Technische Universität Braunschweig

Algorithmen und Datenstrukturen – Übung #5

Mergesort, Master-Theorem, Heapsort

Ramin & Chek-Manh

15.01.2026

Mergesort

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Algorithmus (grob):

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1. Sortiere linke Hälfte
2. Sortiere rechte Hälfte
3. Füge beide Hälften zusammen (merge)

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$$
\begin{aligned}
&\textbf{function } \text{MERGESORT}(A,\, p,\, r)\\
&\quad \textbf{if } p < r \textbf{ then}\\
&\qquad q \leftarrow \left\lfloor \frac{p+r}{2} \right\rfloor\\
&\qquad \text{MERGESORT}(A,\, p,\, q)\\
&\qquad \text{MERGESORT}(A,\, q+1,\, r)\\
&\qquad \text{MERGE}(A,\, p,\, q,\, r)
\end{aligned}
$$

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Mergesort - Laufzeit

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Sei $T(n)$ die Laufzeit von Mergesort für n Elemente

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Zusammen ergibt das:

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + O(n)$$

Lösen von Rekursionsgleichungen

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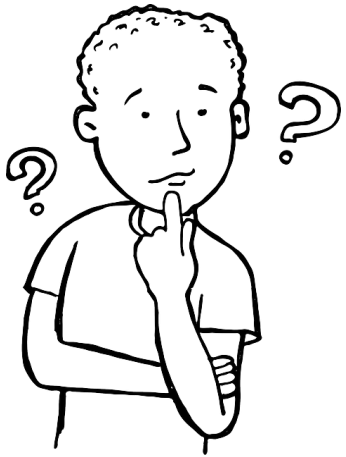

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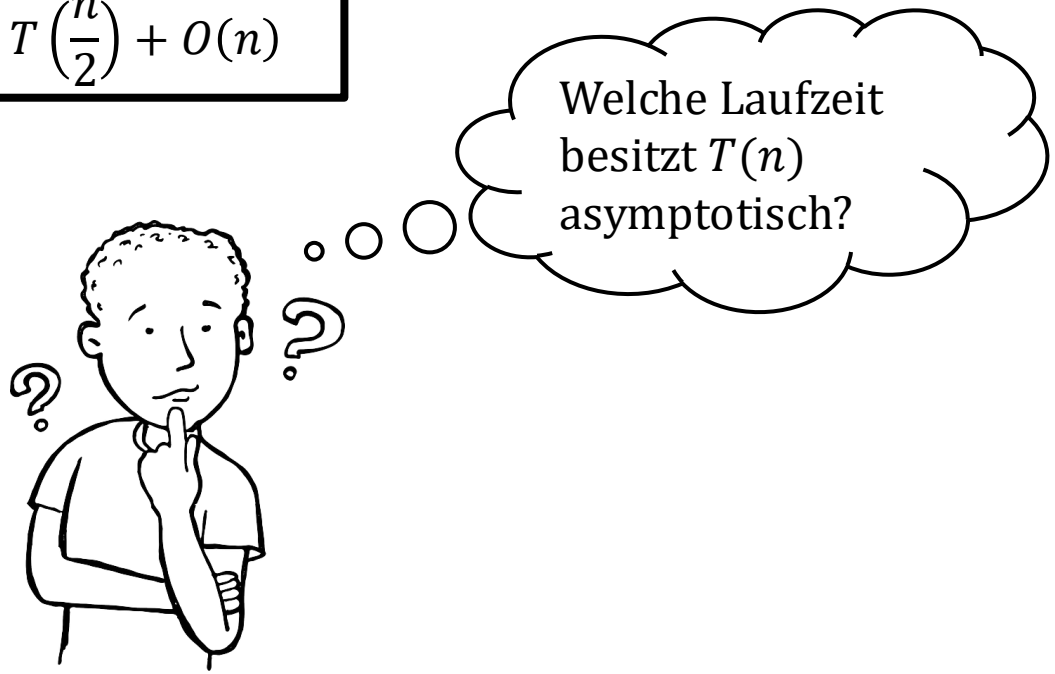

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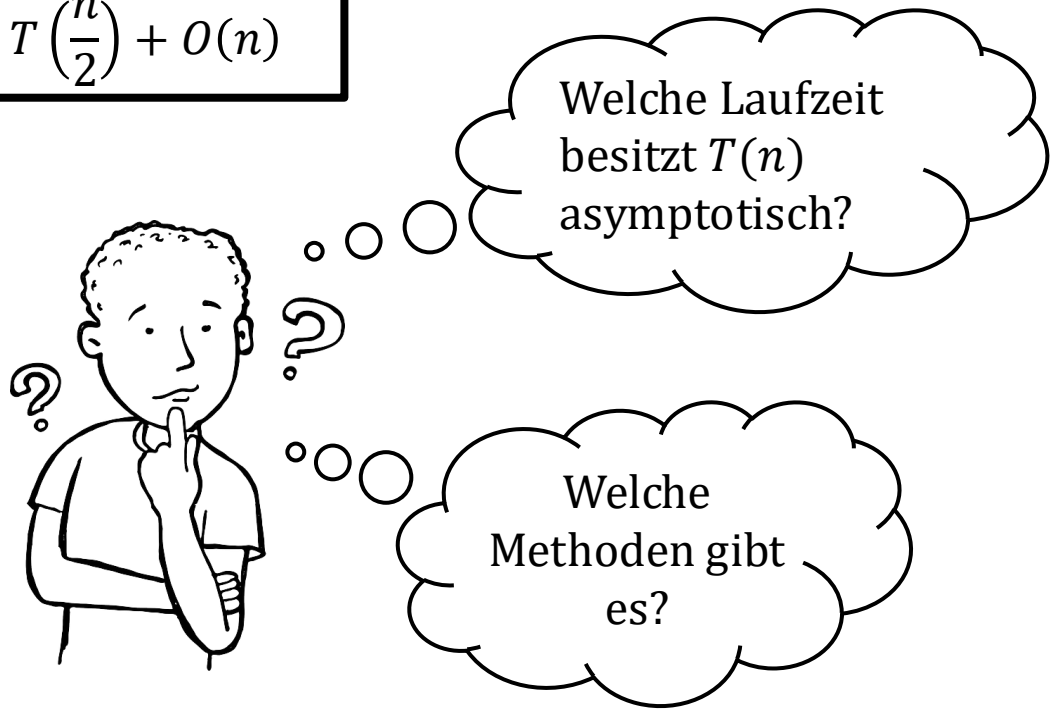

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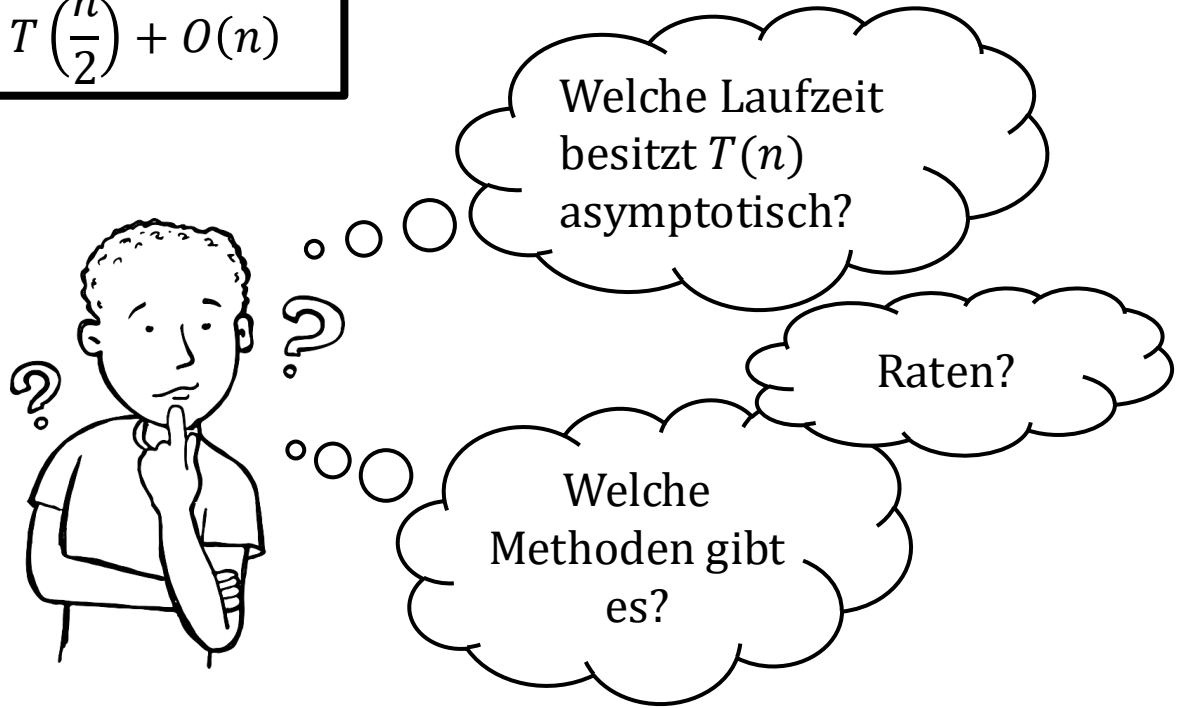

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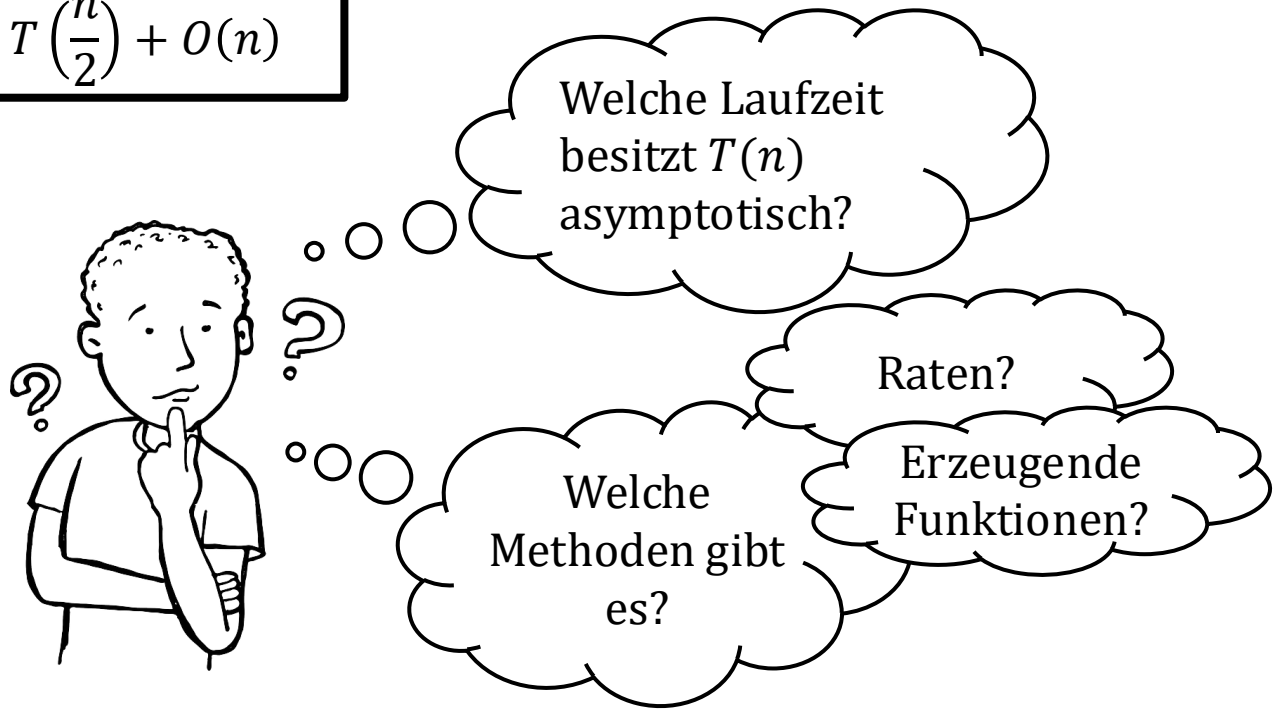

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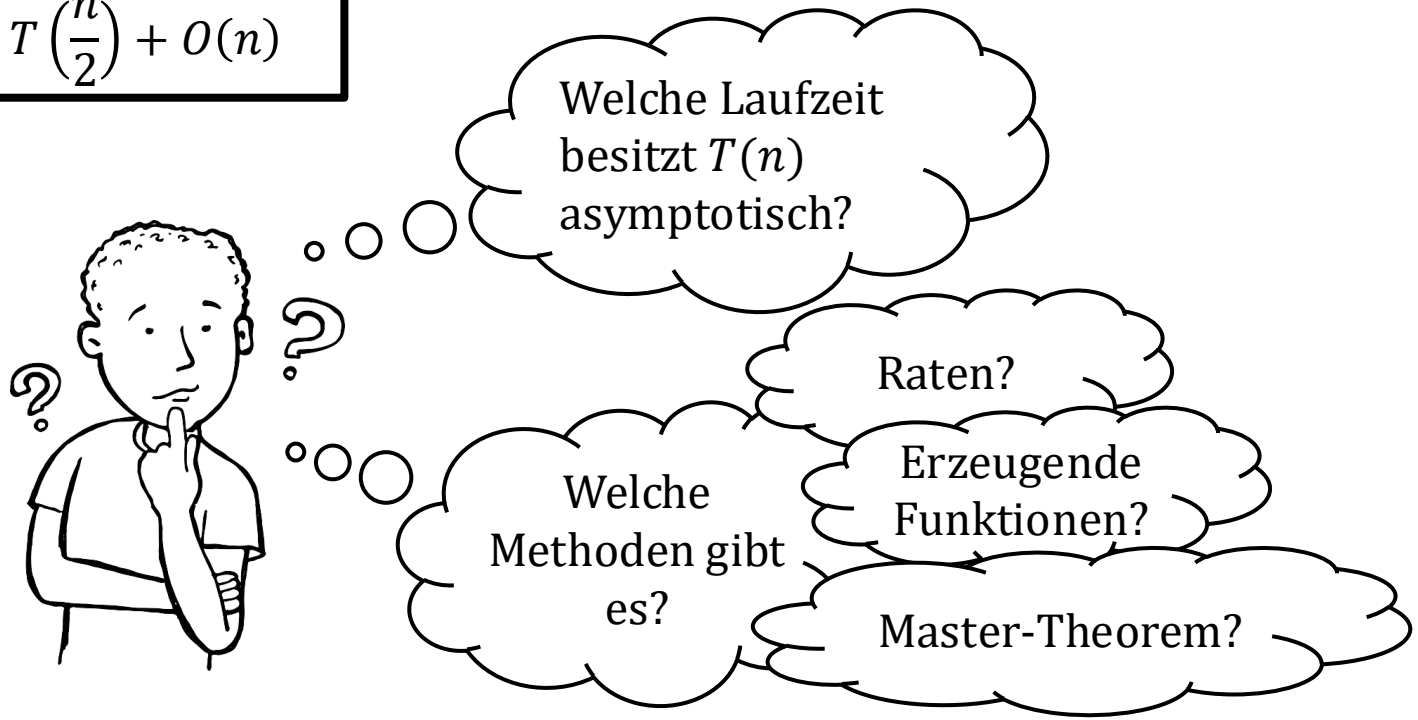

