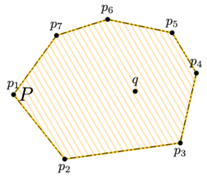


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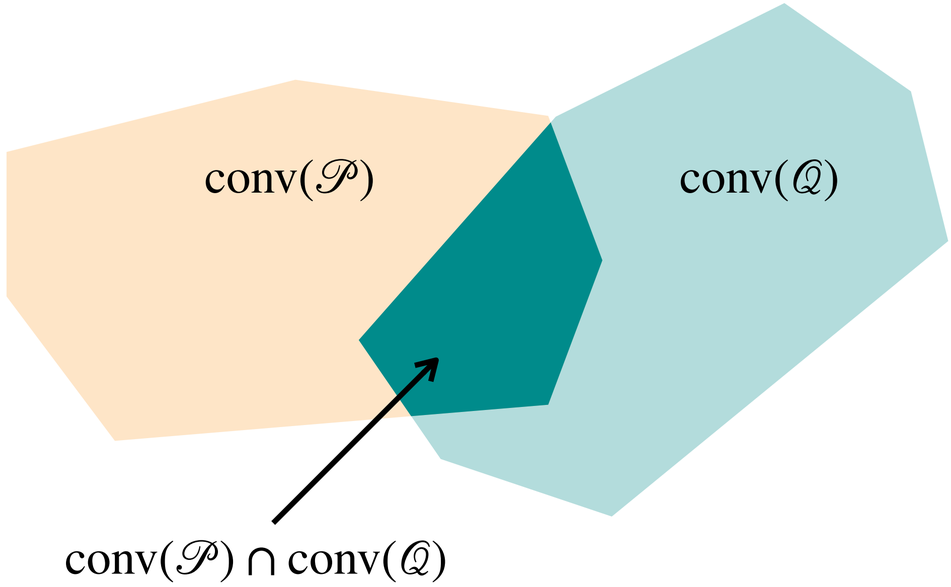
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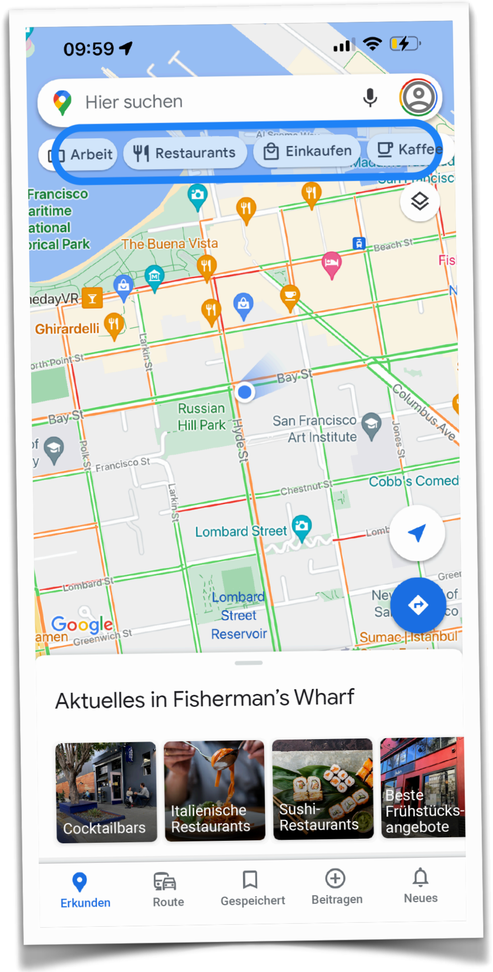
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Point Location Problems

“Where am I?”

- Given geometric information such as a map in the plane, how can we decide **where** we are?



