Computational Geometry

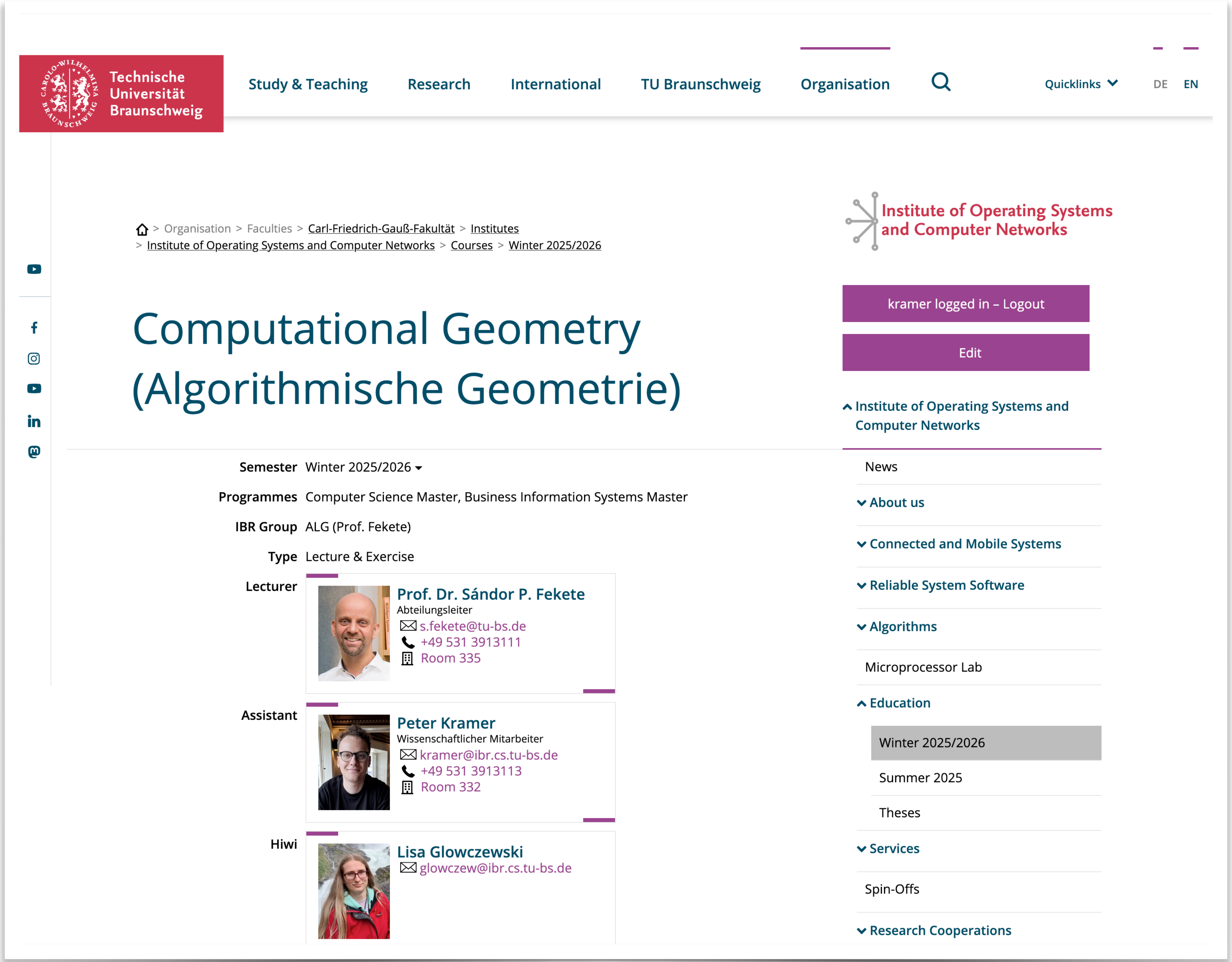
Tutorial #1 — Organisation and Convex Hulls