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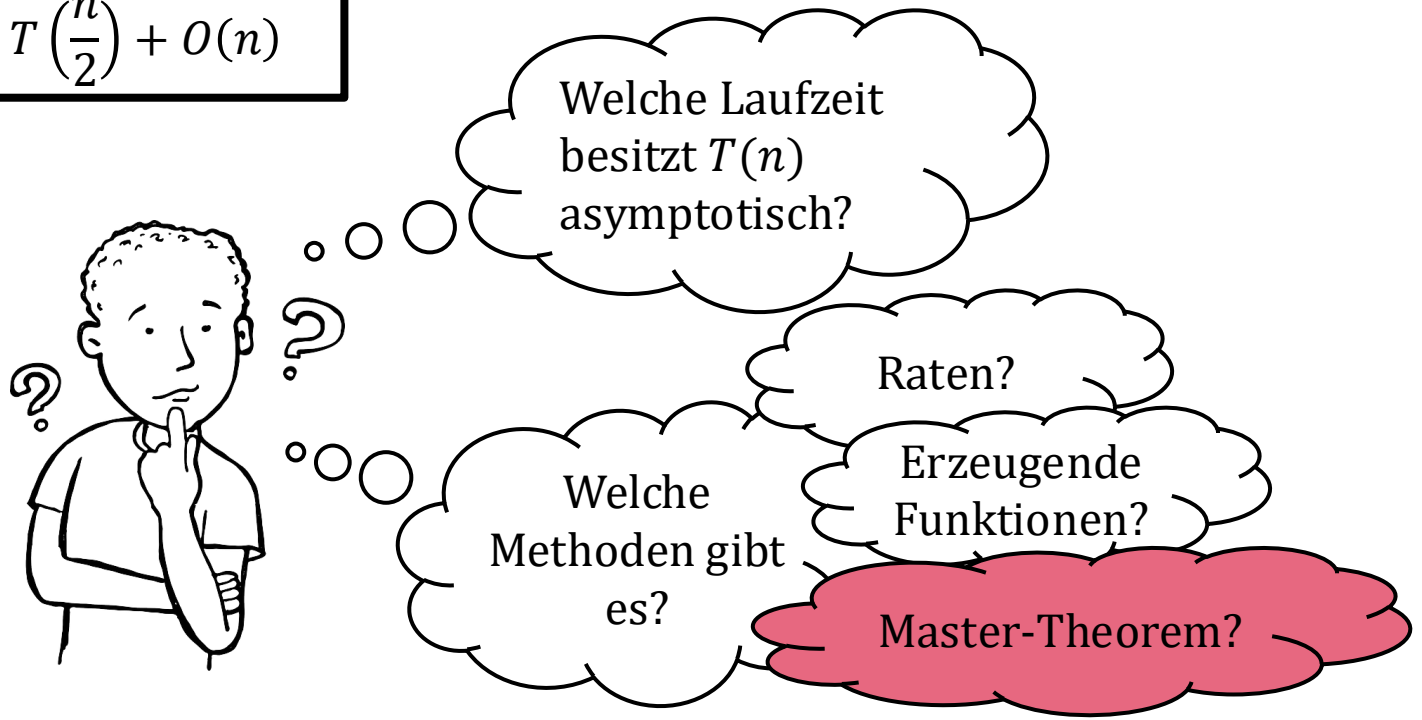

Master-Theorem

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Satz: Sei $T: \mathbb{N} \to \mathbb{R}$ eine Funktion der Form

$$T(n) \coloneqq \sum_{i=1}^{m} T(\alpha_i \cdot n) + \Theta\left(n^k\right),$$

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wobei $m \in \mathbb{N}$, $k \in \mathbb{R}$ und $0 < \alpha_i < 1$ für alle $i \in \{1, \dots, m\}$.

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Dann gilt

$$T(n) \in \begin{cases} \Theta\left(n^k\right), & \text{falls } \sum_{i=1}^{m} \alpha_i^k < 1 \\ \Theta\left(n^k \log n\right), & \text{falls } \sum_{i=1}^{m} \alpha_i^k = 1 \\ \Theta(n^c) \text{ mit } \sum_{i=1}^{m} \alpha_i^c = 1, & \text{falls } \sum_{i=1}^{m} \alpha_i^k > 1 \end{cases}$$

Master-Theorem (Beispiele)

Beispiel 1

$$U(n) = 3 \cdot U\left(\frac{n}{3}\right) + 6 \cdot U\left(\frac{n}{6}\right) + 7n \log n - 8n + 15n^2$$

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Also:

$m = 9, k = 2$ und $\alpha_1 = \cdots = \alpha_3 = \frac{1}{3}, \alpha_4 = \cdots = \alpha_9 = \frac{1}{6}$

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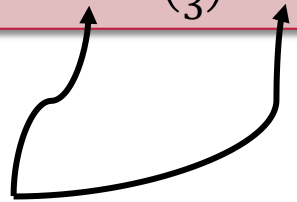

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Beispiel 1

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$$\sum_{i=1}^{9} \alpha_i^2 = 3 \cdot \left(\frac{1}{3}\right)^2 + 6 \cdot \left(\frac{1}{6}\right)^2 = \frac{1}{3} + \frac{1}{6} < \frac{1}{2} + \frac{1}{2} = 1$$

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$\Theta(n^2)$

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folgt $U(n) \in \Theta(n^2)$

Beispiel 2

$$V(n) = 7n^2 + 5n^3 + 27 \cdot V\left(\frac{n}{3}\right) - 7n$$

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$$\sum_{i=1}^{27} \alpha_i^3 = 27 \cdot \left(\frac{1}{3}\right)^3 = \frac{27}{27} = 1$$

Beispiel 2

$$V(n) = 7n^2 + 5n^3 + 27 \cdot V\left(\frac{n}{3}\right) - 7n$$

Also:

$m = 27, k = 3$ und $\alpha_1 = \cdots = \alpha_{27} = \frac{1}{3}$

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folgt $V(n) \in \Theta(n^3 \log n)$

Beispiel 3

$$W(n) = 9 \cdot W\left(\frac{n}{3}\right) + 7n$$

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Also:

$W(n) \in \Theta(n^2)$

Beispiel 4

$$U(n) = \frac{1}{2} \cdot U\left(\frac{n}{3}\right) + 6 \cdot U\left(\frac{n}{6}\right) + 7n \log n - 8n + 15n^2$$

Beispiel 4

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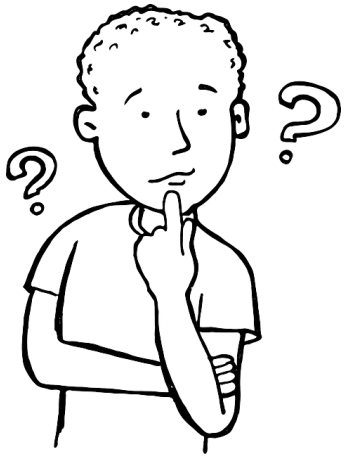

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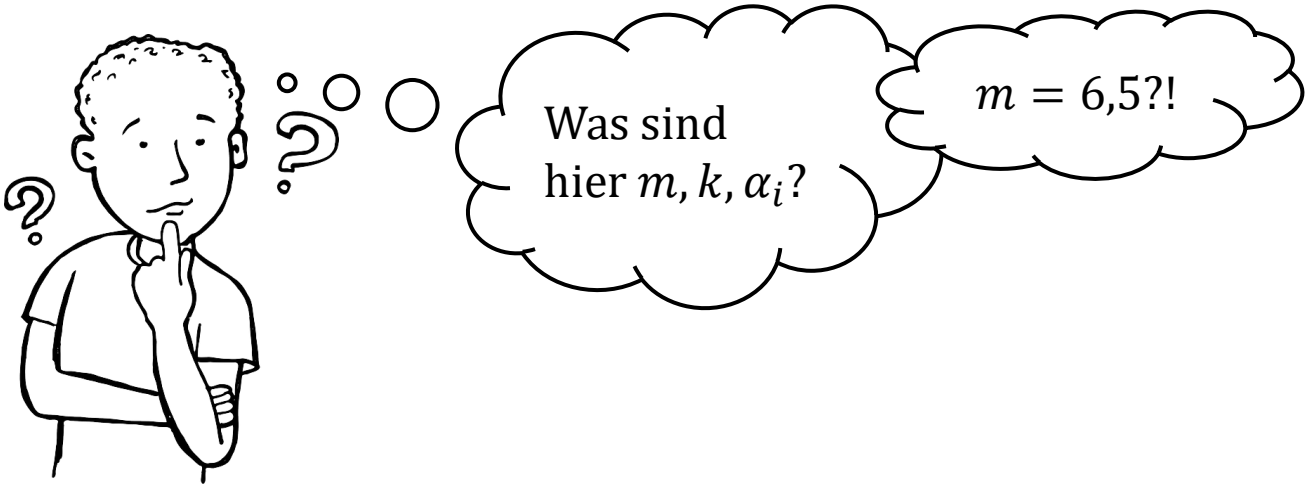

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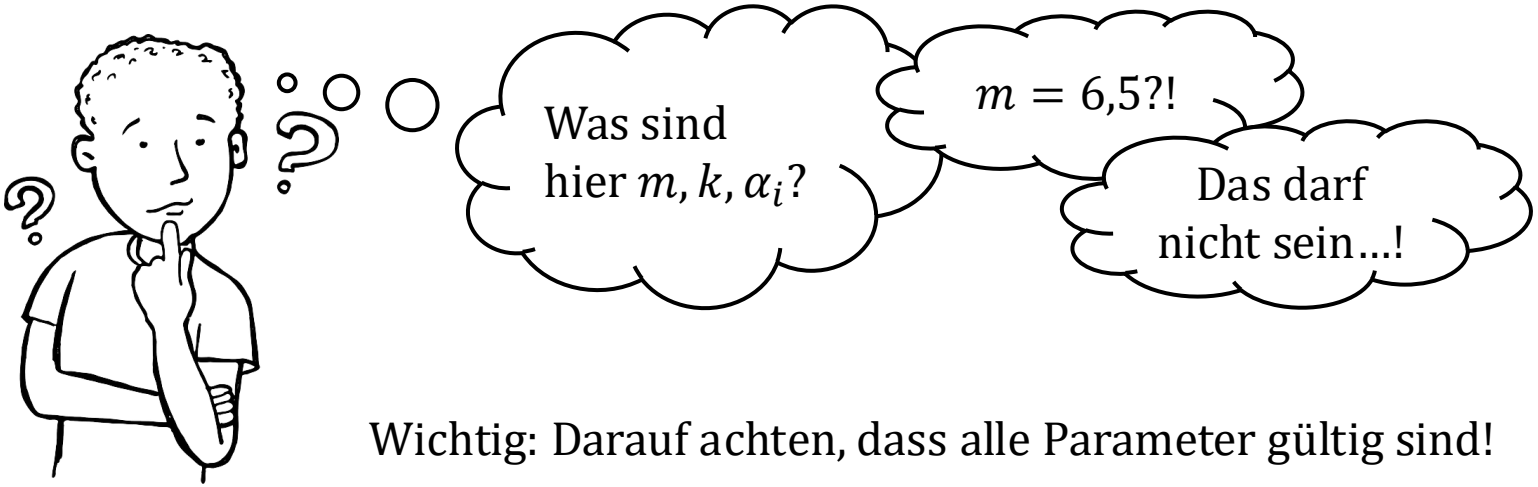
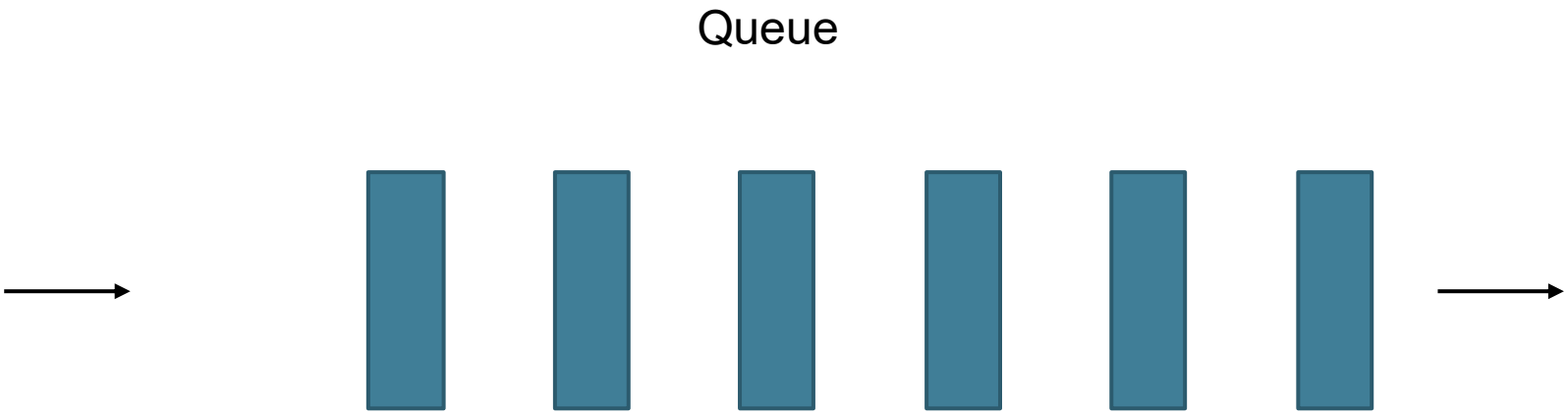


Wichtig: Darauf achten, dass alle Parameter gültig sind!

(cooles Zeugs mit)

Max-Heaps

Priority Queue



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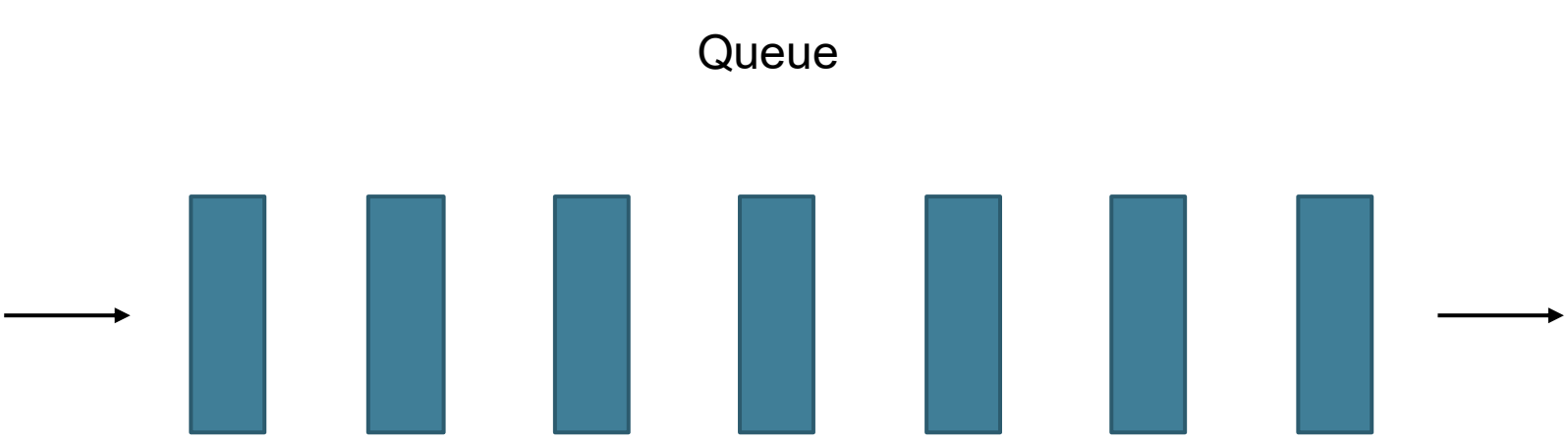