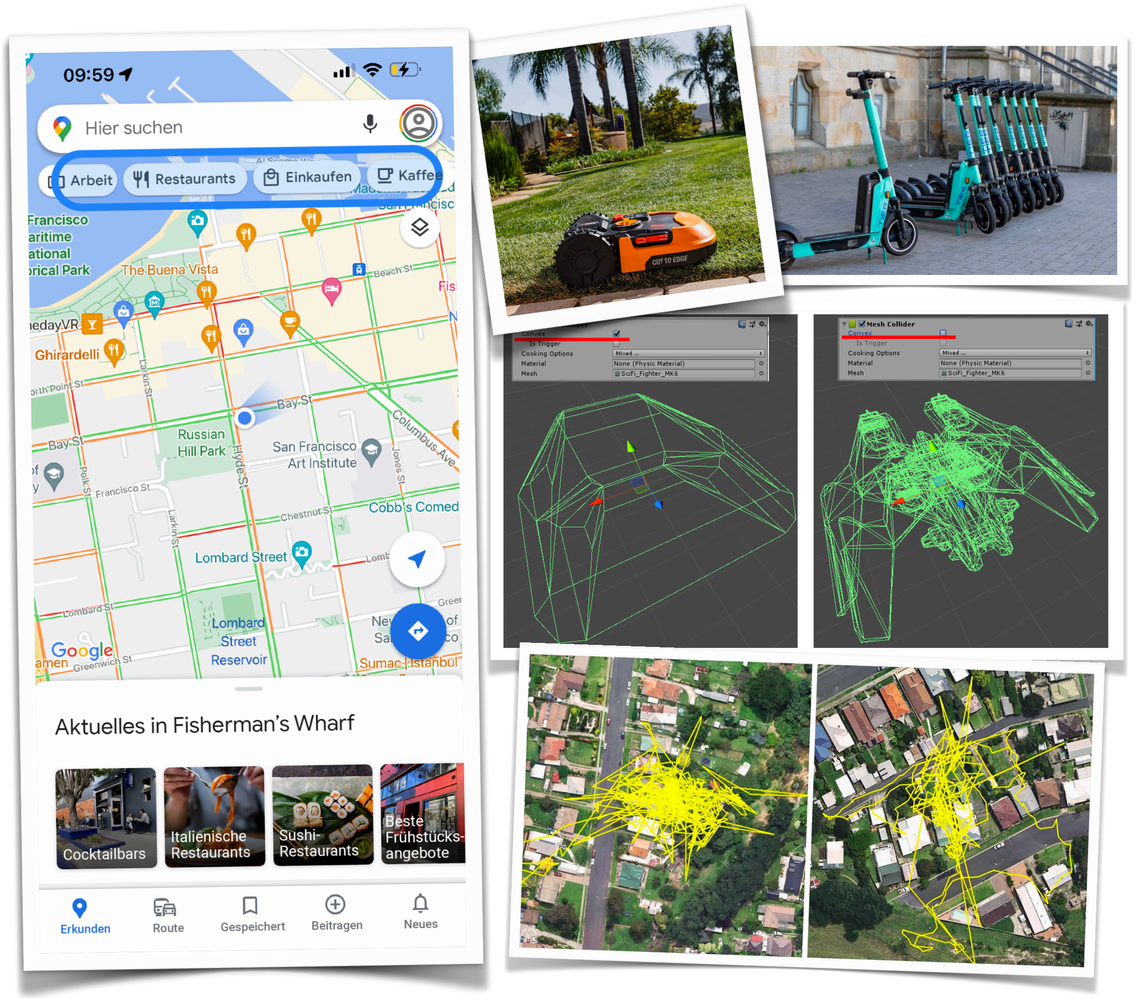
Point Location Problems

“Where am I?”

- Given geometric information such as a map in the plane, how can we decide **where** we are?

- *“In which country am I right now?”*
  “Can I leave this scooter here?”
  “Do these virtual objects collide?”

