Computational Geometry

Tutorial #6

Institut für Betriebssysteme
und Rechnerverbund
Algorithmik

Peter Kramer | *January 15th, 2026*

Convex hull of simple polygons

Information Processing Letters 25 (1987) 11–12
North-Holland

20 April 1987

ON-LINE CONSTRUCTION OF THE CONVEX HULL OF A SIMPLE POLYLINE

Avraham A. MELKMAN *

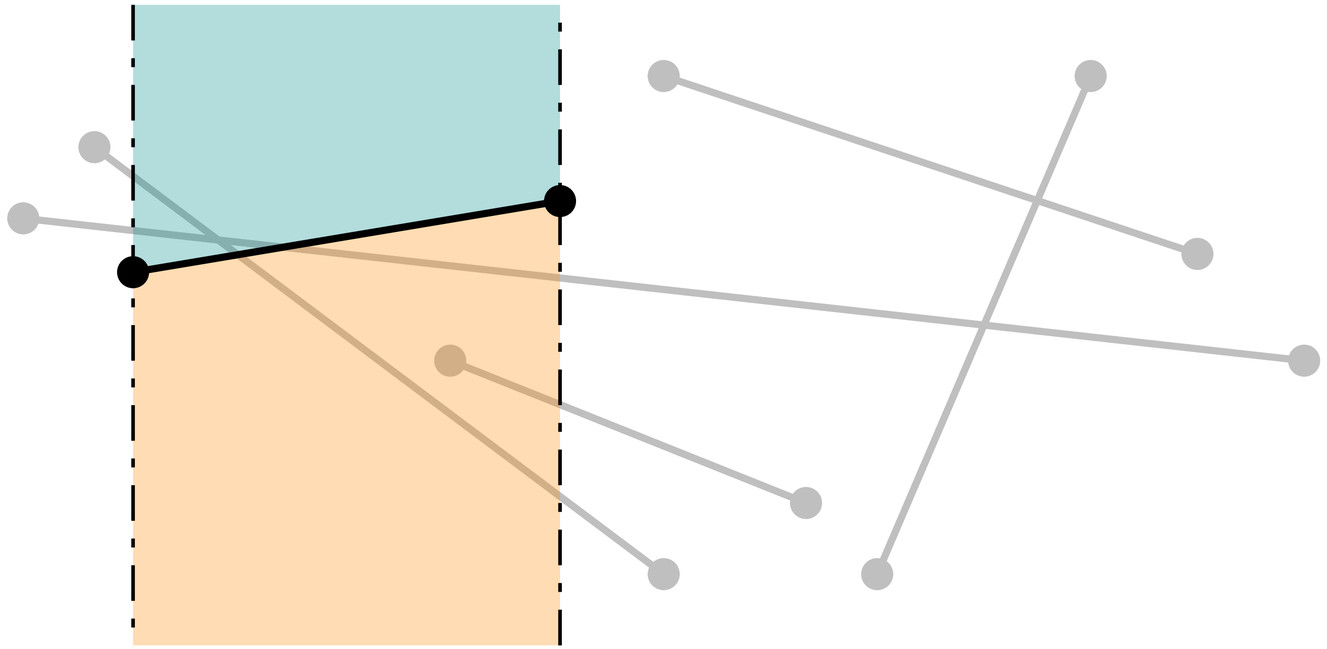
Ben Gurion University of the Negev, Beer Sheva, Israel

Communicated by Alan Shaw
Received 2 April 1985
Revised 27 November 1985 and 9 September 1986