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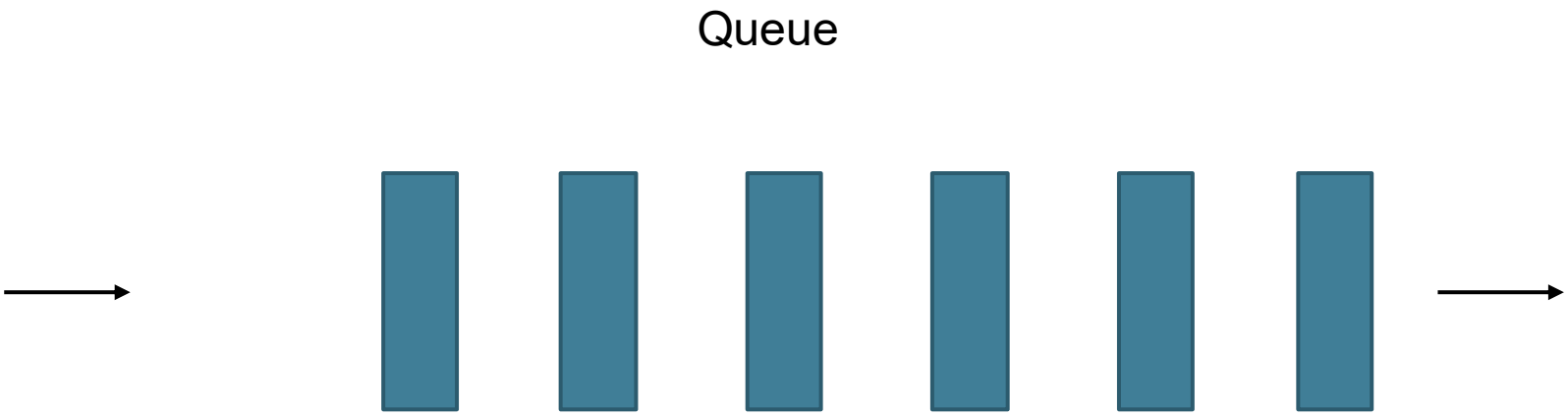

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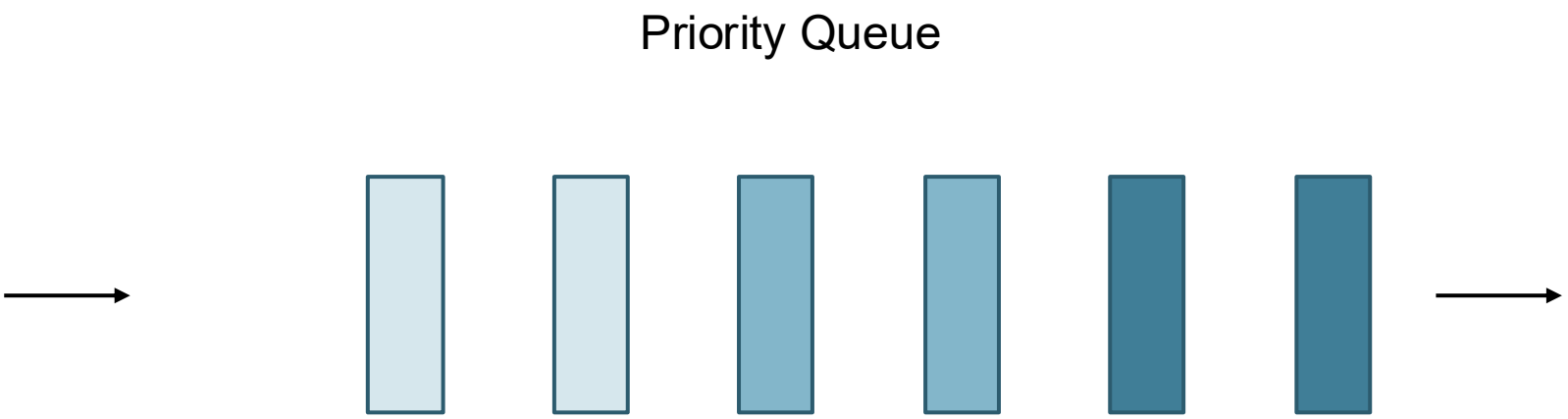

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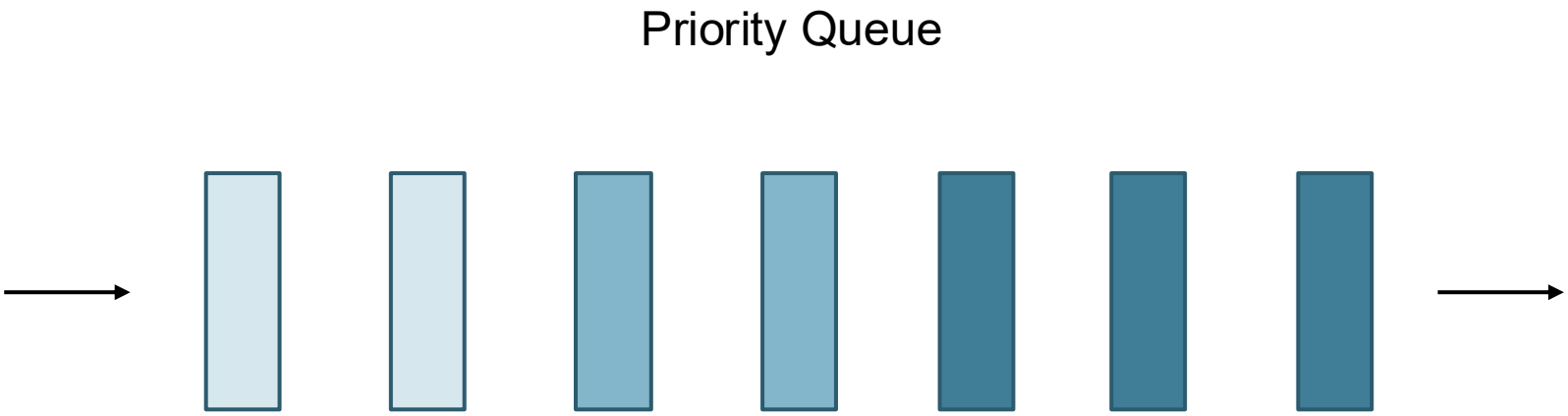

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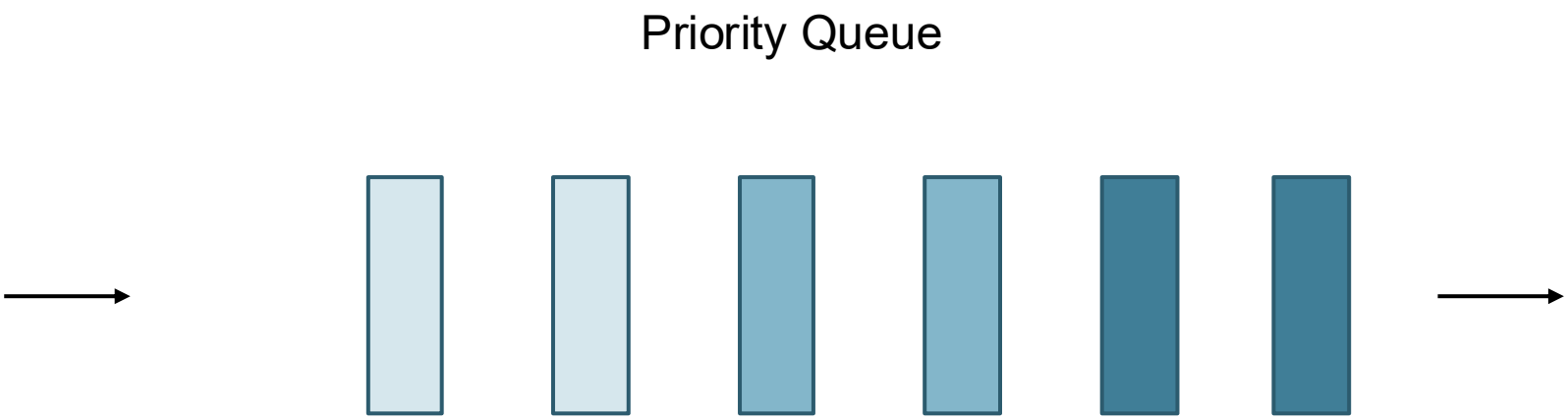

Operationen

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$Insert(A, x)$: Füge x zu A hinzu.

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$max(A)$: Gebe das größte Element x aus.

$extract\_max(A)$: … und lösche es aus A

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Heaps werden in der Regel als Arrays gespeichert:

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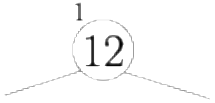

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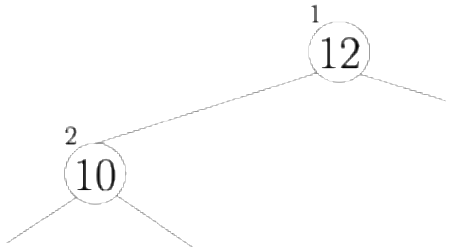

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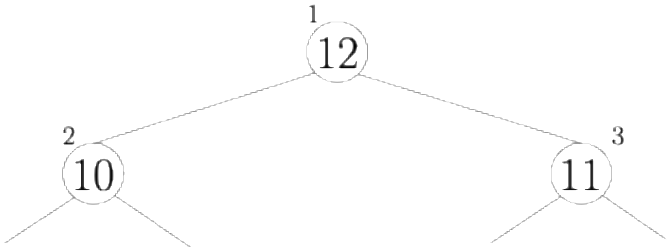

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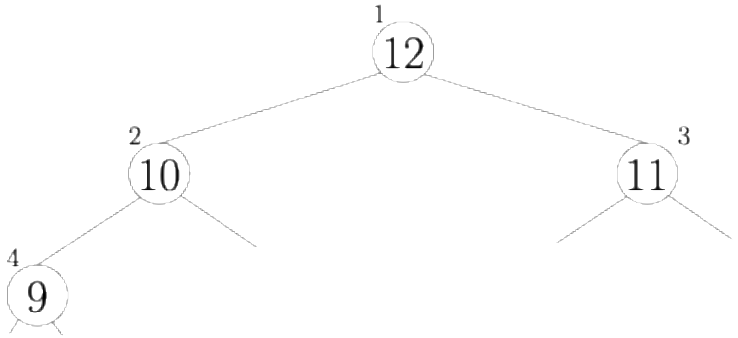

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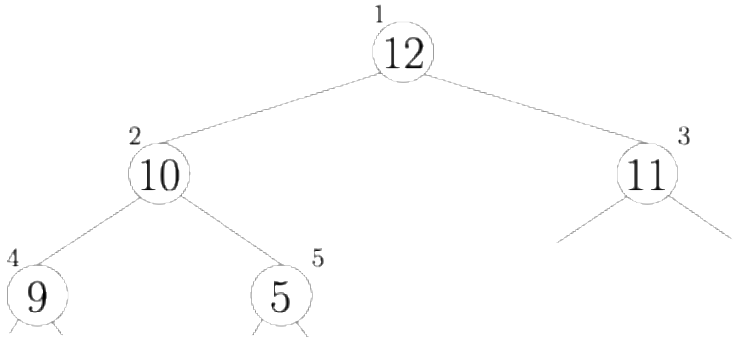

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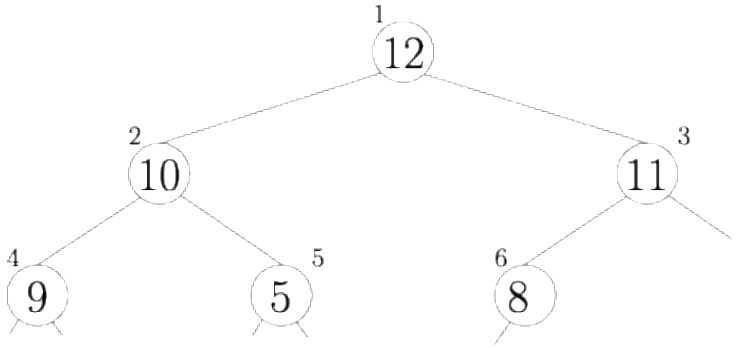

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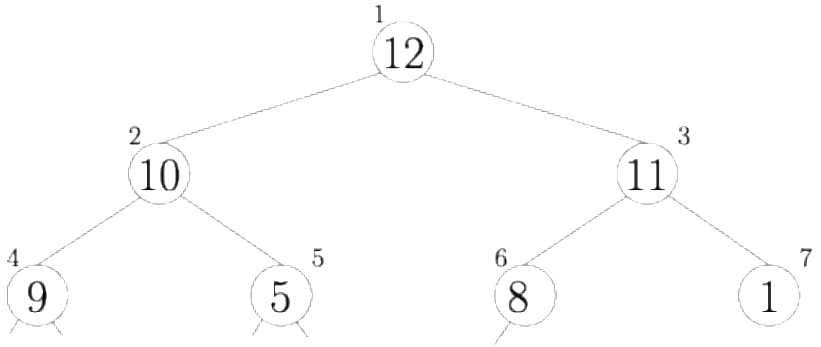

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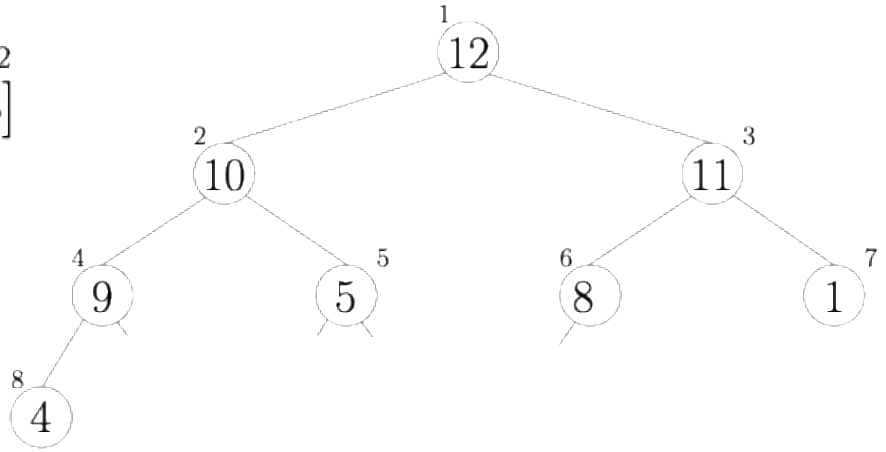

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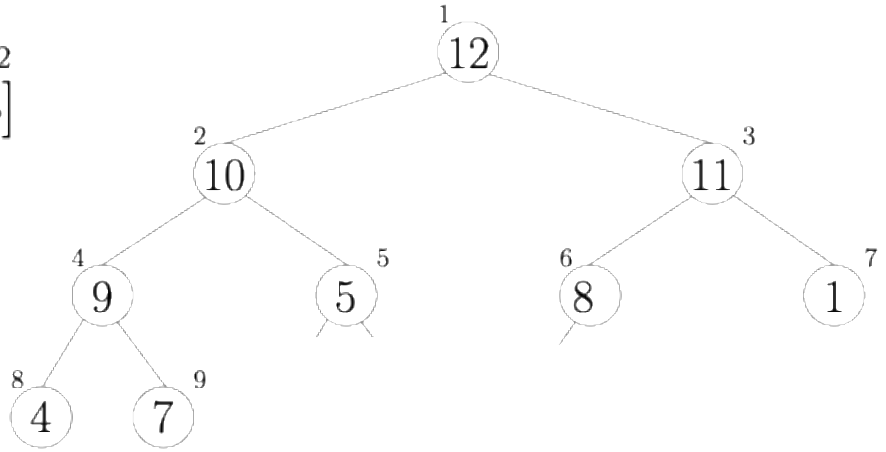

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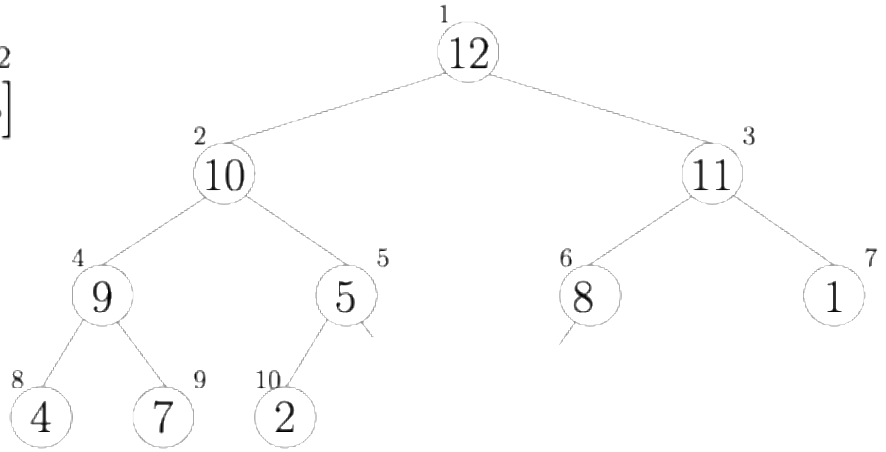