Institut für Betriebssysteme
und Rechnerverbund
Algorithmik

Peter Kramer | November 6th, 2025

Organisation

Website



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Computational Geometry (Algorithmische Geometrie)
Semester Winter 2025/2026
Programmes Computer Science Master, Business Information Systems Master
IBR Group ALG (Prof. Fekete)
Type Lecture & Exercise
Lecturer
Prof. Dr. Sándor P. Fekete
Abteilungsleiter
s.fekete@tu-bs.de
+49 531 3913111
Room 335
Assistant
Peter Kramer
Wissenschaftlicher Mitarbeiter
kramer@ibr.cs.tu-bs.de
+49 531 3913113
Room 332
Hiwi
Lisa Glowczewski
glowczew@ibr.cs.tu-bs.de
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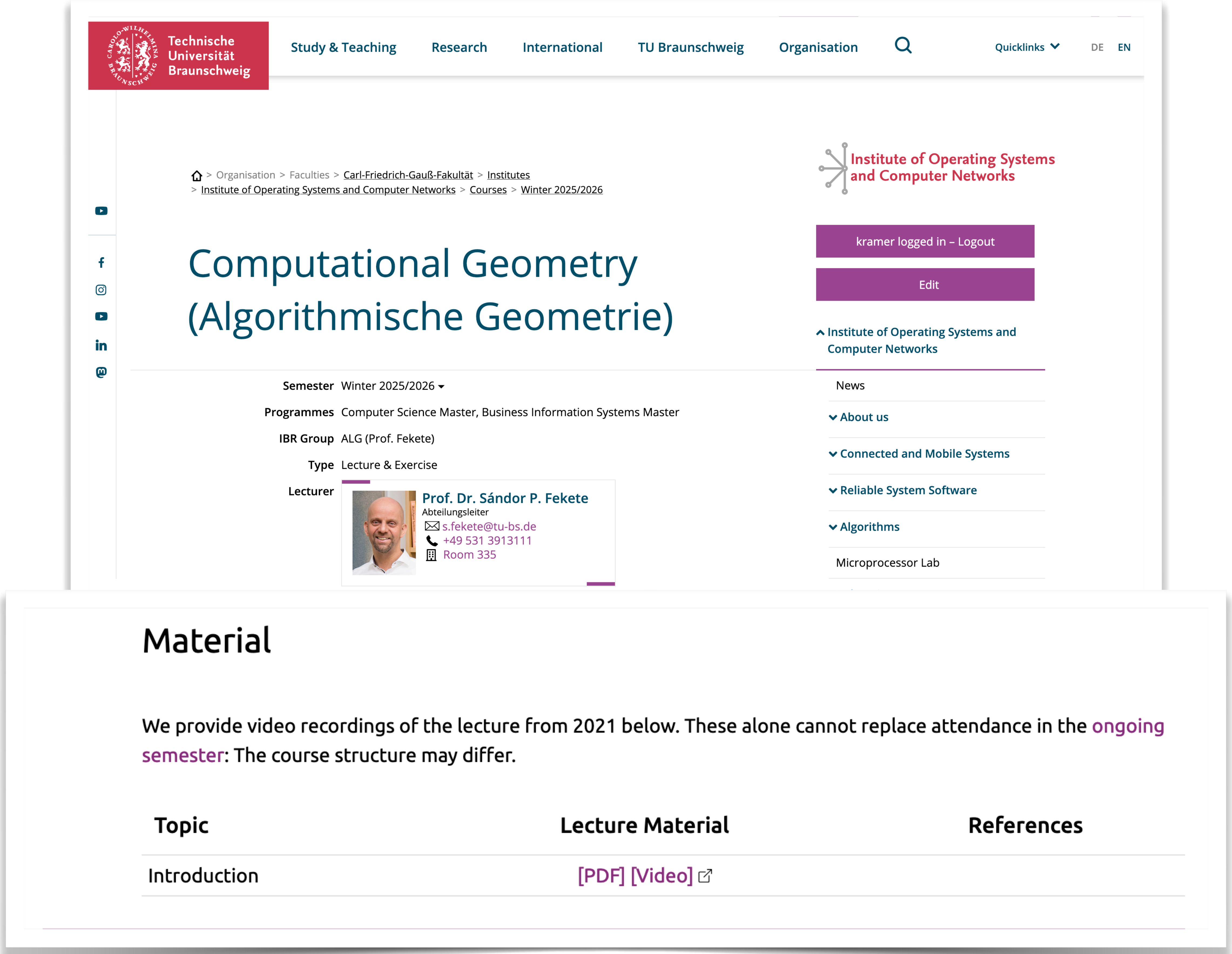

