

Computational Geometry – Sheet 2 **Winter 2026/2027**

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Due 19.11.2026
Discussion 26.11.2026

Please submit your individual handwritten answers, using the box in front of `IZ 338` before *the tutorial timeslot on the due date above. Make sure to include your* full name, matriculation number, *and the* programme *that you are enrolled in, e.g., computer science (Master).*

You may submit your solutions in English or German. We encourage you to avoid using AI tools such as LLMs to solve these exercises.

Exercise 1 (Closest pair on a circle). (10 points)

Let $\mathcal{P}$ be a set of $n \geq 3$ points on the unit circle. That is, for every point $p \in P$, we have $\|p\|_2 = 1$.

a) Prove that all closest pairs in $\mathcal{P}$ are pairs of consecutive points on the circle (they occur successively in the circular order of points).

b) Use the claim from a) to find an algorithm that finds a closest pair in $\mathcal{O}(n \log n)$ time.

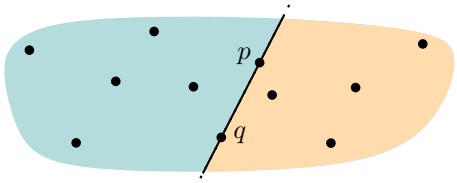
c) Does the claim from a) hold if the points are vertices of an arbitrary convex polygon?

Exercise 2 (Partitioning Points). (10 points)

Let $\mathcal{P}$ now be a set of $n \geq 3$ points in the plane, in general position (recall *Definition E1*).

a) Prove that there exist points $p, q \in \mathcal{P}$ that divide $\mathcal{P}$ evenly based on left-/rightTurn:

$$|\{\, r \in \mathcal{P} \mid \text{leftTurn}(p, q, r) = \texttt{true} \,\}| = {}^{|\mathcal{P}|}/_2 \pm 1.$$

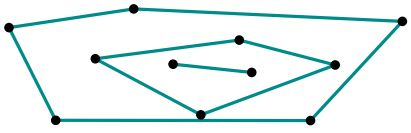


b) Design an algorithm that finds p and q in $\mathcal{O}(n)$ time for $n = |\mathcal{P}|$.

Exercise 3 (Convex layers). (10 points)

The *convex layers* of a finite point set $\mathcal{P}$ in the plane correspond to a decomposition of $\mathcal{P}$ into nested, convex polygons (*layers*). The outermost layer L_0 consists exactly of the extremal points defining $\text{conv}(P)$. The next layer is recursively defined as points defining $\text{conv}(P \setminus L_0)$, meaning

$$L_i = \mathcal{P} \cap \delta\,\text{conv}\Big(\mathcal{P} \setminus \bigcup_{j \in [0,i]} L_j\Big).$$



Design an algorithm which computes the convex layers of n points in the Euclidean plane, in $\mathcal{O}(n^2)$ time. Briefly argue its runtime and correctness.