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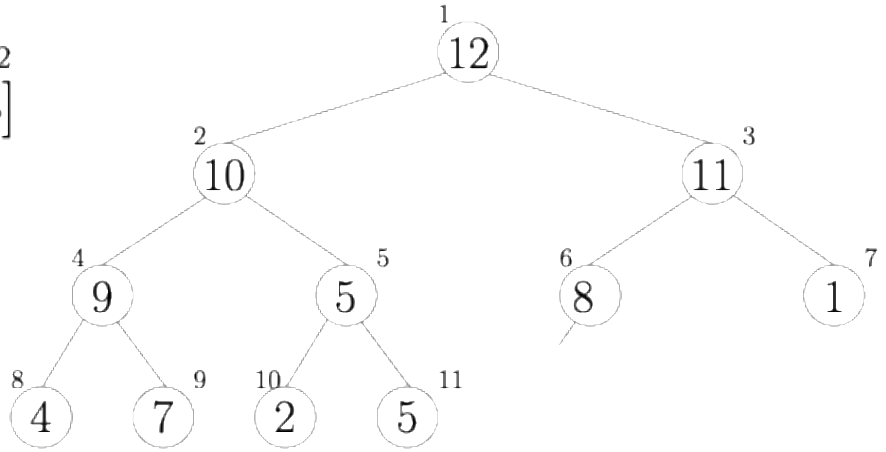

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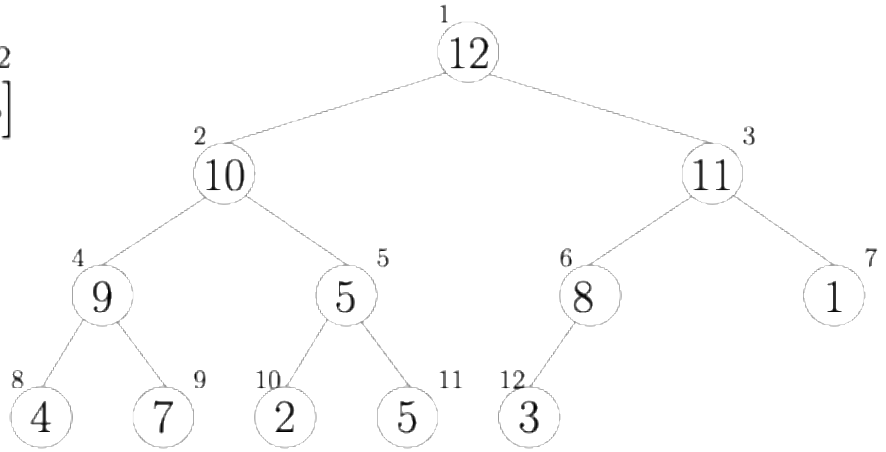

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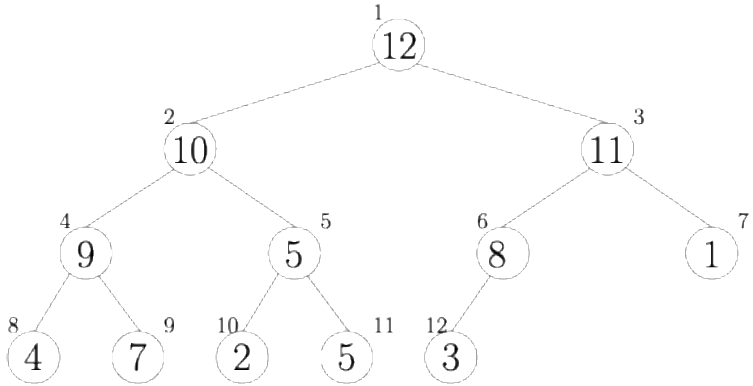

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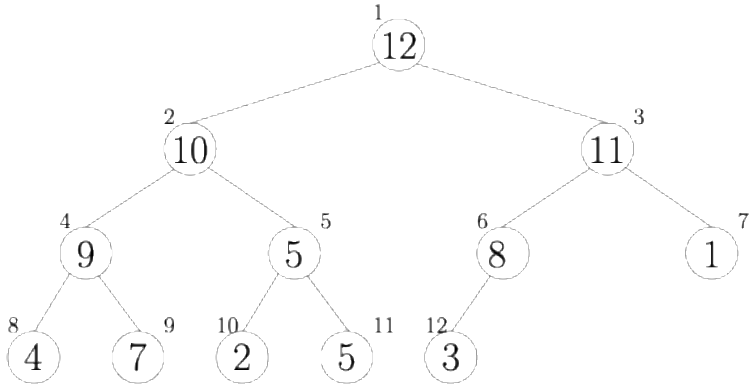
Wurzel: Erstes Element ($i = 1$).



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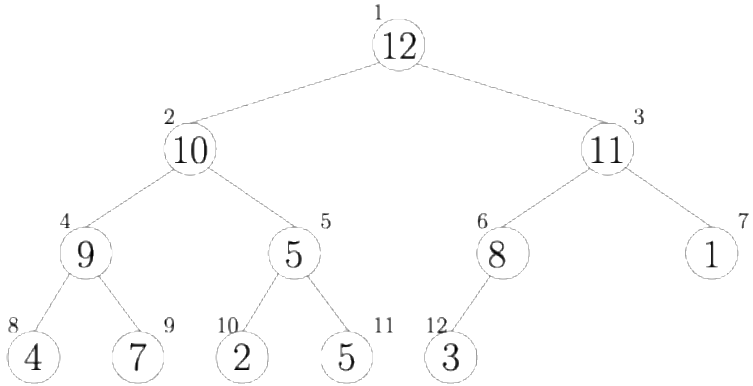
$parent(i)$: $i/2$.



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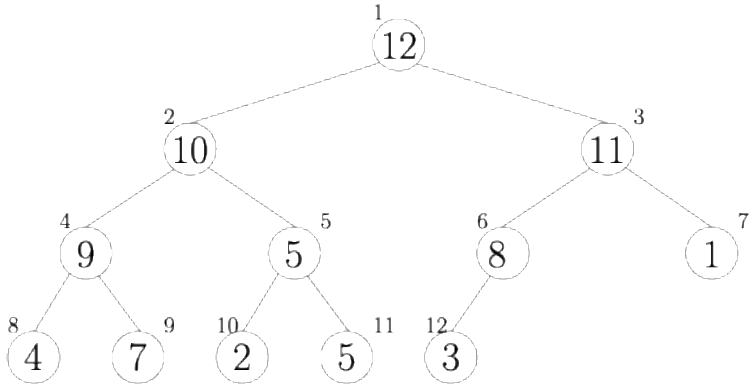
Wurzel: Erstes Element ($i = 1$).

$parent(i)$: $i/2$.

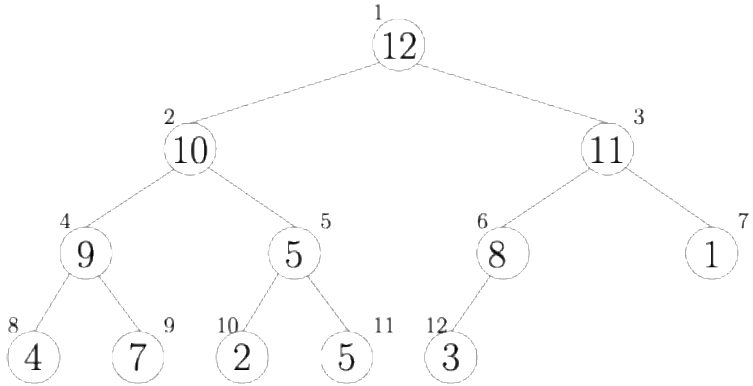
$left(i)$: $2i$; $right(i)$: $2i + 1$.



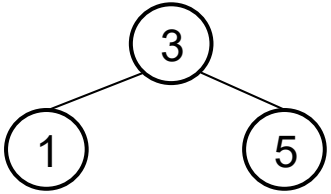
Max-Heaps

$$A = \overset{1}{[12}, \overset{2}{10}, \overset{3}{11}, \overset{4}{9}, \overset{5}{5}, \overset{6}{8}, \overset{7}{1}, \overset{8}{4}, \overset{9}{7}, \overset{10}{2}, \overset{11}{5}, \overset{12}{3}]$$

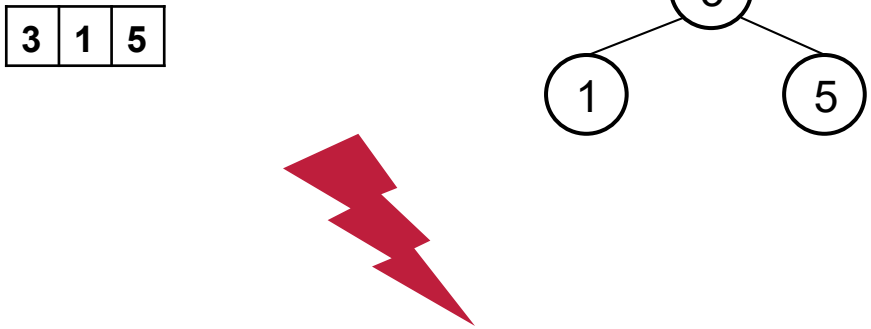
Der Schlüssel jedes Knotens ist mindestens so groß wie die seiner beiden Kinder.



Max-Heaps



Max-Heaps



Heap-Opertations

Heap-Opertations

$build\_max\_heap(A)$: konstruiert einen Max Heap aus

Heap-Opertations

$build\_max\_heap(A)$: konstruiert einen Max Heap aus einem unsortierten Array

Heap-Opertations

$build\_max\_heap(A)$: konstruiert einen Max Heap aus einem unsortierten Array

$max\_heapify$: Repariere die Heap Eigenschaft in einem Teilbaum.

Heap-Opertations

$build\_max\_heap(A)$: konstruiert einen Max Heap aus einem unsortierten Array

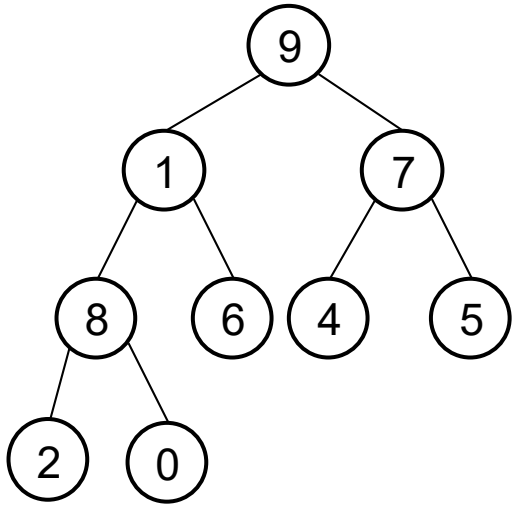
$max\_heapify$: Repariere die Heap Eigenschaft in einem Teilbaum.