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s.fekete@tu-bs.de
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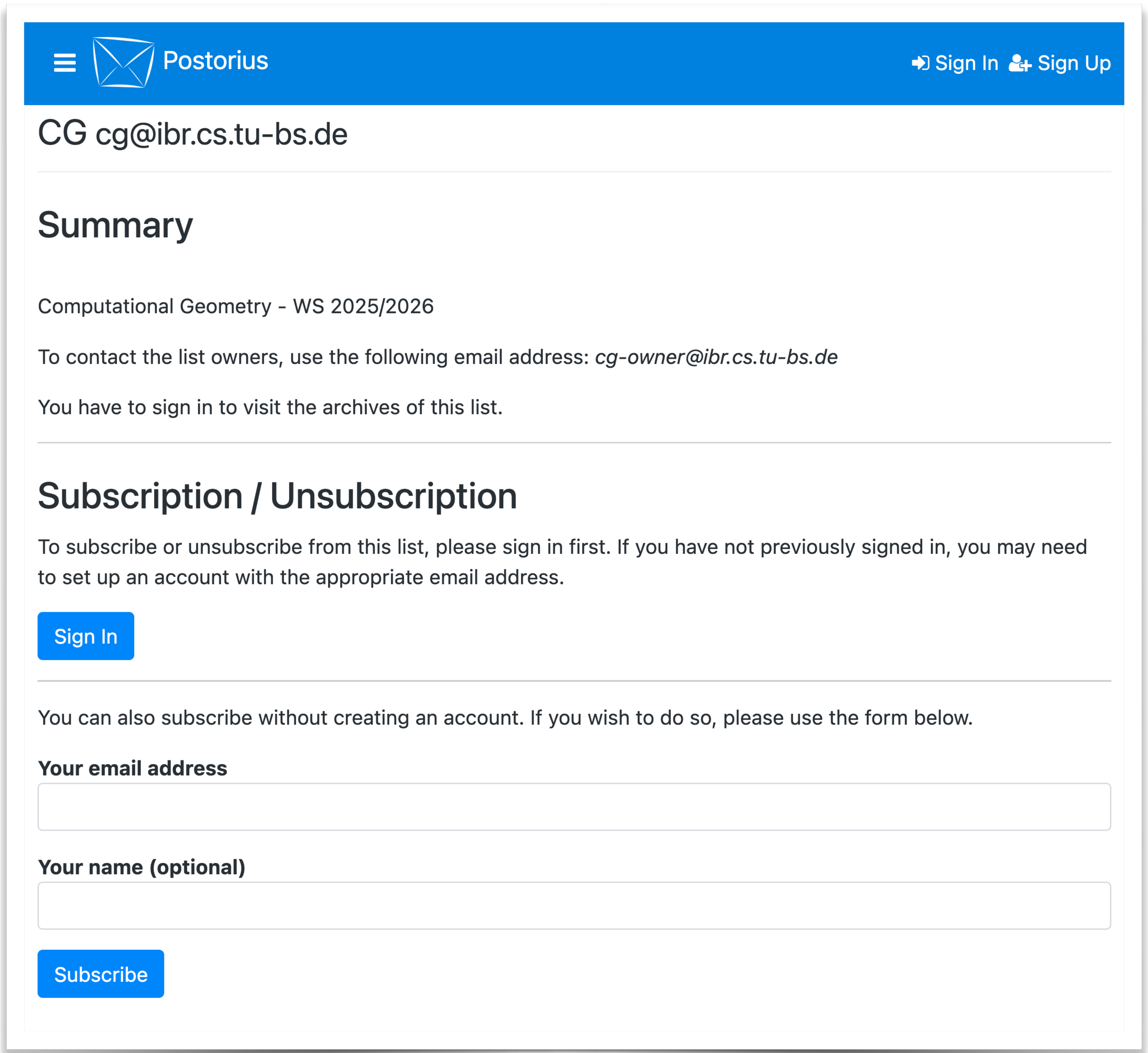
Room 335
Material
We provide video recordings of the lecture from 2021 below. These alone cannot replace attendance in the ongoing semester: The course structure may differ.
Topic
Lecture Material
References
Introduction
[PDF] [Video]



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Summary
Computational Geometry - WS 2025/2026
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Tutorial

- **Every Thursday at 3pm** in this room, either a big or a small tutorial.
  - Depending on attendance, we might switch rooms.

