

Computational Geometry – Sheet 1 **Winter 2026/2027**

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Due 05.11.2026
Discussion 12.11.2026

Please submit your individual handwritten answers, using the box in front of `IZ 338` before *the exercise timeslot on the due date above. Make sure to include your* full name, matriculation number, *and the* programme *that you are enrolled in, e.g., computer science (Master).*

You may submit your solutions in English or German. We encourage you to avoid using AI tools such as LLMs to solve these exercises.

Exercise 1. (5 points)

Refer to *Definitions 2.2 and 2.3* from the lecture. Prove or disprove: The intersection of two convex hulls according to *Definition 2.3* is itself convex for all finite sets $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}^2$, i.e.,

$$\text{conv}(\text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q})) = \text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q}).$$

(Hint: Review the properties of convex hulls and combinations carefully.)

Definition E1 (General position). When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edges cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

Exercise 2 (Point in Convex Polygon Problem). (15 points)

Using only the known predicates, design an $\mathcal{O}(\log n)$ algorithm for the following problem. Given a convex polygon $P = (p_1, p_2, \ldots, p_n)$ with $p_i \in \mathbb{R}^2$ and an additional point $q \in \mathbb{R}^2$, does P contain q? You may assume that the point set $\{p_1, p_2, \ldots, p_n, q\}$ obeys general position.

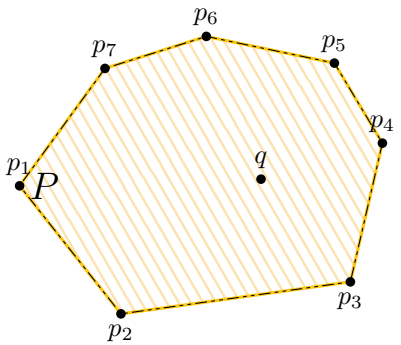


Figure 1: Example: the point q is located inside the convex polygon P.

(Hint: Let $1 < i < n$. When you use an orientation test on p_1, p_i, and q, what does this tell you? For instance, if you test $\mathit{rightTurn}(p_1, p_4, q)$ and obtain `true`, what can you say about q with respect to P?)

Exercise 3 (Diameter of a Point Set). **(5+10 points)**

Given a set $\mathcal{P}$ of n points in the Euclidean plane, two points $p, q \in \mathcal{P}$ are a *farthest pair* in $\mathcal{P}$ if

$$\forall\, u, v \in \mathcal{P}: \quad \|p - q\|_2 \;\geq\; \|u - v\|_2.$$

The Euclidean distance between p and q is then also called the *diameter* $\operatorname{diam}(\mathcal{P})$.

a) Prove that all farthest pairs in $\mathcal{P}$ are vertices of the convex hull $\operatorname{conv}(\mathcal{P})$.

b) Design an $\mathcal{O}(n)$ time algorithm that approximates $\operatorname{diam}(\mathcal{P})$ within some factor $c \geq 1$. Your algorithm should return a value $d \in \mathbb{R}_{\geq 0}$ such that $d \leq \operatorname{diam}(\mathcal{P}) \leq d \cdot c$.

Briefly explain the approximation factor c and the linear runtime.