Max Heapify

Als Array

9	1	7	8	6	4	5	2	0

Als Baum

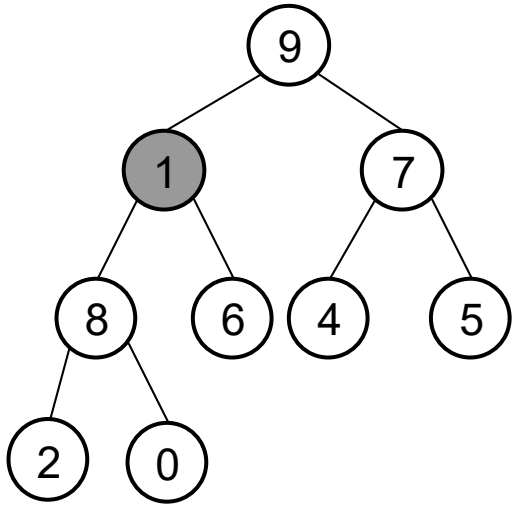


Max Heapify

Als Array

9	1	7	8	6	4	5	2	0

Als Baum

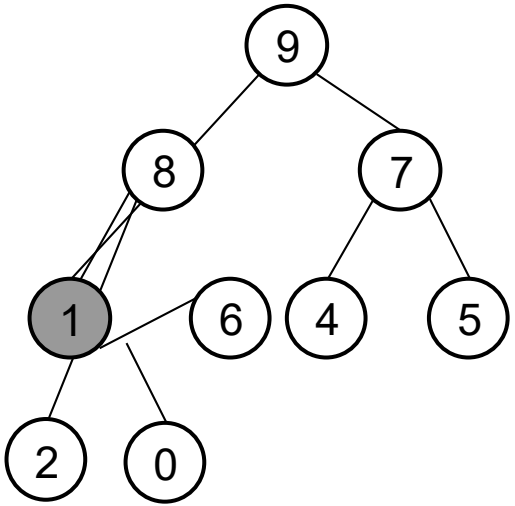


Max Heapify

Als Array

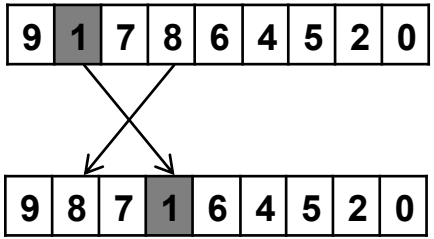
9	1	7	8	6	4	5	2	0

Als Baum

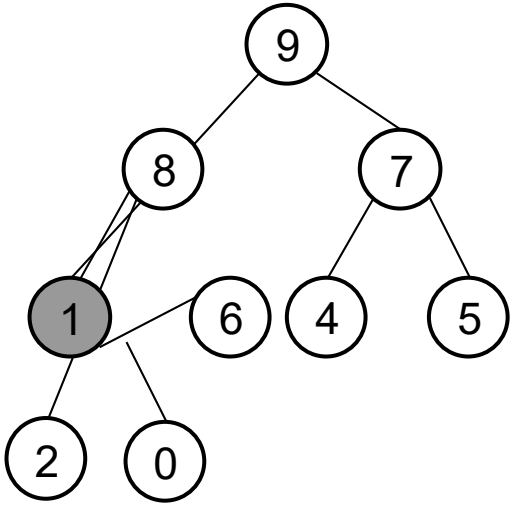


Max Heapify

Als Array

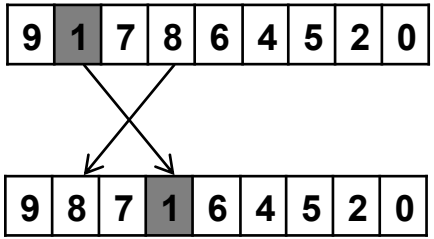


Als Baum

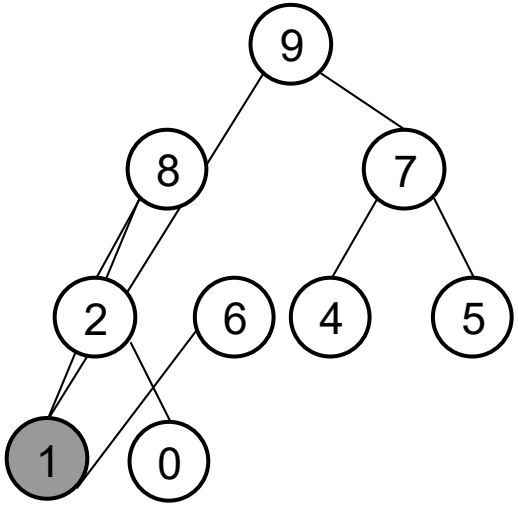


Max Heapify

Als Array

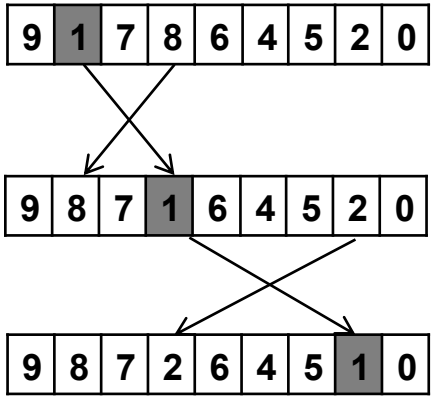


Als Baum

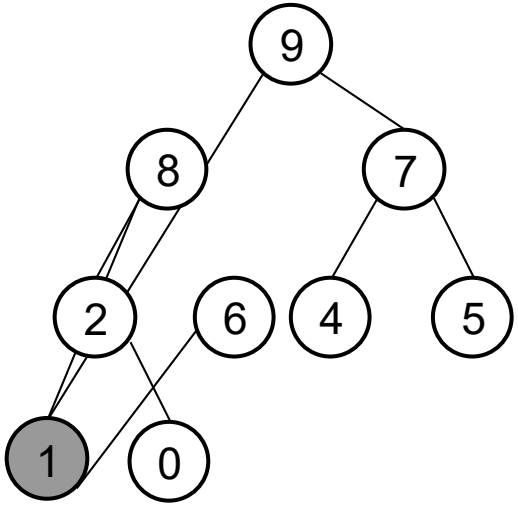


Max Heapify

Als Array

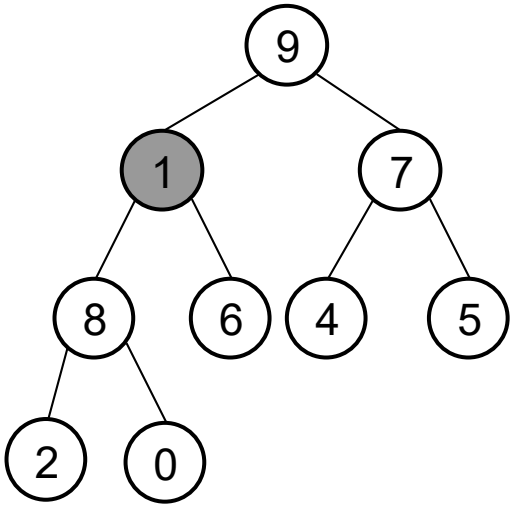


Als Baum



Max Heapify

Wir nehmen an, dass die Teilbäume $left(i)$ und $right(i)$ bereits Max Heaps sind.



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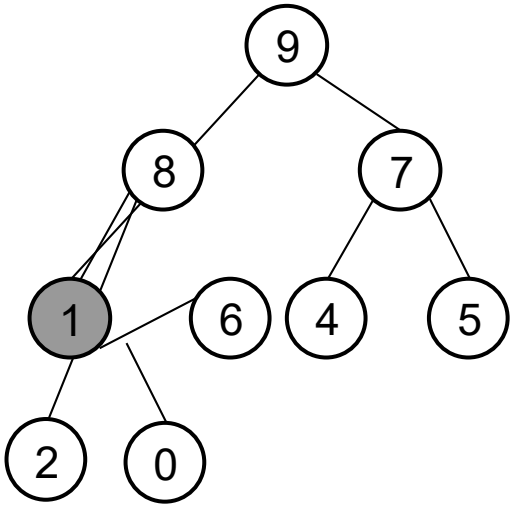

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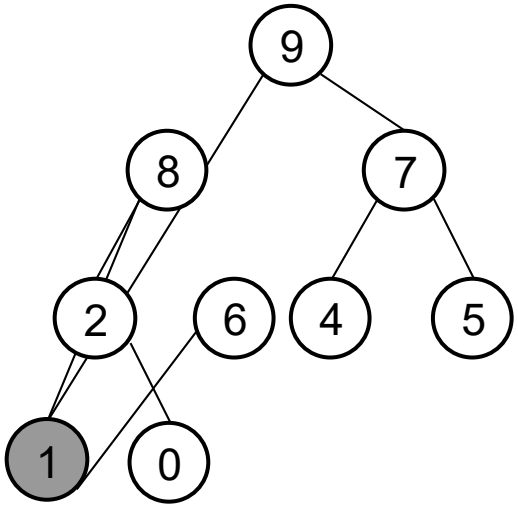
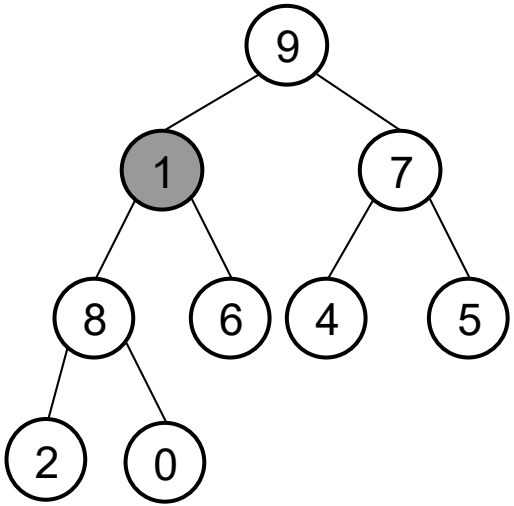
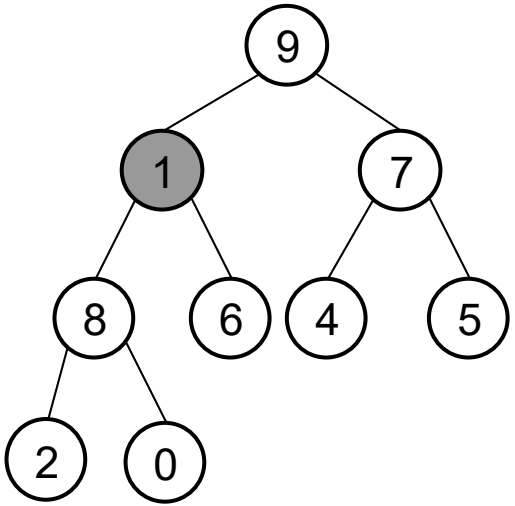


Max Heapify(A,i)



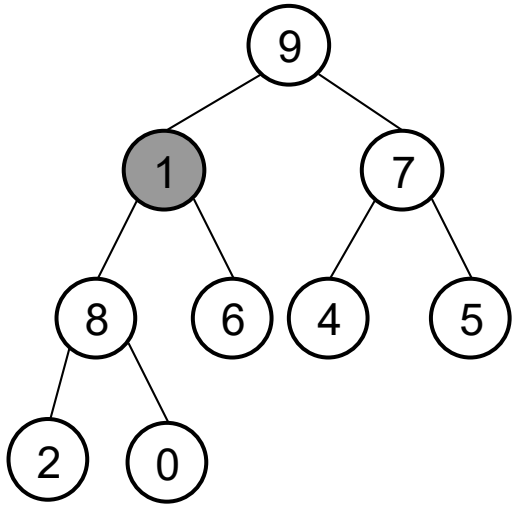
Max Heapify(A,i)

Height $h \in O(\log n)$



Max Heapify(A,i)

Height $h \in O(\log n)$
$\rightarrow max\_heapify(A, i) \in O(\log n)$



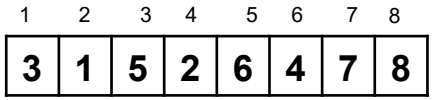
Max Heapify(A,i)

Height $h \in O(\log n)$
$\rightarrow max\_heapify(A, i) \in O(\log n)$

```
1:  function MAX-HEAPIFY(A, i)
2:      ℓ := links(i), r := rechts(i)
3:      if ℓ ≤ heap-größe[A] und A[ℓ] > A[i] then
4:          max := ℓ
5:      else
6:          max := i
7:      if r ≤ heap-größe[A] und A[r] > A[max] then
8:          max := r
9:      if max ≠ i then
10:         Vertausche A[max]und A[i]
11:         MAX-HEAPIFY(A, max)
```

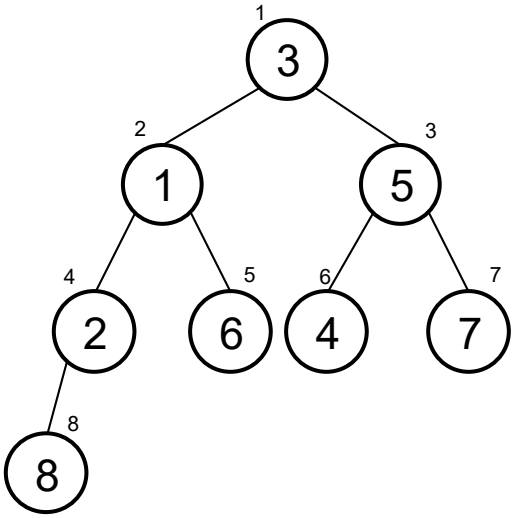
Build-Max-Heap

Build-Max-Heap



Build-Max-Heap

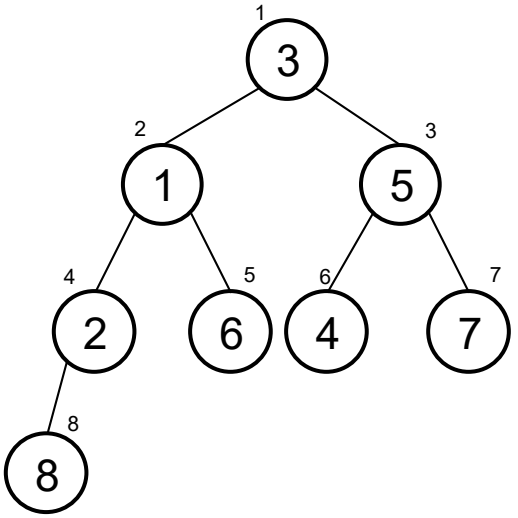
1	2	3	4	5	6	7	8
3	1	5	2	6	4	7	8



Build-Max-Heap

1	2	3	4	5	6	7	8
3	1	5	2	6	4	7	8

function BUILD-MAX-HEAP(A)
  heap-größe[A] := länge[A]
  for $i = \lfloor \frac{\text{länge}[A]}{2} \rfloor$ **down to** 1 **do**
    MAX-HEAPIFY(A, i)



Build-Max-Heap

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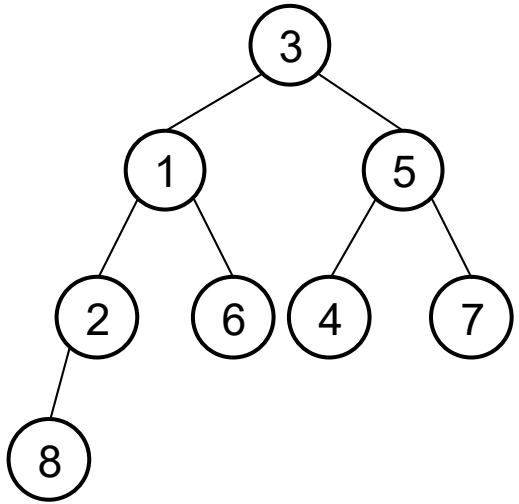
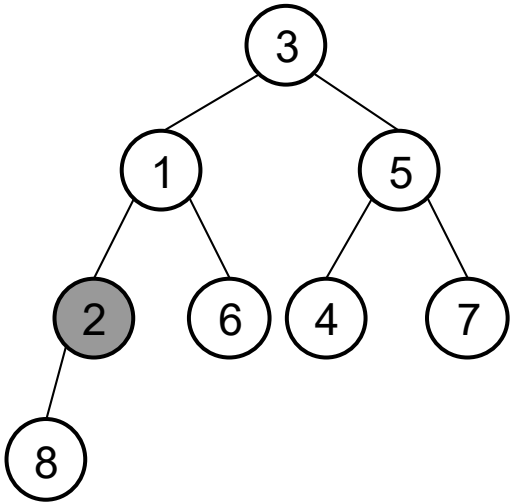


Build-Max-Heap

3	1	5	2	6	4	7	8



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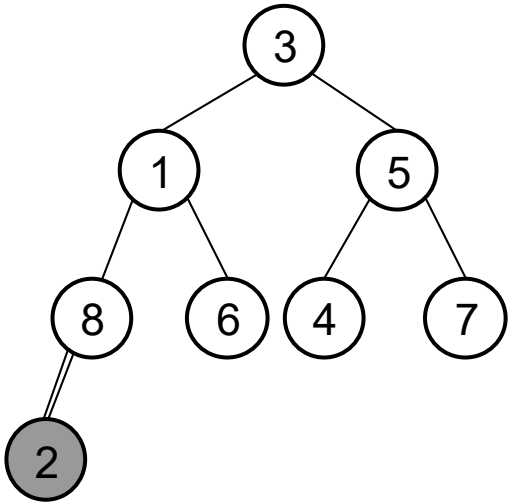
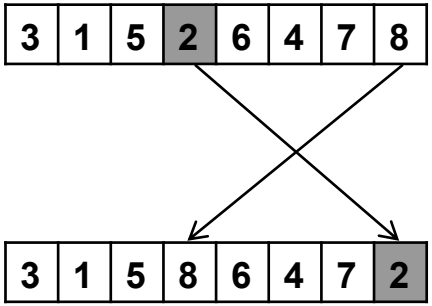
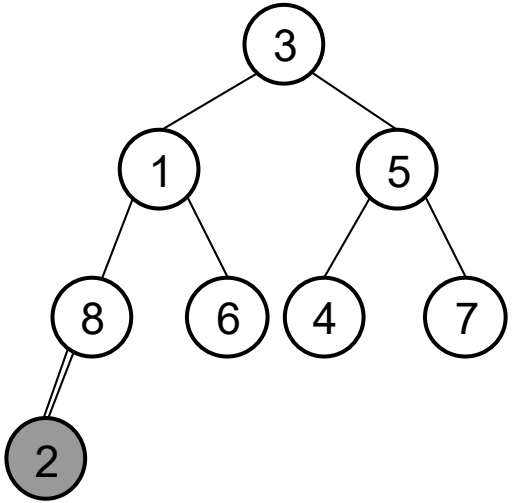


