



Algorithmen und Datenstrukturen

WS 2025/26

Prof. Dr. Sándor Fekete

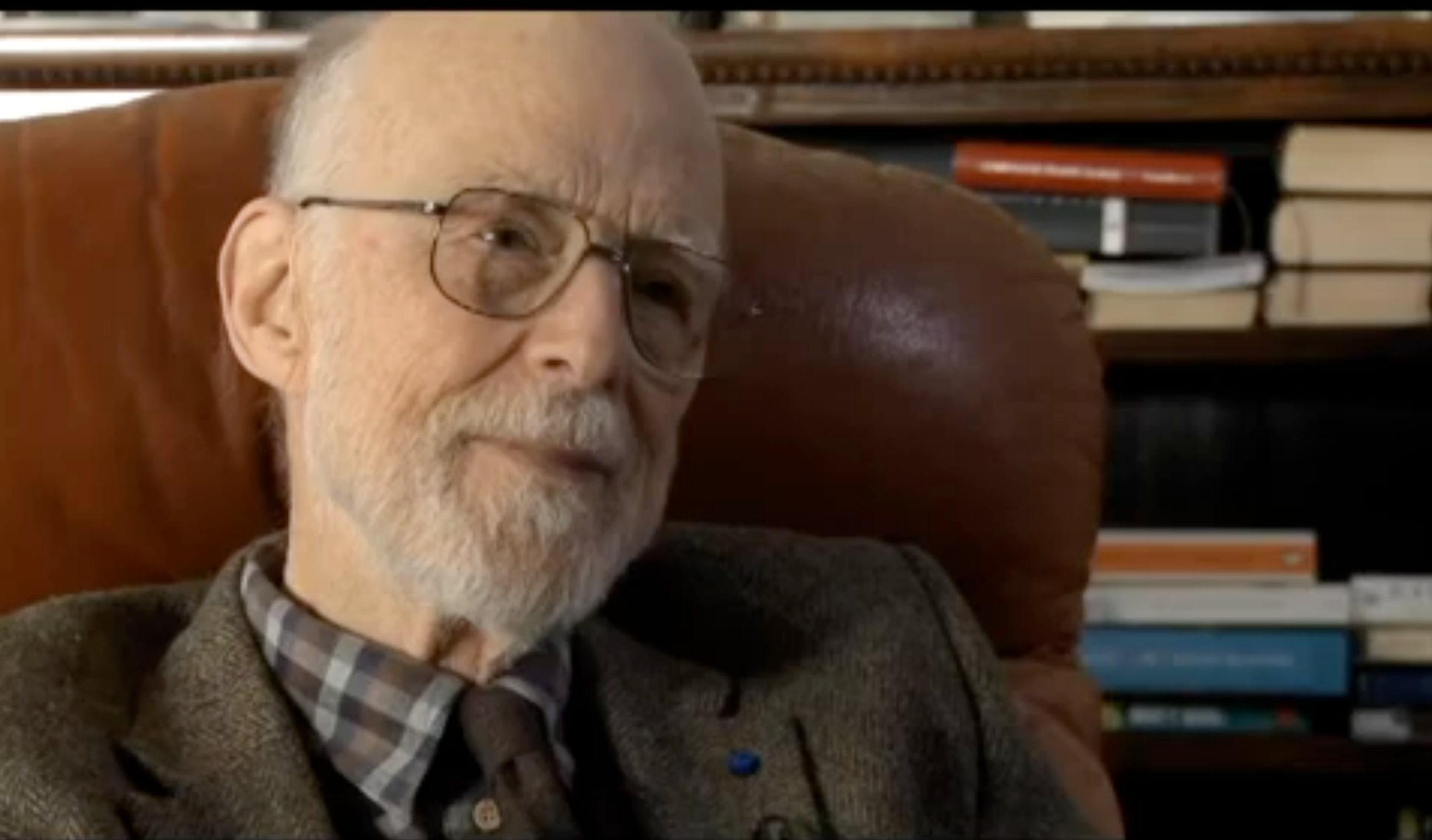


UNIVERSITY OF
LIVERPOOL





Tony Hoare (1960)



ALGORITHM 64

QUICKSORT

C. A. R. HOARE

Elliott Brothers Ltd., Borehamwood, Hertfordshire, Eng.