Intersections of line segments

Peter Kramer | *January 15th, 2026*

Intersections of line segments
Algorithm design



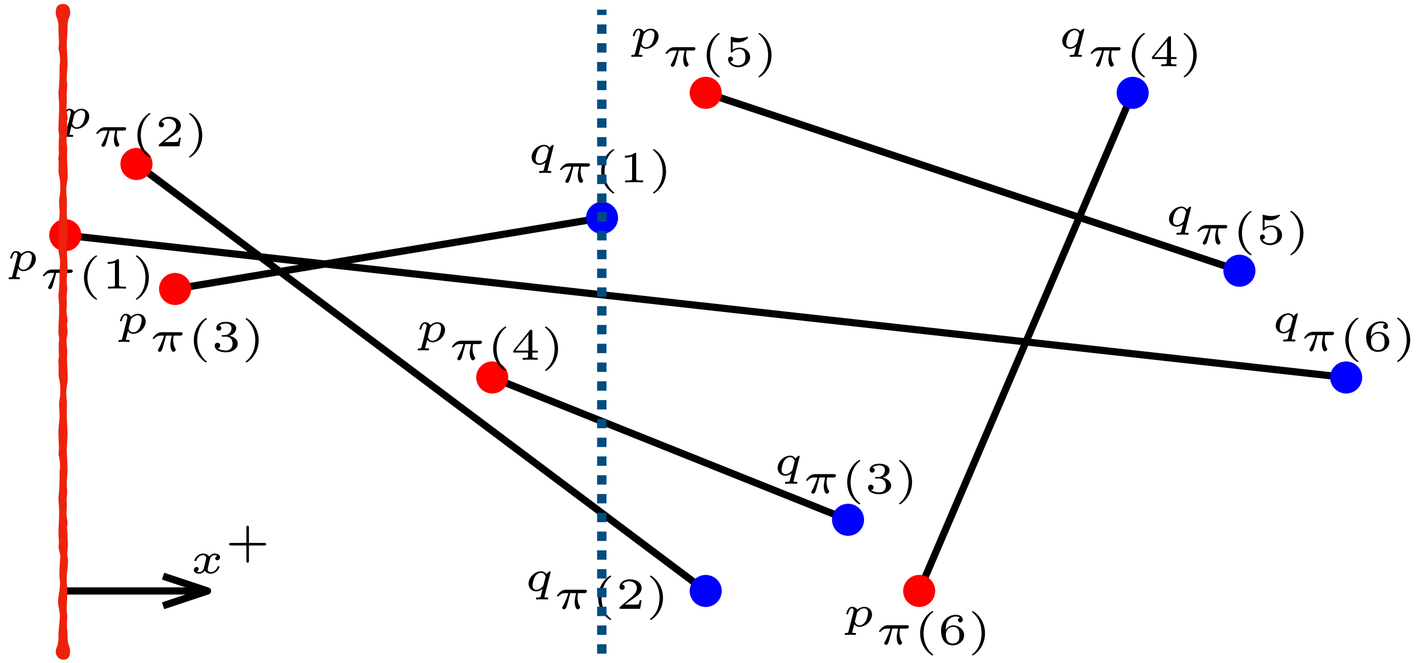
- **Given:** Endpoints $(p_1, q_1), \ldots, (p_n, q_n)$ of n line segments $\overline{p_i q_i}$ in the plane.
- **Wanted:** Intersections of segments, so …
  - … the number of intersections k, and
  - … the involved edges of each crossing.

- *When do two lines intersect (criteria)?*
  How many crossings can there be?
  Is there structure to this problem?

- **Goal:** Construct a sweep-line algorithm that computes this in $\mathcal{O}((n + k)\log n)$ time.

Intersections of line segments

Sweep-Line Algorithm



(i) Orient segments such that $x(p_i) \leq x(q_i)$.

(ii) Sort p and q by their x-coordinates.
Let $p_{\pi(i)}$ be the element rank i (ith lowest).
These define start and end events.