- **Big tutorial:** Apply and expand upon concepts from the lecture.
- **Small tutorial:** Homework discussion.

October						
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

November						
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

- 13 Lectures
- 7 Big Tutorials
- 5 Small Tutorials
- 4 Sheets

December						
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

January						
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	20	23	24	25
26	27	28	29	30	31	

February						
						1
2	3	4	29	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Tutorial

- **Every Thursday at 3pm** in this room, either a big or a small tutorial.
  - Depending on attendance, we might switch rooms.

- **Big tutorial:** Expand upon and put concepts from the lecture to use.
- **Small tutorial:** Homework discussion.

- **Studienleistung:** Four homework sheets.
  - Biweekly assignments to solve in **groups of two or three**.
  - Requires **50% of total possible points** on the sheets; roughly 75 points.

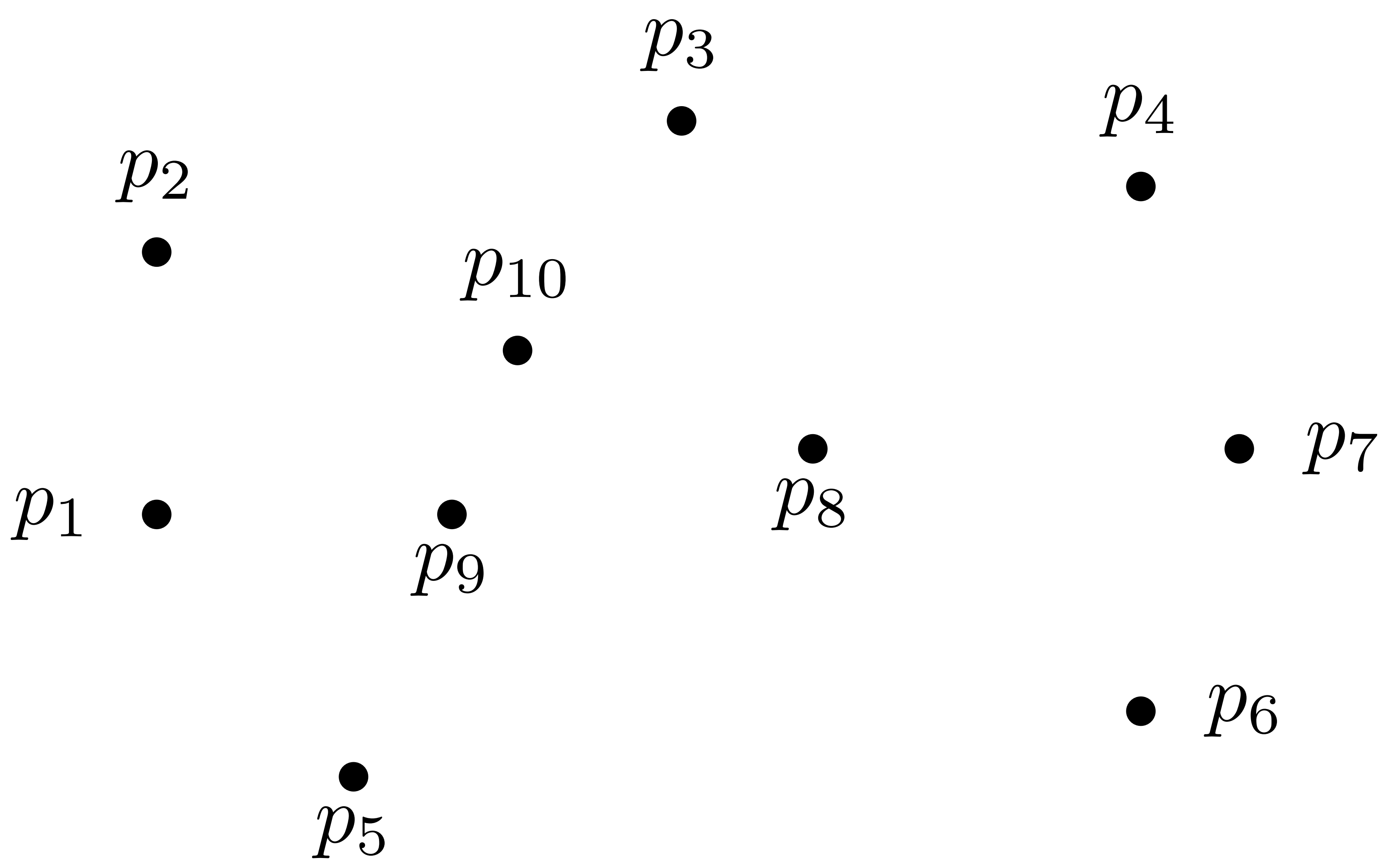
Convex Hulls

Peter Kramer | November 6th, 2025

The convex hull

… is a point set? A convex polygon? A subset?