procedure quicksort (A,M,N); **value** M,N;

array A; **integer** M,N;

comment Quicksort is a very fast and convenient method of sorting an array in the random-access store of a computer. The entire contents of the store may be sorted, since no extra space is required. The average number of comparisons made is $2(M-N) \ln(N-M)$, and the average number of exchanges is one sixth this amount. Suitable refinements of this method will be desirable for its implementation on any actual computer;

begin **integer** I,J;

if $M < N$ **then begin** partition (A,M,N,I,J);

quicksort (A,M,J);

quicksort (A, I, N)

end

end quicksort

5.5 Quicksort

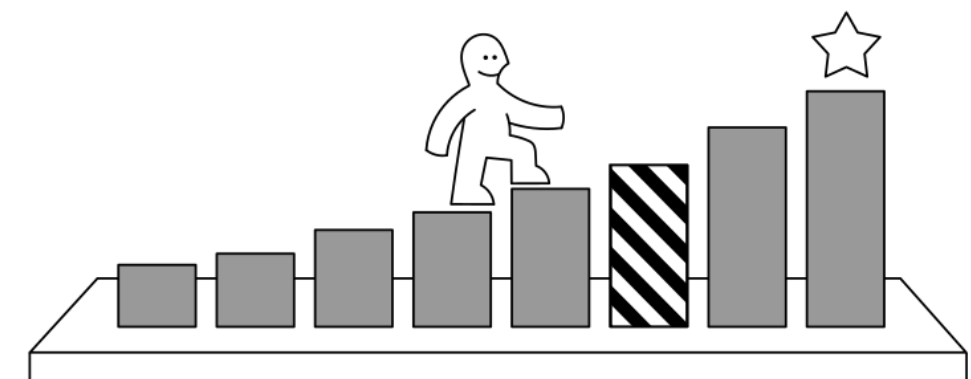
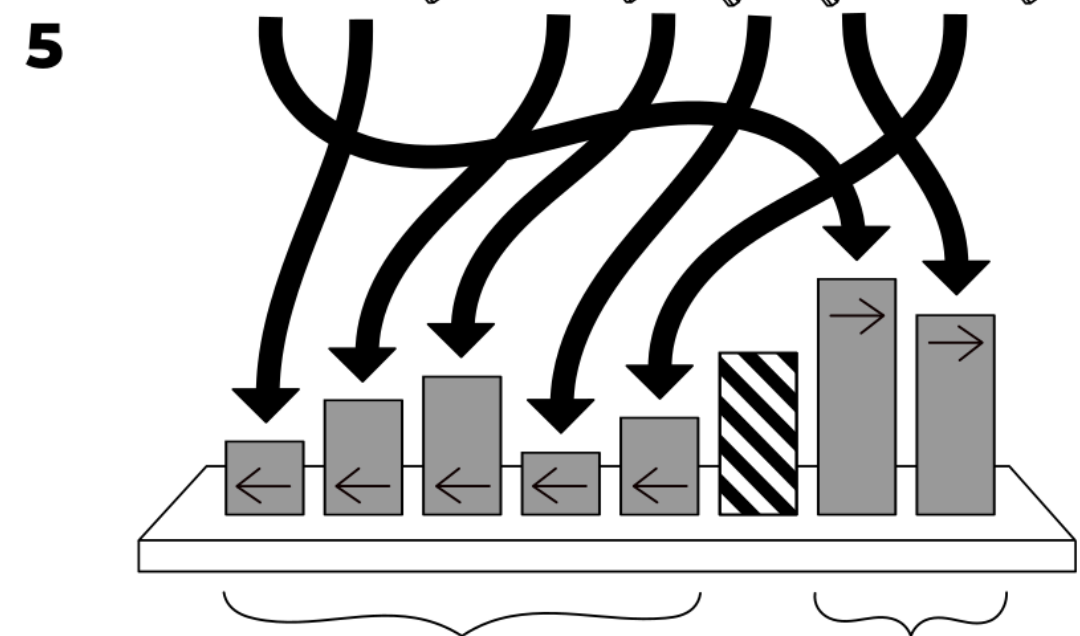
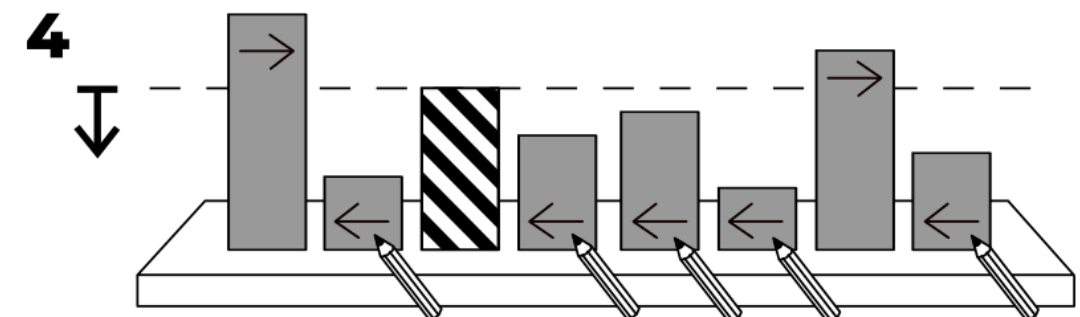
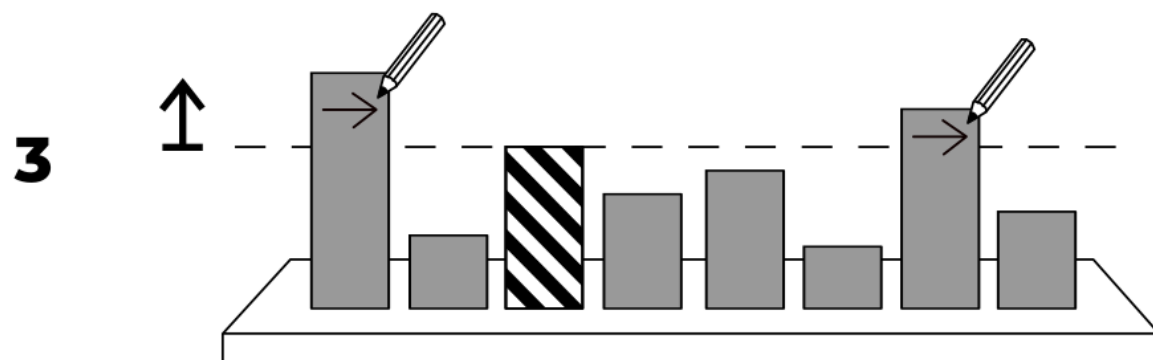
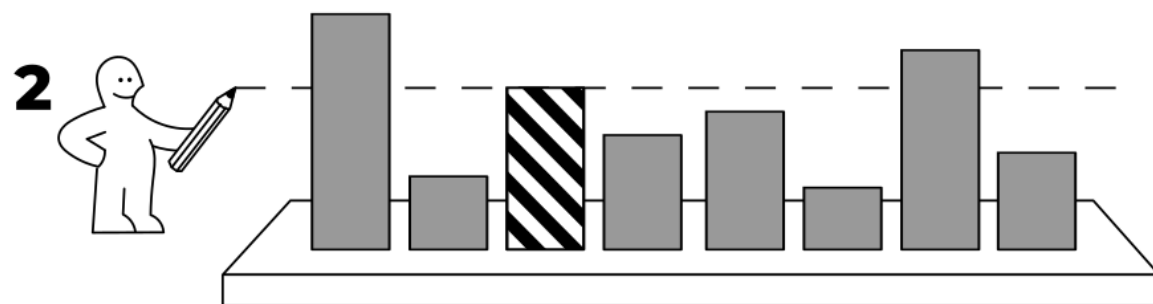
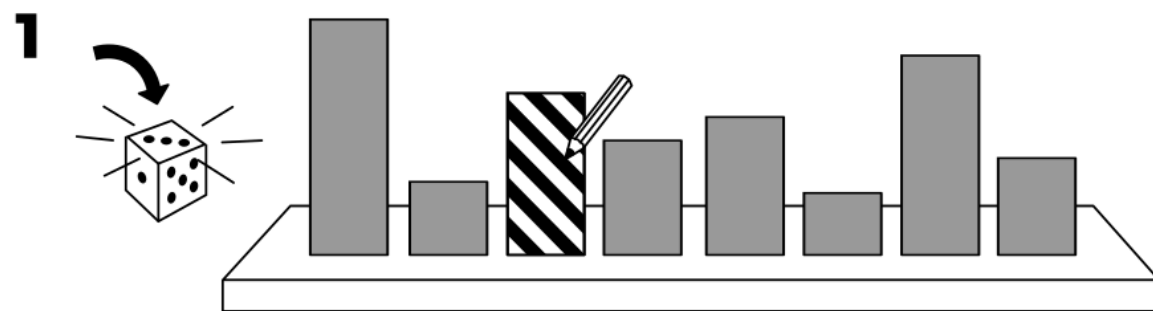
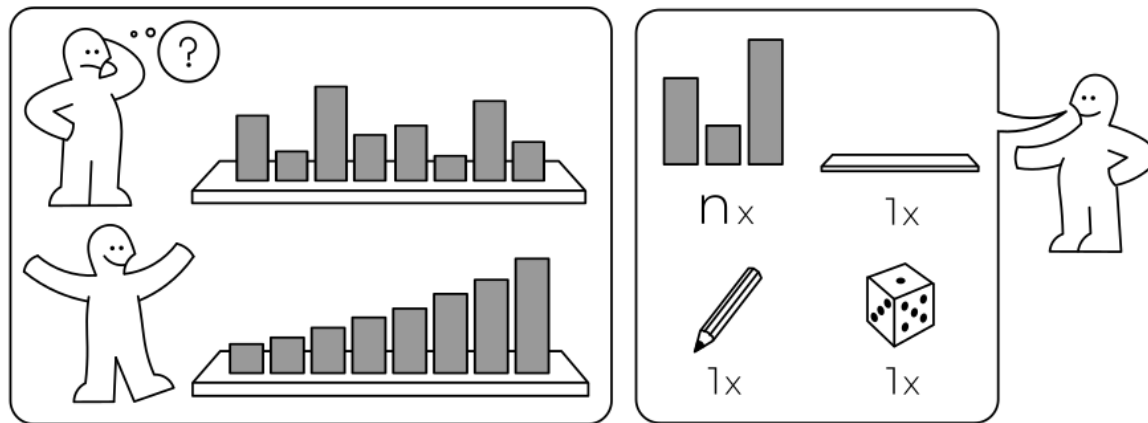
Grundideen:

- **Divide and Conquer**
- **Jeweils Aufteilung in zwei Teilarrays**
- **Rekursiv: Sortieren der Teilarrays**
- ***Kein Merge-Schritt!***
- **Stattdessen Aufteilung der Teilarrays anhand eines “Pivot”-Elements, das die Menge in kleinere und größere Elemente teilt.**
- **Balance der Aufteilung vorher nicht absehbar.**

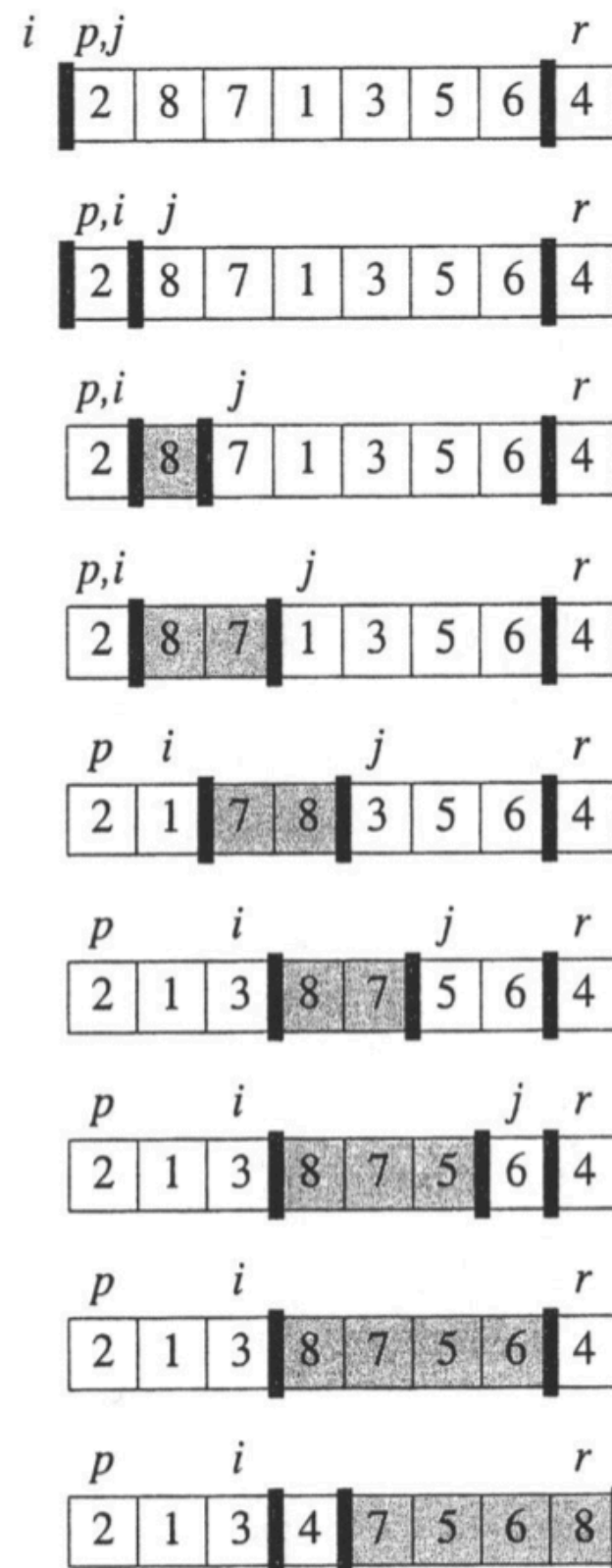
KVICK SÖRT

idea-instructions.com/quick-sort/
v1.1, CC by-nc-sa 4.0

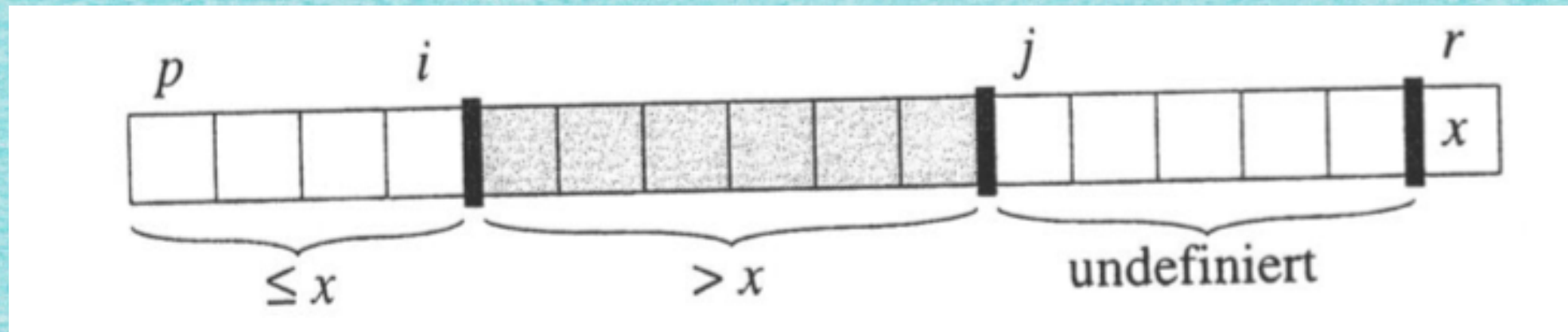
IDEA



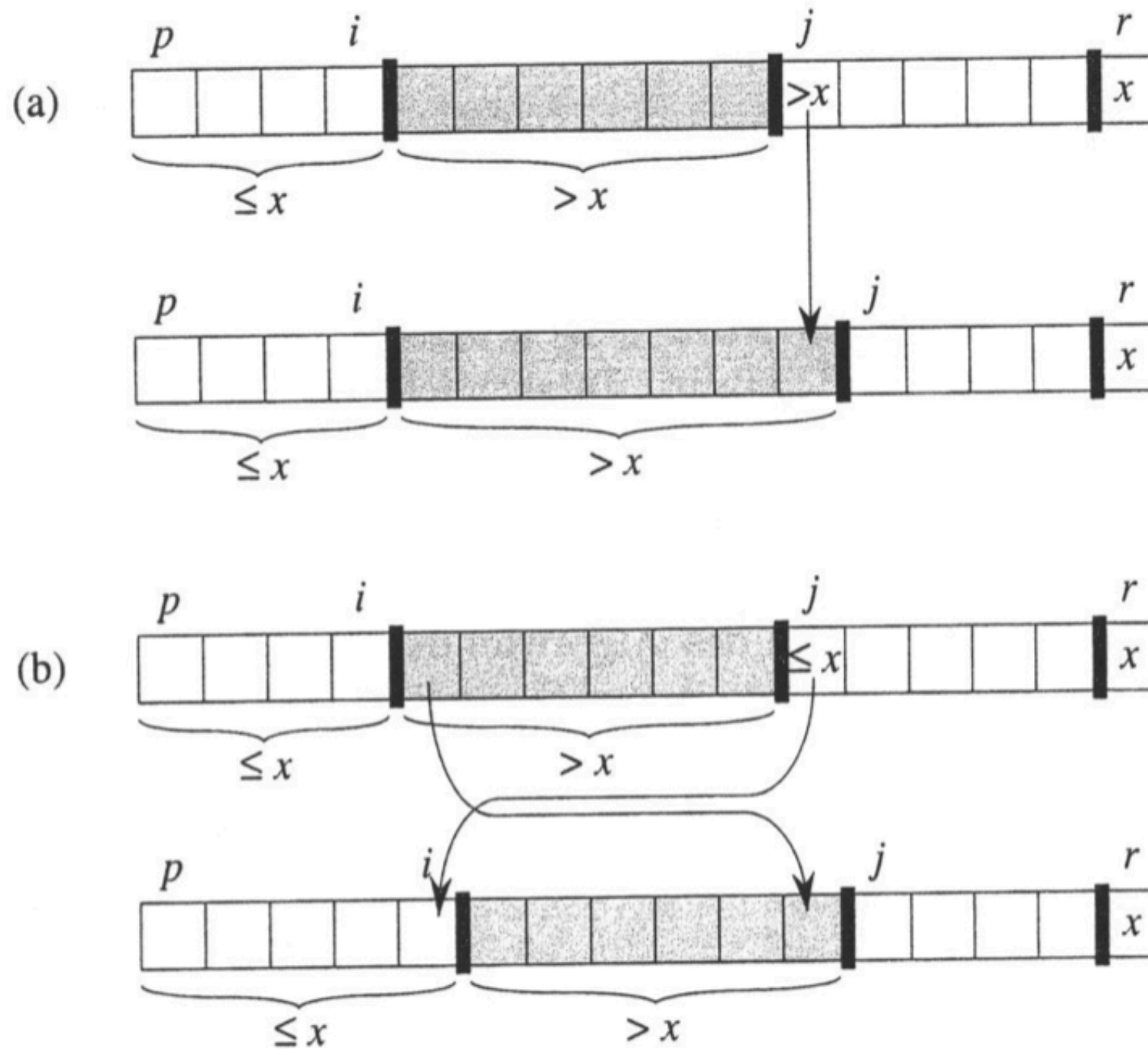
5.5.1 Ablauf Quicksort