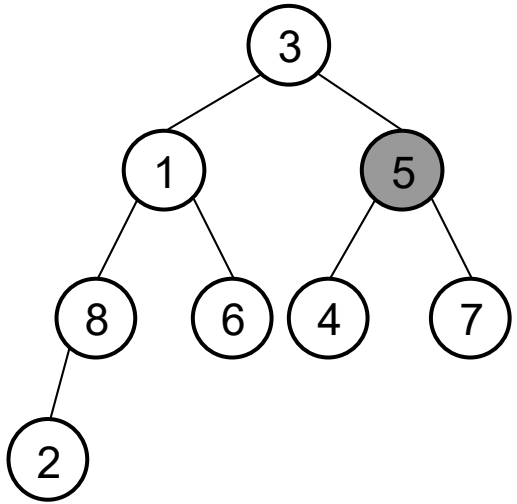
Build-Max-Heap





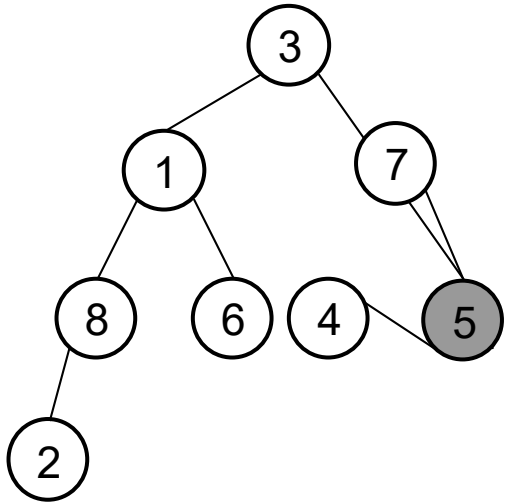
Build-Max-Heap

3	1	5	8	6	4	7	2

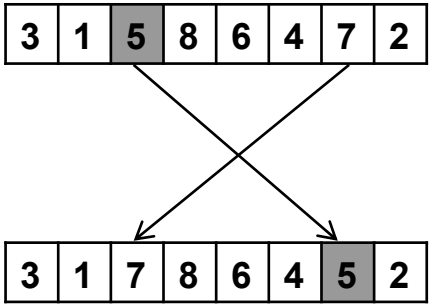


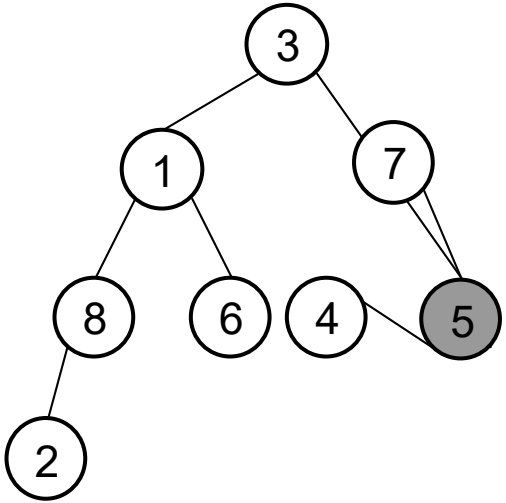
Build-Max-Heap

3	1	5	8	6	4	7	2



Build-Max-Heap





Build-Max-Heap

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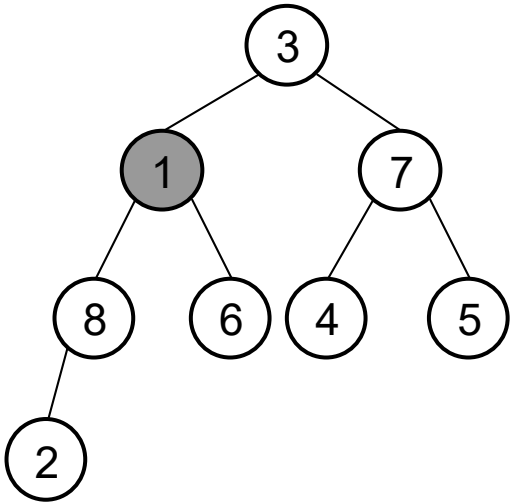

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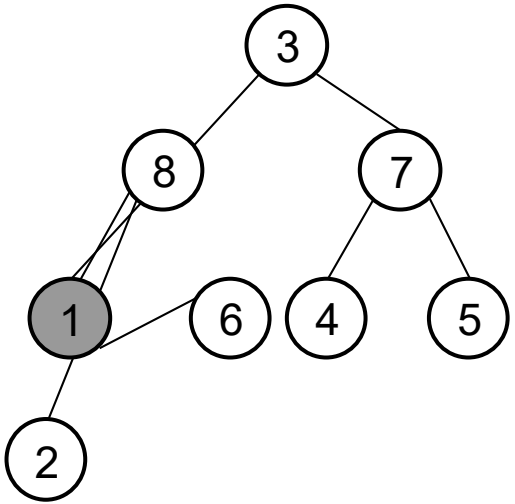
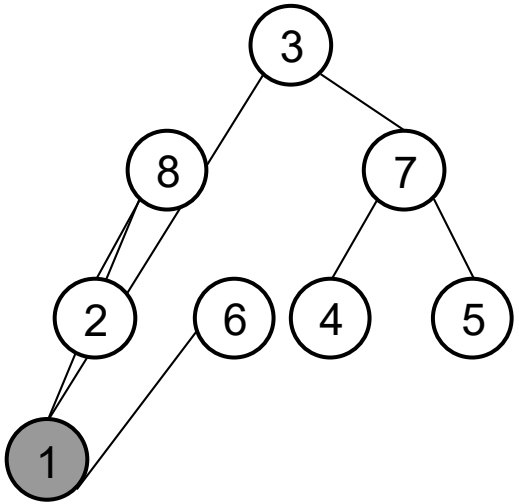
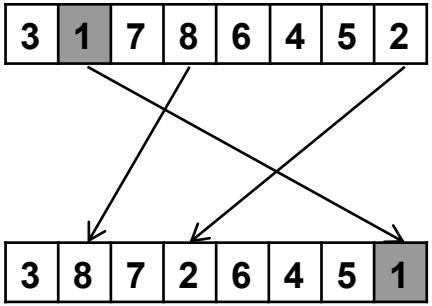


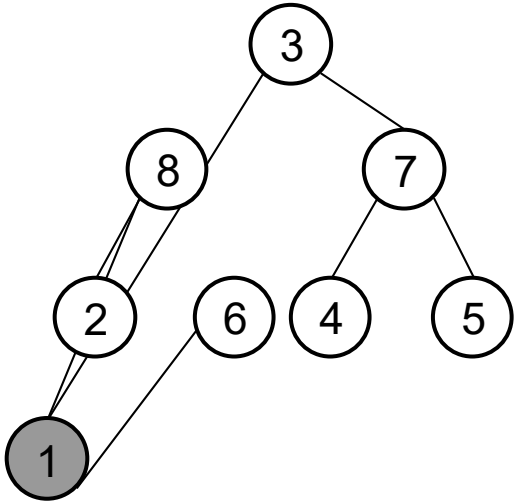
Build-Max-Heap

3	1	7	8	6	4	5	2



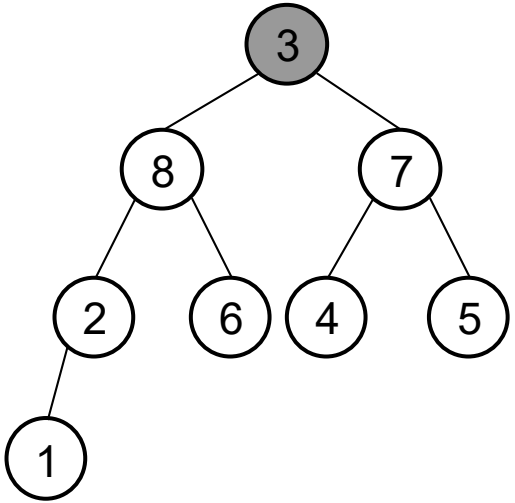
Build-Max-Heap





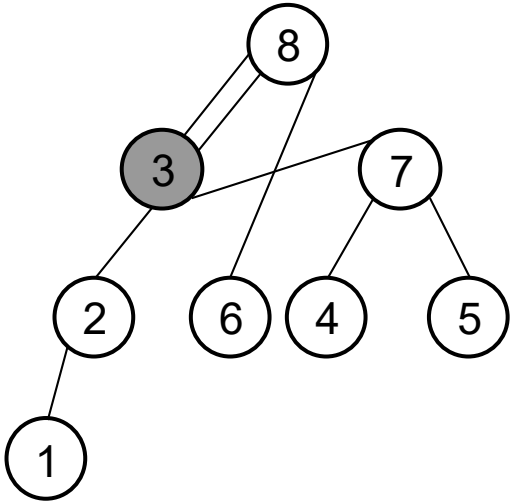
Build-Max-Heap

3	8	7	2	6	4	5	1



Build-Max-Heap

3	8	7	2	6	4	5	1



Build-Max-Heap

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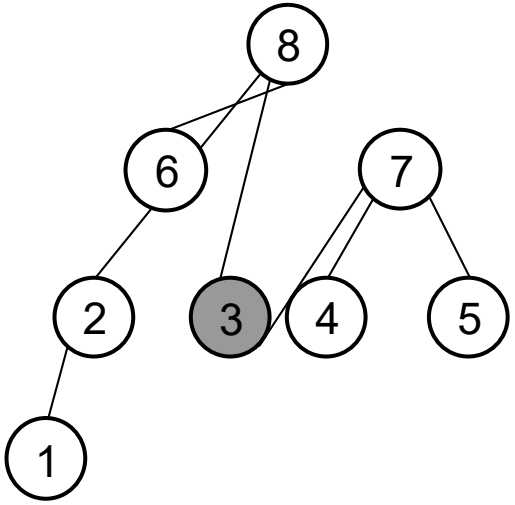

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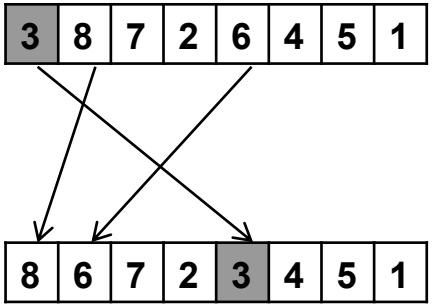

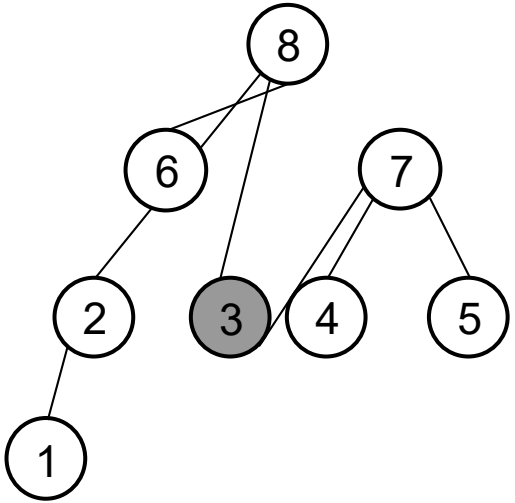

Build-Max-Heap

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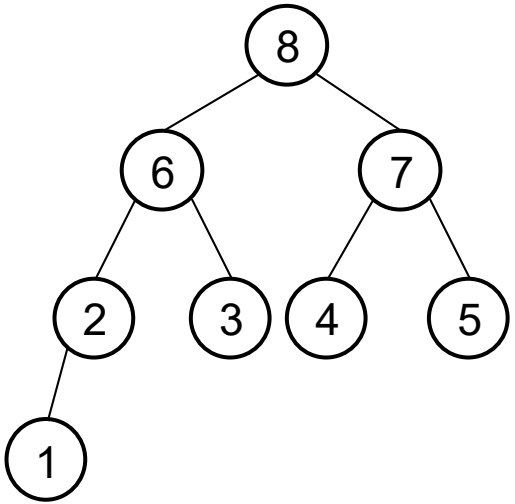
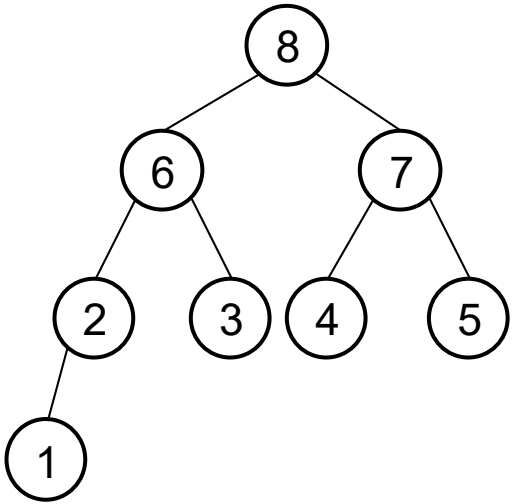


Build-Max-Heap

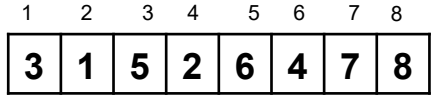
Fertig!

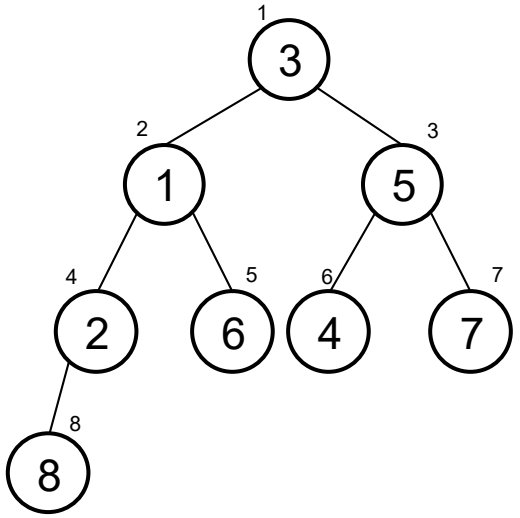
8	6	7	2	3	4	5	1



Build-Max-Heap (Laufzeit)

function BUILD-MAX-HEAP(A)
  heap-größe$[A]$:= länge$[A]$
  for $i = \lfloor \frac{\text{länge}[A]}{2} \rfloor$ **down to** 1 **do**
    MAX-HEAPIFY(A, i)





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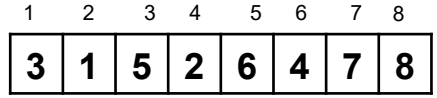
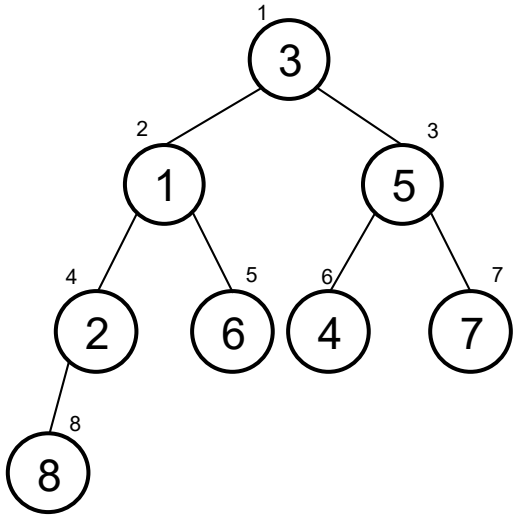
        MAX-HEAPIFY(A, i)

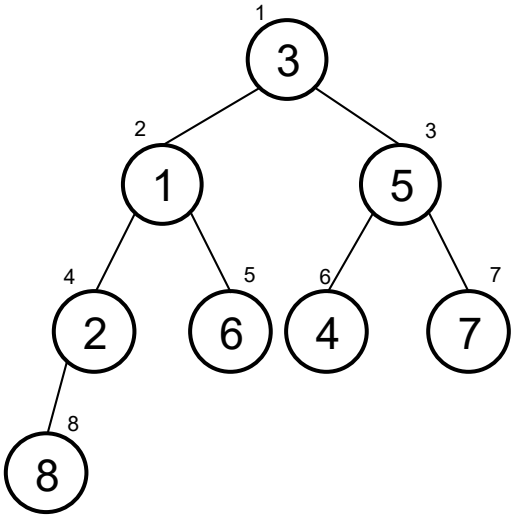
$$n \cdot max\_heapify \rightarrow O(n \log n)$$





Build-Max-Heap (Laufzeit) Besser!

1	2	3	4	5	6	7	8
3	1	5	2	6	4	7	8



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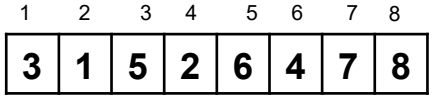
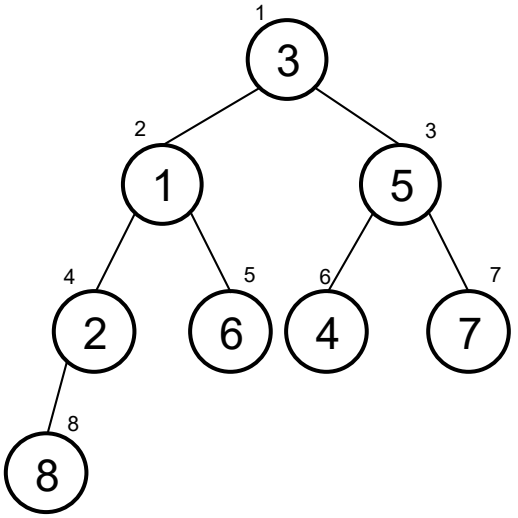
Für Blätter benötigt $max\_heapify$ $O(1)$ Operationen.