- For a finite point set $\mathscr{P} := \{p_1, p_2, \ \ldots, p_n\} \subset \mathbb{R}^2, \mathrm{conv}(\mathscr{P}) = \ \ldots ?$



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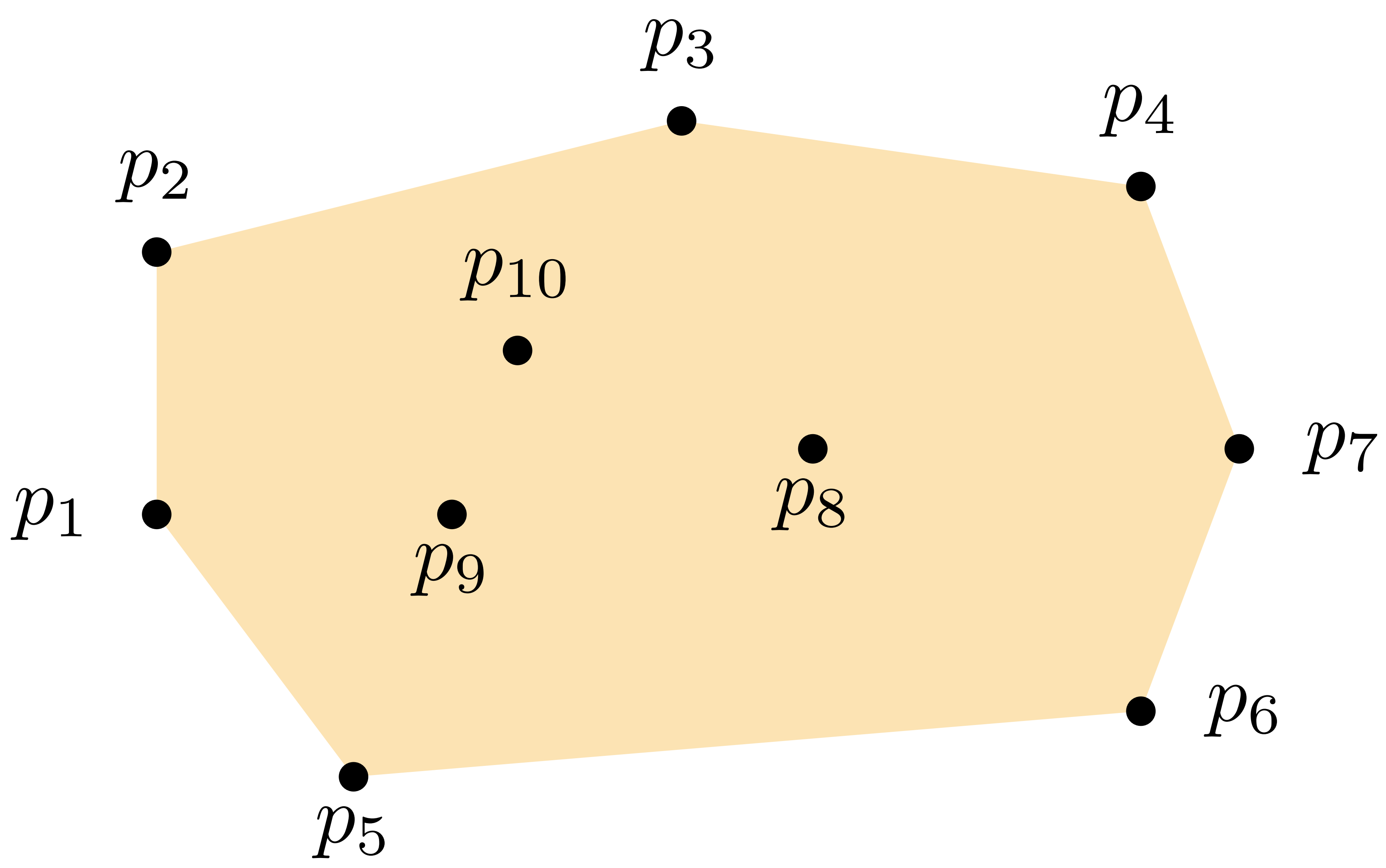