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5.5.1 Ablauf Quicksort



5.5.2 Algorithmische Beschreibung

Algorithmus 5.14

INPUT: Subarray von $A=[1,\dots,n]$,
der bei Index p beginnt und bei Index r endet, d.h. $A[p,\dots,r]$
OUTPUT: Sortierter Subarray

```
QUICKSORT( $A, p, r$ )  
1  if  $p < r$   
2    then  $q \leftarrow \text{PARTITION}(A, p, r)$   
3         QUICKSORT( $A, p, q - 1$ )  
4         QUICKSORT( $A, q + 1, r$ )
```


Subroutine 5.15

INPUT: Subarray von $A=[1,\dots,n]$, d.h. $A[p,\dots,r]$

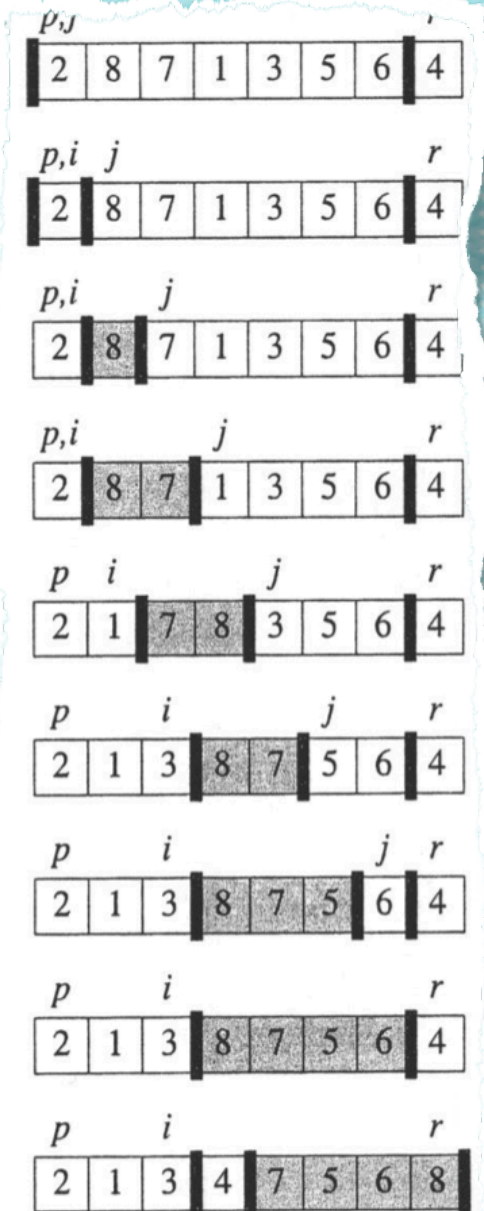