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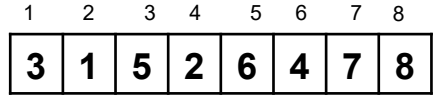
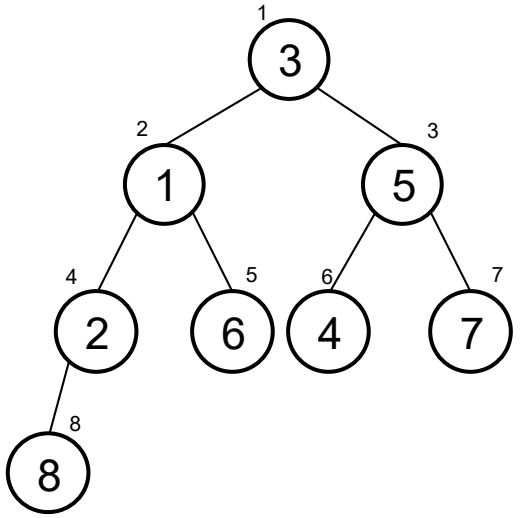
Generell: Knoten der Höhe l benötigen $O(l)$ Operationen.





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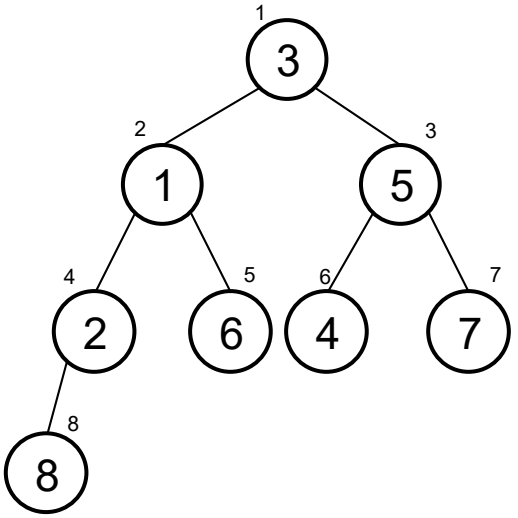
$n/4$ Knoten der Höhe 1, $n/8$ Knoten der Höhe 2,…, und 1 Knoten der Höhe $\log n$.



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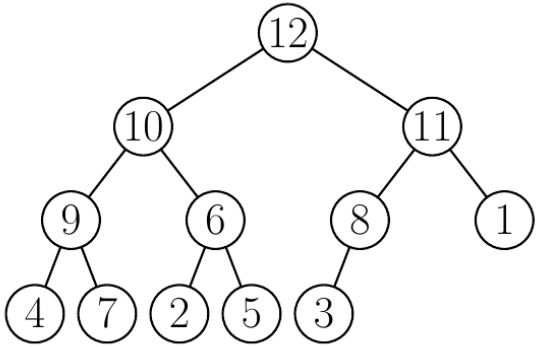

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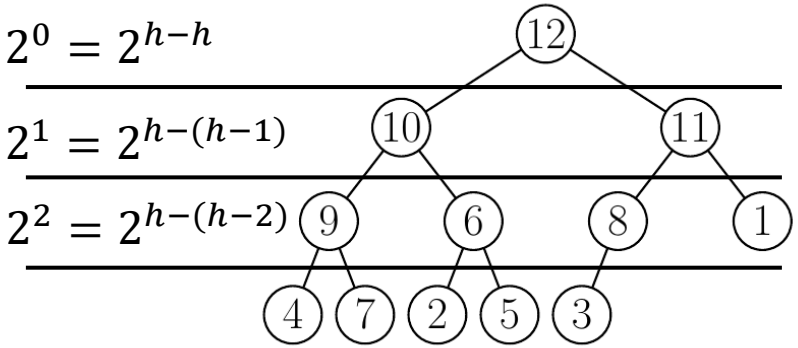

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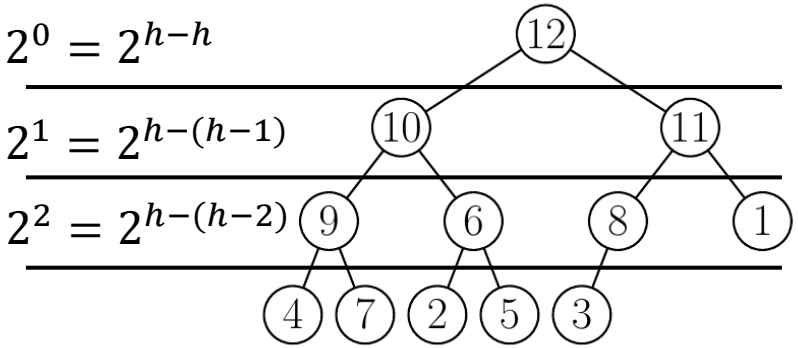

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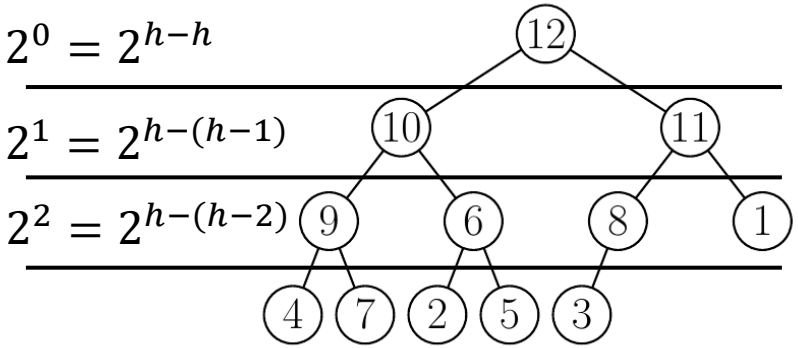

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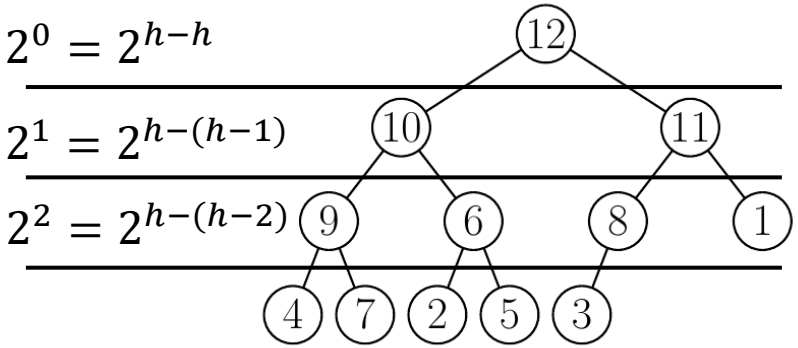

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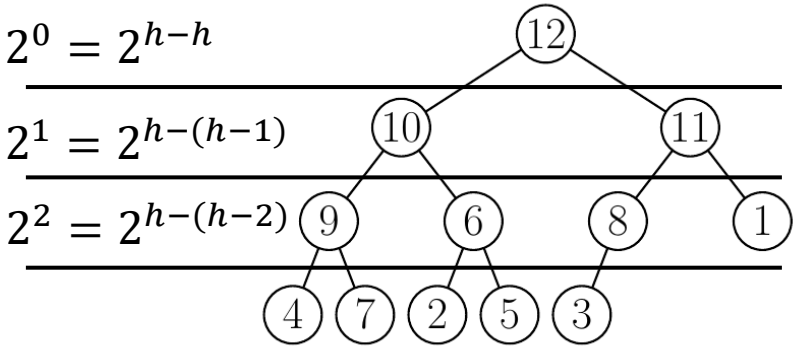

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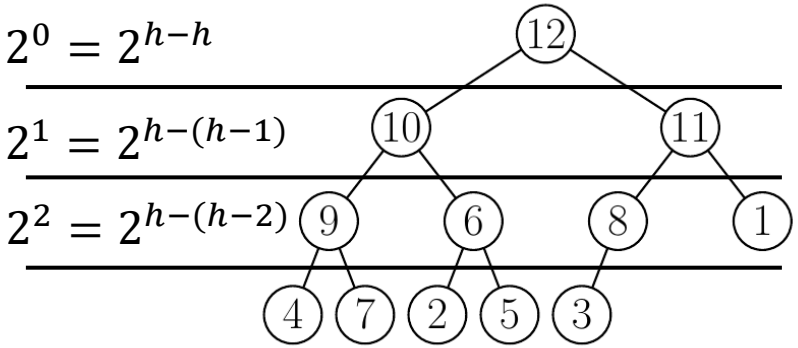

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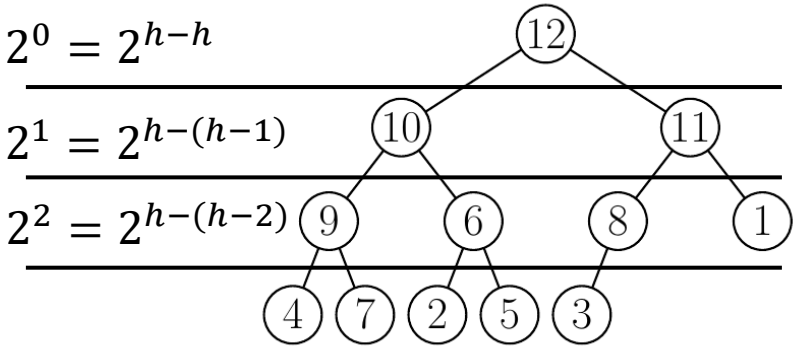

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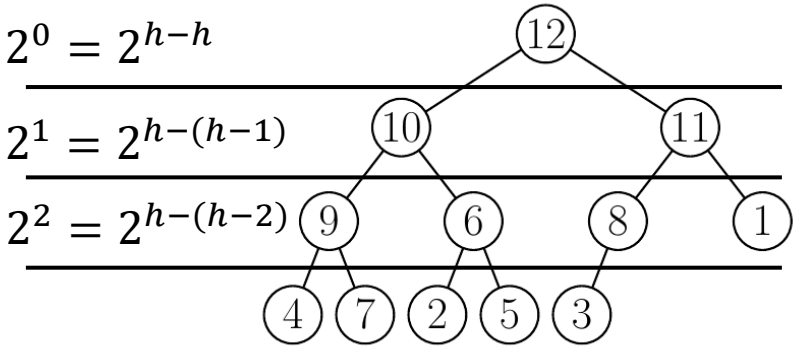

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Heapsort

Heapsort

8	6	7	2	3	4	5	1

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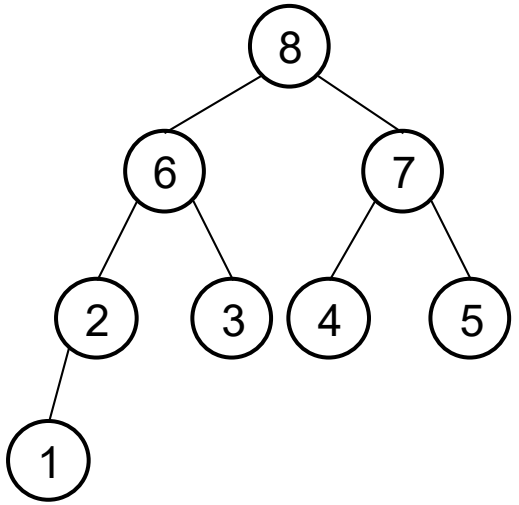

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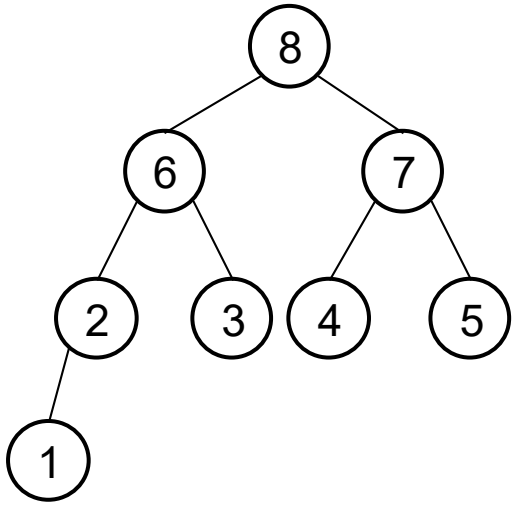

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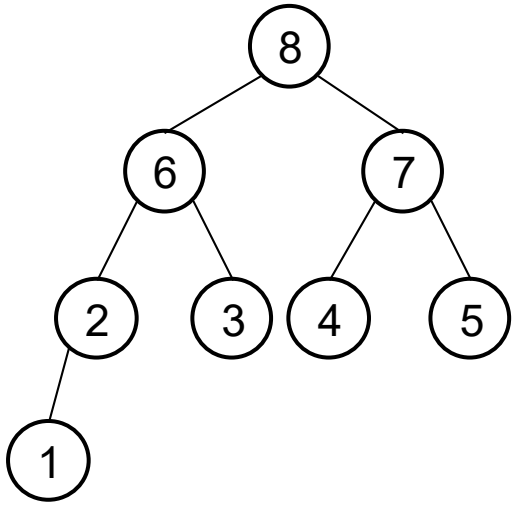

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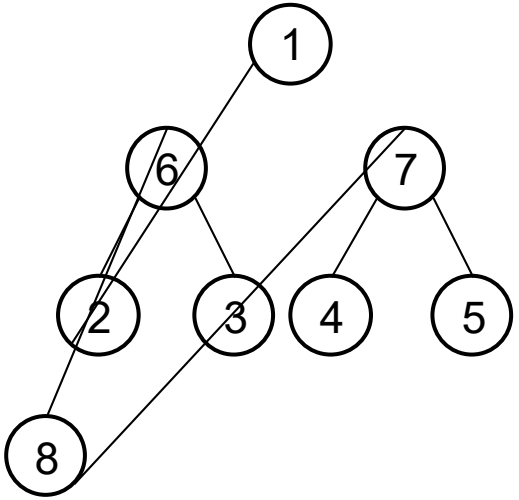

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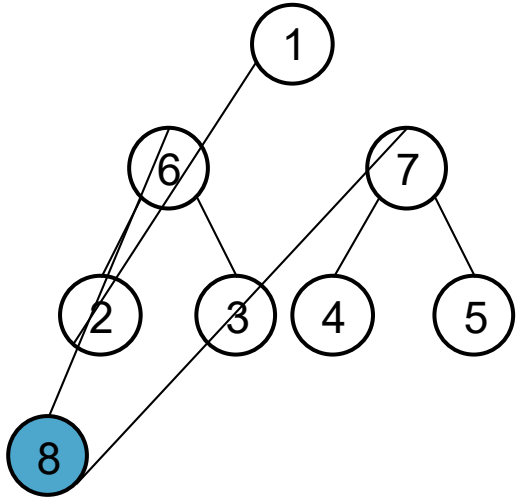

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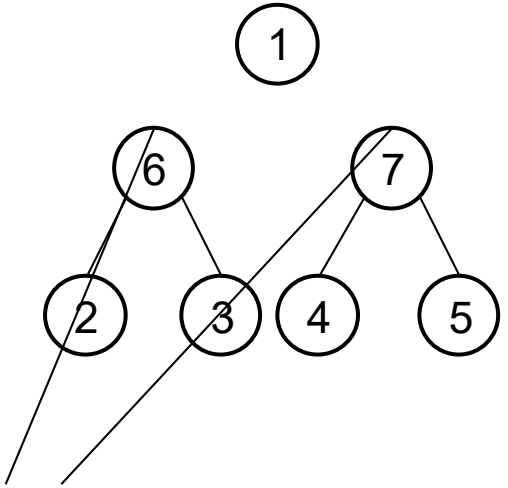