- ***“Is point p inside of region P?”***



Applications: Geofencing, Navigation, Simulation Software, Outlier Detection, …

Computational Geometry – Sheet 1 **Winter 2026/2027**

Prof. Dr. Sándor P. Fekete — **Due** 05.11.2026

Peter Kramer — **Discussion** 12.11.2026

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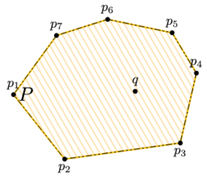


Figure 1: Example: the point q is located inside the convex polygon P.

(Hint: Let $1 < i < n$. When you use an orientation test on p_1, p_i, and q, what does this tell you? For instance, if you test *rightTurn*(p_1, p_4, q) and obtain `true`, what can you say about q with respect to P?)

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Definition E1 (General position). When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edges cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

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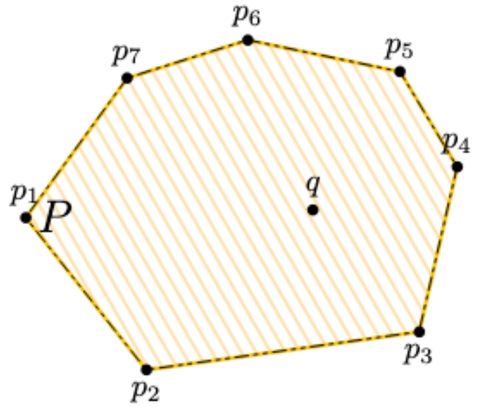


Figure 1: Example: the point q is located inside the convex polygon P.

Exercise 3 (Diameter of a Point Set). **(5+10 points)**

Given a set $\mathcal{P}$ of n points in the Euclidean plane, two points $p, q \in \mathcal{P}$ are a *farthest pair* in $\mathcal{P}$ if

$$\forall\, u, v \in \mathcal{P}: \quad \|p - q\|_2 \;\geq\; \|u - v\|_2.$$

The Euclidean distance between p and q is then also called the *diameter* $\operatorname{diam}(\mathcal{P})$.

a) Prove that all farthest pairs in $\mathcal{P}$ are vertices of the convex hull $\operatorname{conv}(\mathcal{P})$.

b) Design an $\mathcal{O}(n)$ time algorithm that approximates $\operatorname{diam}(\mathcal{P})$ within some factor $c \geq 1$. Your algorithm should return a value $d \in \mathbb{R}_{\geq 0}$ such that $d \leq \operatorname{diam}(\mathcal{P}) \leq d \cdot c$.

Briefly explain the approximation factor c and the linear runtime.