OUTPUT: Zwei Subarrays $A[p,\dots,q-1]$ und $A[q+1,\dots,r]$
mit $A[i] \leq A[q]$ und $A[q] < A[j]$ für $i=p,\dots,q-1$ und $j=q+1,\dots,r$

PARTITION(A, p, r)

```

1   $x \leftarrow A[r]$ 
2   $i \leftarrow p - 1$ 
3  for  $j \leftarrow p$  to  $r - 1$ 
4      do if  $A[j] \leq x$ 
5          then  $i \leftarrow i + 1$ 
6          vertausche  $A[i] \leftrightarrow A[j]$ 
7  vertausche  $A[i + 1] \leftrightarrow A[r]$ 
8  return  $i + 1$ 

```



5.5.3 Laufzeit von Quicksort

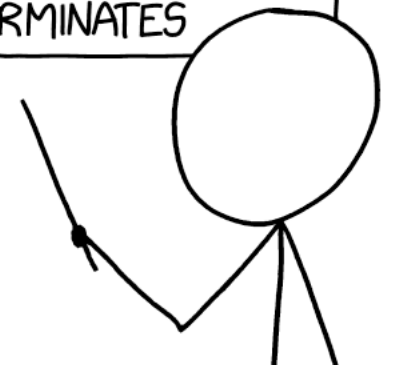
Wie viele Schritte benötigt Quicksort für einen Array der Länge n ?