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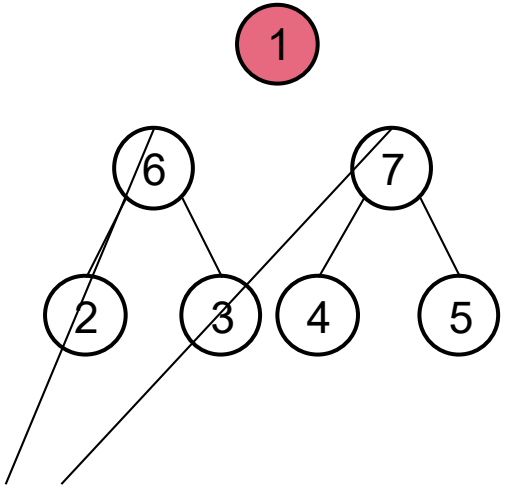

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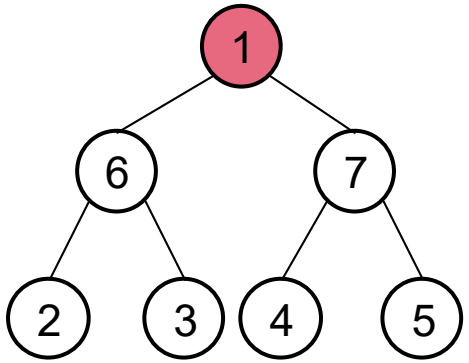

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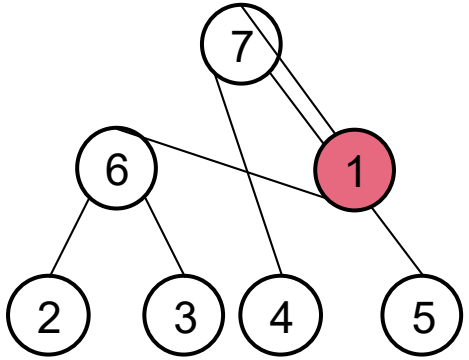

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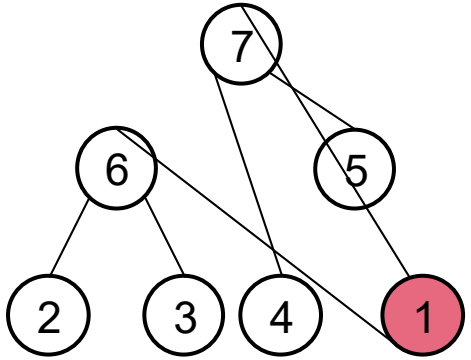

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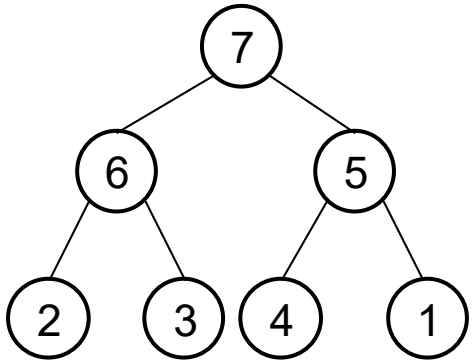

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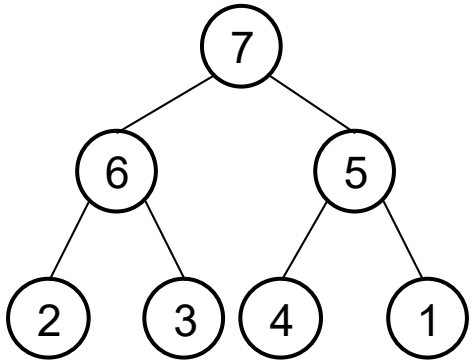

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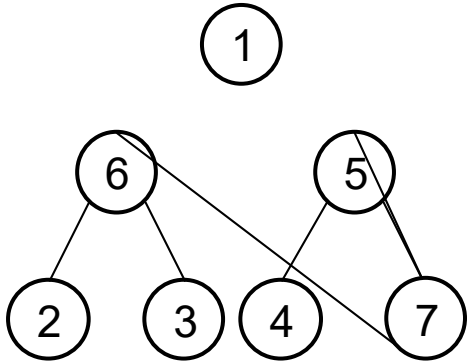

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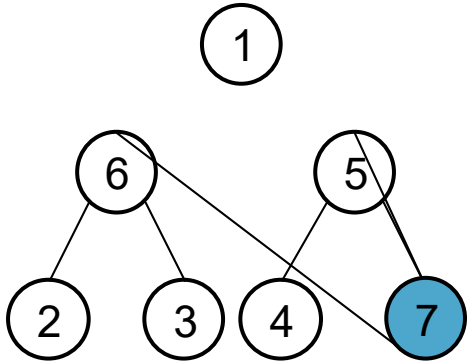

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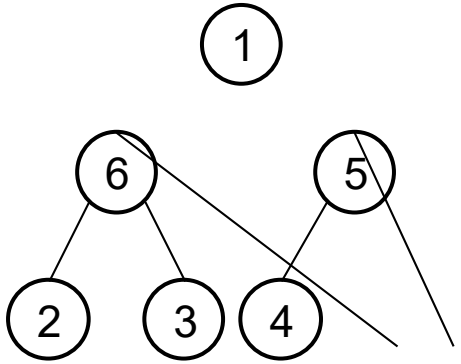

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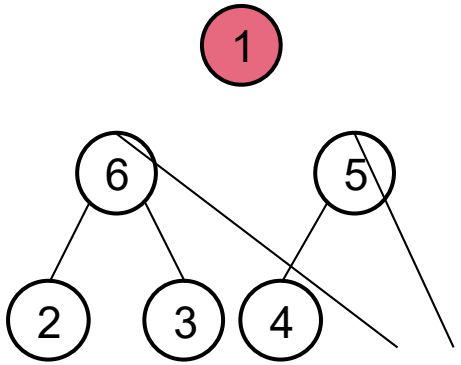

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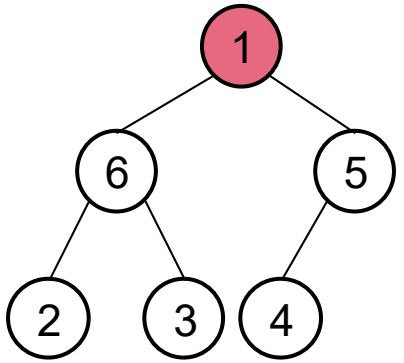

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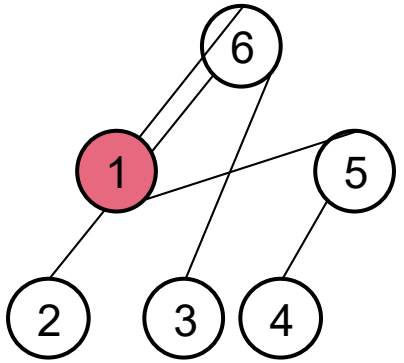

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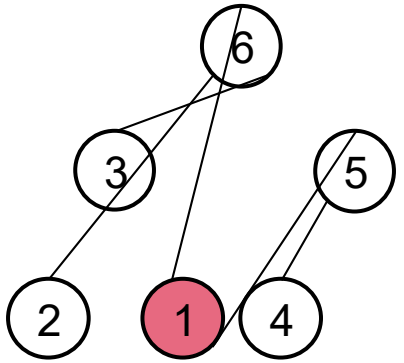

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end function
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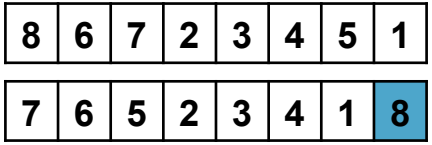
Heapsort

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Heapsort

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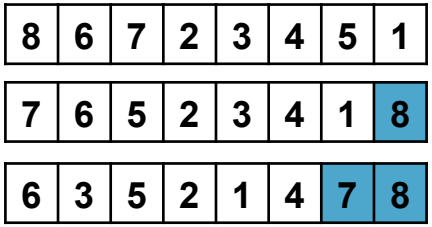

Heapsort

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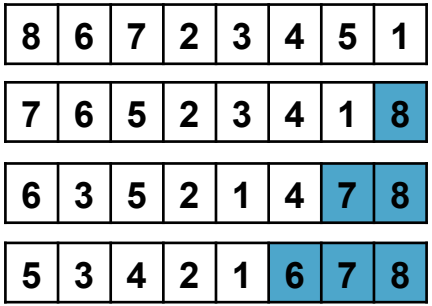

Heapsort

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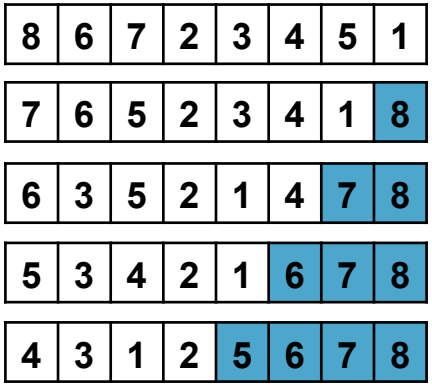

Heapsort

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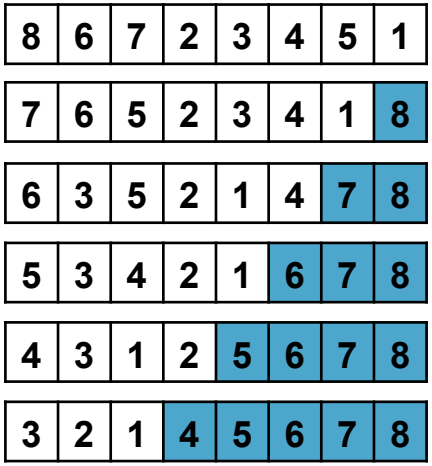

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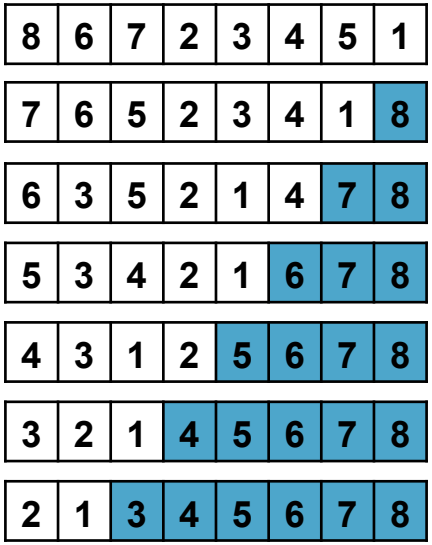

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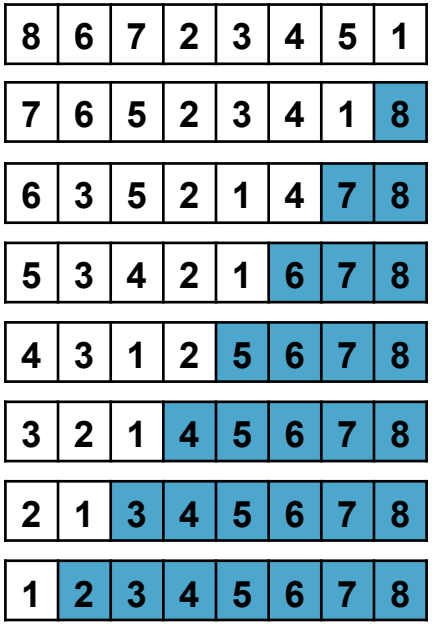

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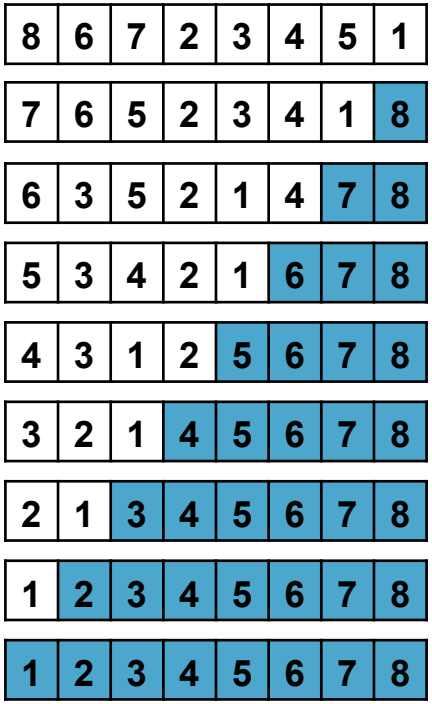

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Heapsort – Laufzeit

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function SORT(A)
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        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap:

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap: $O(n)$

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen:

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen: $O(1)$

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen: $O(1)$
2. Verkleinern:

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen: $O(1)$
2. Verkleinern: $O(1)$

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen: $O(1)$
2. Verkleinern: $O(1)$
3. Max-Heapify:

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
        vertausche A[1] und A[i]
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end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen: $O(1)$
2. Verkleinern: $O(1)$
3. Max-Heapify: $O(\log n)$

Heapsort – Laufzeit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ←length(A) downto 2 do
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end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen: $O(1)$
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3. Max-Heapify: $O(\log n)$

Für n Iterationen ist das

Heapsort – Laufzeit

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end function
```

Build-Max-Heap: $O(n)$

In der For-Schleife

1. Vertauschen: $O(1)$
2. Verkleinern: $O(1)$
3. Max-Heapify: $O(\log n)$

Für n Iterationen ist das $O(n \log n)$

Heapsort – Korrektheit

```
function SORT(A)
    BUILD-MAX-HEAP(A)
    for i ← length(A) downto 2 do
        vertausche A[1] und A[i]
        heap-größe[A] ← heap-größe[A] − 1
        MAX-HEAPIFY(A, 1)
    end for
end function
```

Heapsort ist ein Sortierverfahren.

Beweis:

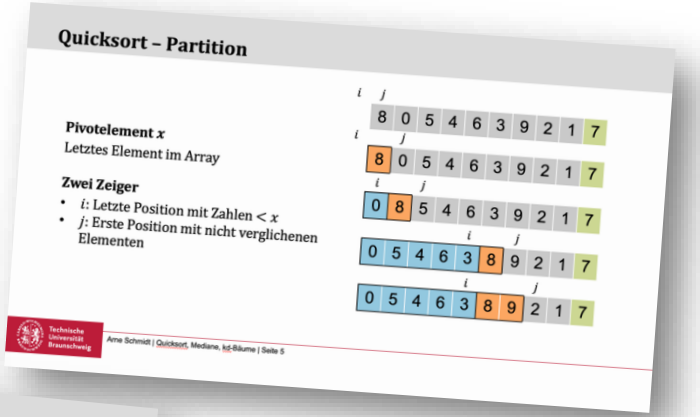
Wir zeigen: Nach der i-ten Iteration besitzen wir wieder einen Max-Heap auf den restlichen Feldern
➔*Wir können das nächstgrößere Element an die richtige Stelle sortieren.*

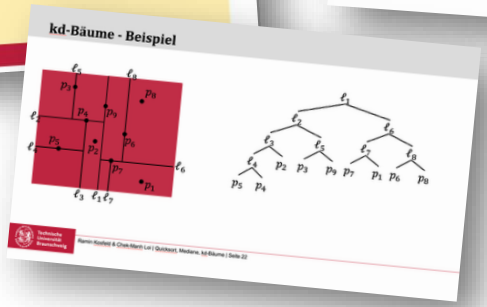
- Zu Beginn besitzen wir einen Max-Heap.
  ➔das größte Element ist an der ersten Stelle.
- Angenommen, wir haben zu Beginn der Iteration i einen Max-Heap auf den aktiven $n - i + 1$ Feldern.
- Nach dem Tauschen des i-größten Elements nach hinten und nach Verkleinern des Arrays:
  ➔U.U. kein Max-Heap vorhanden. (vor Max-Heapify-Aufruf)
- Aber: Jeder Knoten außer Wurzel erfüllt Max-Heap Eigenschaft.
- Also: Max-Heapify auf Wurzelknoten reicht aus, um einen Max-Heap wiederherzustellen!
- → Damit haben wir wieder einen Max-Heap auf den restlichen Feldern.

➔ *Wir tauschen also in jeder Iteration das richtige Element nach hinten!*

Fragen?

... und nächstes Mal ...





... am 29.01.2026
(in 2 Wochen!)