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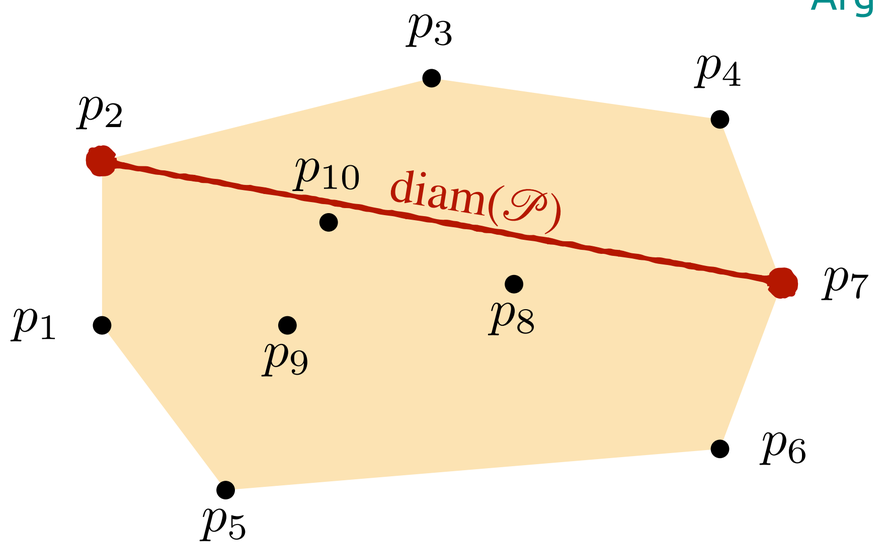
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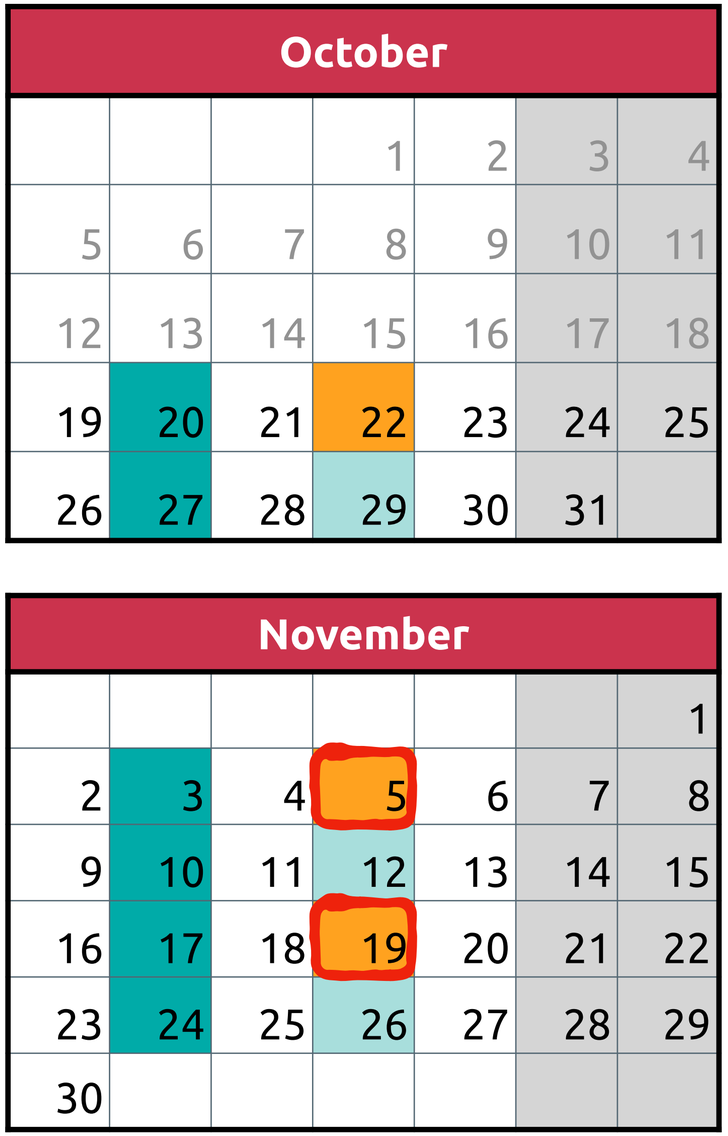


Argue that your algorithm's output is
- at most $c \cdot \operatorname{diam}(\mathcal{P})$, and
- at least $\operatorname{diam}(\mathcal{P})$.

TL;DL

The important parts

- **Sign up for the mailing list!**
- One tutorial a week (except holidays).
- Biweekly homework, submit individually.
- First **sheet is out now**, due in two weeks.
- Material can be found on the website.
  - Slides, videos, and homework sheets.
- Any questions? Ask Peter or Ash:
  - kramer@ibr.cs.tu-bs.de
  - anbar@ibr.cs.tu-bs.de



Extra slides

Big O Notation

Catching up, if necessary.

- If you speak German: There is plenty of material on the course website of “Algorithmen und Datenstrukturen”
- If you don’t: There is plenty of other courseware (e.g., from MIT) available!
- There also exist a variety of short-form explanation videos, such as:

Institut für Betriebssysteme und Rechnerverbund
Algorithmik

MIT OpenCourseWare