Unterscheidung:

- Worst Case
- Best Case
- Average Case

RESULTS OF ALGORITHM COMPLEXITY ANALYSIS:

AVERAGE CASE	$O(N \log N)$
BEST CASE	ALGORITHM TURNS OUT TO BE UNNECESSARY AND IS HALTED, THEN CONGRESS ENACTS SURPRISE DAYLIGHT SAVING TIME AND WE GAIN AN HOUR
WORST CASE	TOWN IN WHICH HARDWARE IS LOCATED ENTERS A GROUNDHOG DAY SCENARIO, ALGORITHM NEVER TERMINATES



5.5.3 Quicksort: Laufzeit

Schlechtester Fall: Pivot liegt extremal, d.h. nach PARTITION ist ein Teilarray $n-1$ lang, der andere hat Länge 0.

$$\begin{aligned} T(n) &= T(n-1) + T(0) + \Theta(n) \\ &= T(n-1) + \Theta(n) . \end{aligned}$$

Man sieht (und beweist leicht):

$$\sum_{i=0}^n \Theta(n-i) = \Theta(n^2)$$

5.5.3 Quicksort: Laufzeit

Beste Fall: Pivot liegt genau in der Mitte, d.h. nach PARTITION haben beide Teilarrays i.W. die Länge $n/2$.

$$T(n) \leq 2T(n/2) + \Theta(n)$$

Man sieht, z.B. mit dem Mastertheorem:

$$\Theta(n \log n)$$

5.5.3 Quicksort: Laufzeit

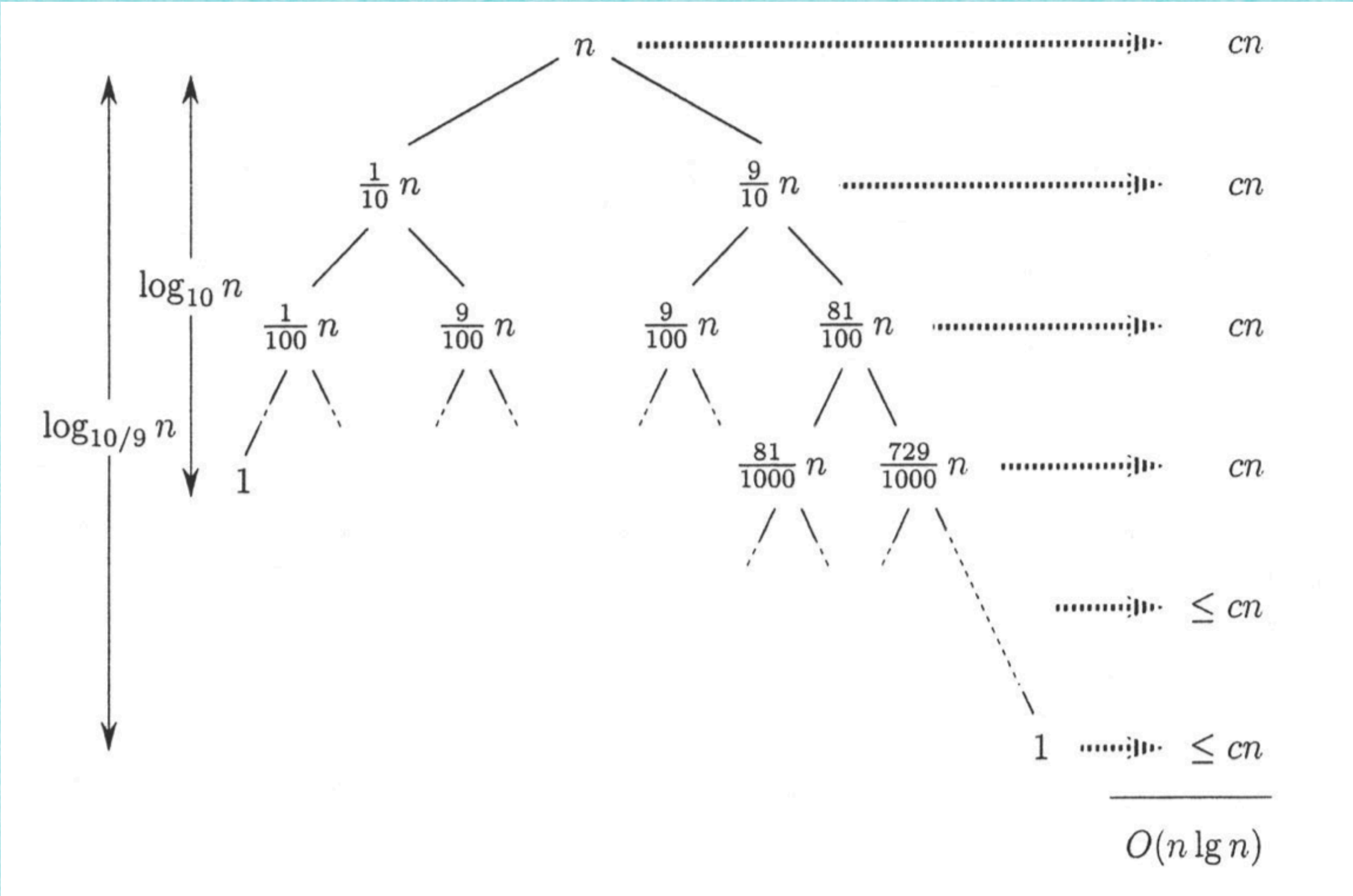
Das funktioniert auch noch, wenn PARTITION ein *annähernd* balanciertes Ergebnis liefert:

$$T(n) \leq T(9n/10) + T(n/10) + cn$$

Man sieht wieder mit dem Mastertheorem:

$$\Theta(n \log n)$$

5.5.3 Quicksort: Laufzeit



5.5.3 Quicksort: Laufzeit

Satz 5.13 (Durchschnittliche Laufzeit von Quicksort)
Für einen n -elementigen Array A hat Quicksort eine erwartete Laufzeit von $O(n \log n)$.

5.5.3 Quicksort: Laufzeit

Warum interessiert das?



*** 1987**

Sebastian Wild



*** 1981**

Zlatan Ibrahimovic

5.5.3 Quicksort: Laufzeit

Warum interessiert das?



* 1987

Sebastian Wild

Average Case Analysis of Java 7's Dual Pivot Quicksort*

Sebastian Wild and Markus E. Nebel

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Abstract. Recently, a new Quicksort variant due to Yaroslavskiy was chosen as standard sorting method for Oracle's Java 7 runtime library. The decision for the change was based on empirical studies showing that on average, the new algorithm is faster than the formerly used classic Quicksort. Surprisingly, the improvement was achieved by using a dual pivot approach, an idea that was considered not promising by several theoretical studies in the past. In this paper, we identify the reason for this unexpected success. Moreover, we present the first precise average case analysis of the new algorithm showing e.g. that a random permutation of length n is sorted using $1.9n \ln n - 2.46n + \mathcal{O}(\ln n)$ key comparisons and $0.6n \ln n + 0.08n + \mathcal{O}(\ln n)$ swaps.

1 Introduction

Due to its efficiency in the average, Quicksort has been used for decades as general purpose sorting method in many domains, e.g. in the C and Java standard libraries or as UNIX's system sort. Since its publication in the early 1960s by Hoare [1], classic Quicksort (Algorithm 1) has been intensively studied and many modifications were suggested to improve it even further, one of them being the following: Instead of partitioning the input file into two subfiles separated by a single pivot, we can create s partitions out of $s - 1$ pivots.

Sedgewick considered the case $s = 3$ in his PhD thesis [2]. He proposed and analyzed the implementation given in Algorithm 2. However, this dual pivot Quicksort variant turns out to be clearly inferior to the much simpler classic algorithm. Later, Hennequin studied the comparison costs for any constant s in his PhD thesis [3], but even for arbitrary $s \geq 3$, he found no improvements that would compensate for the much more complicated partitioning step¹. These negative results may have discouraged further research along these lines.

Recently, however, Yaroslavskiy proposed the new dual pivot Quicksort implementation as given in Algorithm 3 at the Java core library mailing list². He

* This research was supported by DFG grant NE 1379/3-1.

¹ When s depends on n , we basically get the Samplesort algorithm from [4], [5], [6] or [7] show that Samplesort can beat Quicksort if hardware features are exploited. [2] even shows that Samplesort is asymptotically optimal with respect to comparisons. Yet, due to its inherent intricacies, it has not been used much in practice.

² The discussion is archived at <http://permalink.gmane.org/gmane.comp.java.openjdk.core-libs.devel/2628>.

5.5.3 Quicksort: Laufzeit

Warum interessiert das?



* 1987

Sebastian Wild

20th European Symposium on
Algorithms - ESA 2012

September 10-12, Ljubljana, Slovenia



	Comparisons
Classic Quicksort (Algorithm 1)	$2(n+1)\mathcal{H}_{n+1} - \frac{8}{3}(n+1)$ $\approx 2n \ln n - 1.51n + \mathcal{O}(\ln n)$
Sedgewick (Algorithm 2)	$\frac{32}{15}(n+1)\mathcal{H}_{n+1} - \frac{856}{225}(n+1) + \frac{3}{2}$ $\approx 2.13n \ln n - 2.57n + \mathcal{O}(\ln n)$
Yaroslavskiy (Algorithm 3)	$\frac{19}{10}(n+1)\mathcal{H}_{n+1} - \frac{711}{200}(n+1) + \frac{3}{2}$ $\approx 1.9n \ln n - 2.46n + \mathcal{O}(\ln n)$

“Best Paper Award”

Mehr an der Tafel!

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