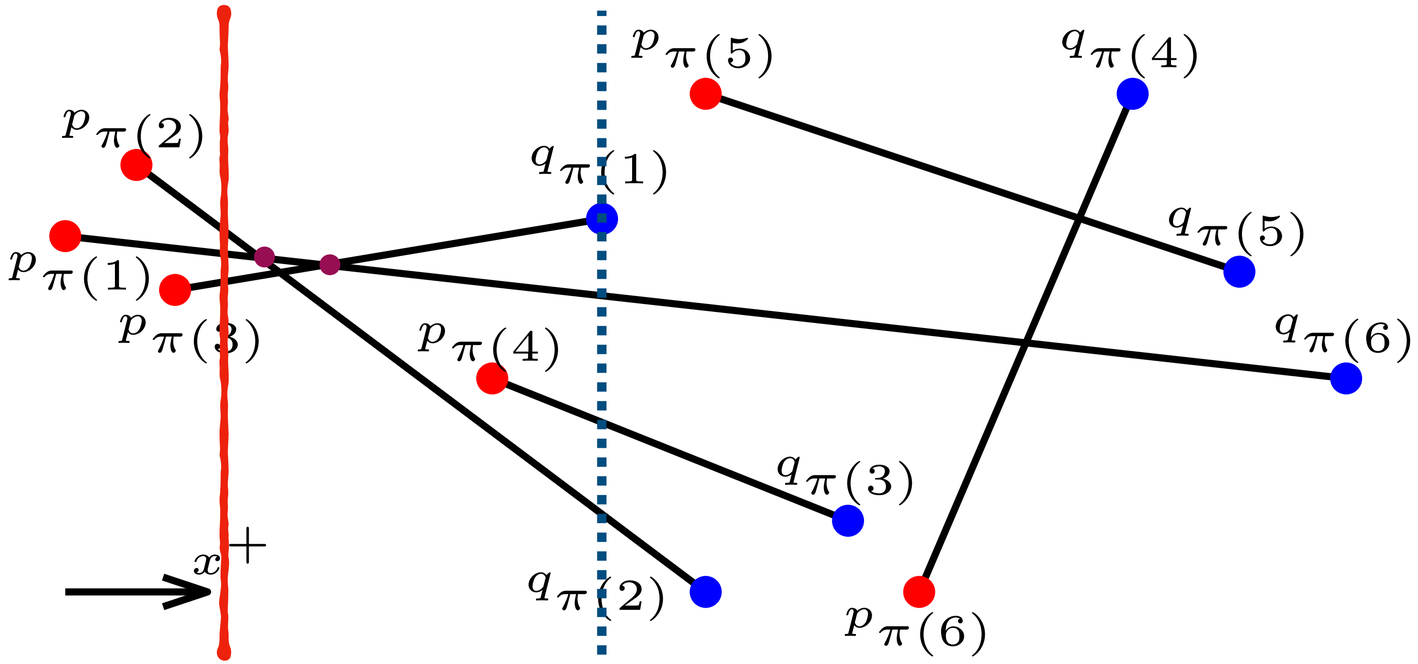
(iii) Let $i, j = 1$, initialise a BST T (sweep state) and a priority queue C (crossings).

(iv) Sweep the vertical bisector at $\pi(i)$ along the x-axis, tracking intersecting segments in y-monotone order using the BST.
This order changes exactly at a crossings.

(board).

Intersections of line segments

Sweep-Line Algorithm



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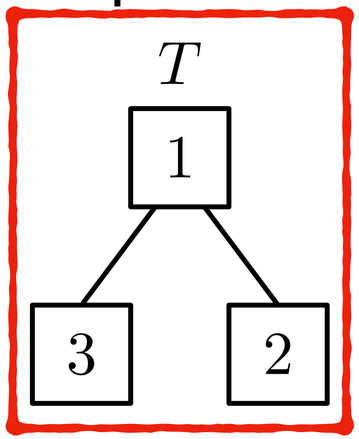
Crossings:

Each $(c, a, b) \in C$ with

- $c \in \mathbb{R}^2$ position
- $a \in [1,n]$ segment A
- $b \in [1,n]$ segment B
- **Priority** $x(c)$.

Sweep state:



Thank you