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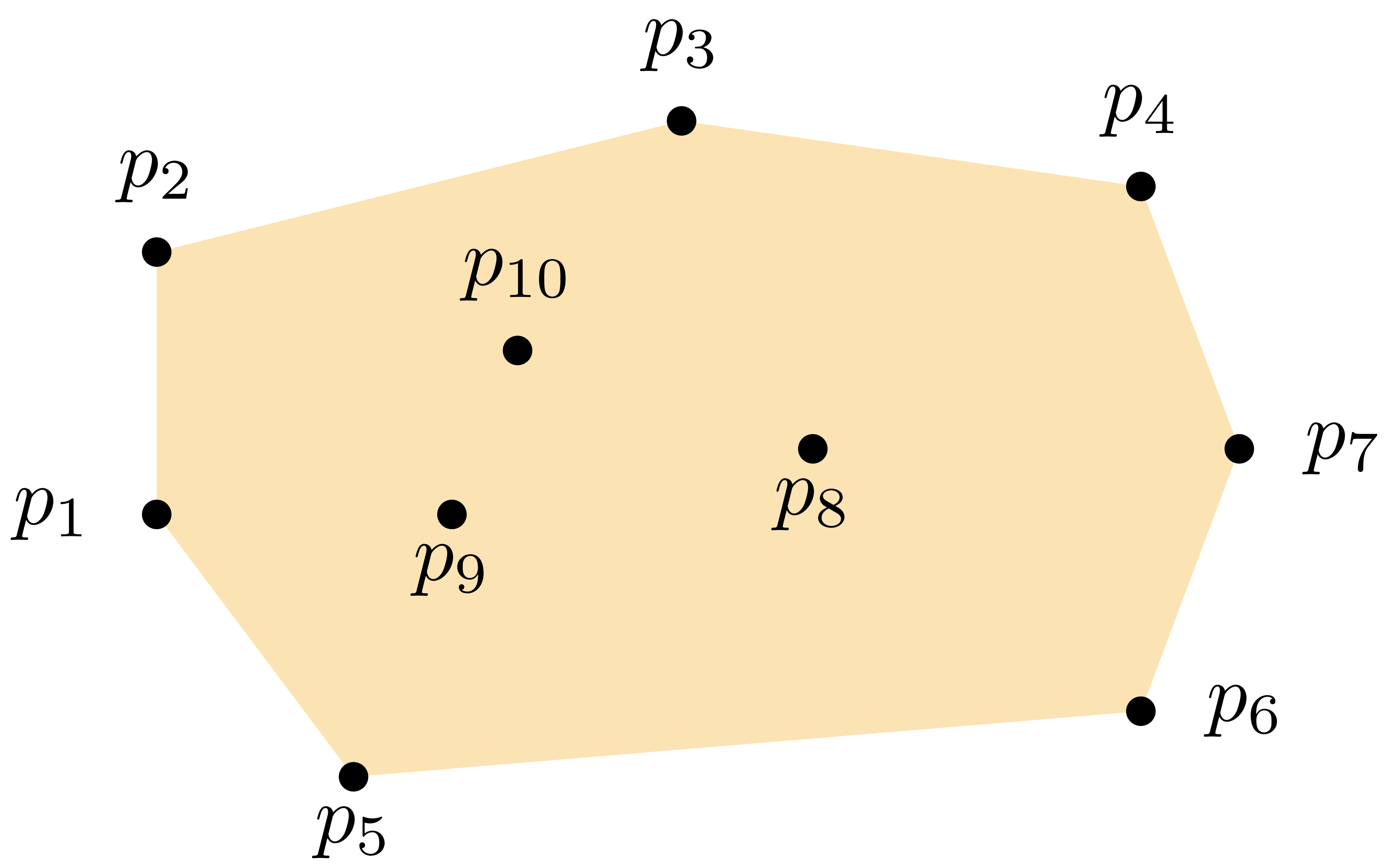
- *Definition 2.3:* $\mathrm{conv}(\mathscr{P}) = \{x \in \mathbb{R}^2 \mid x$ is a convex combination of $\mathscr{P}\}$.



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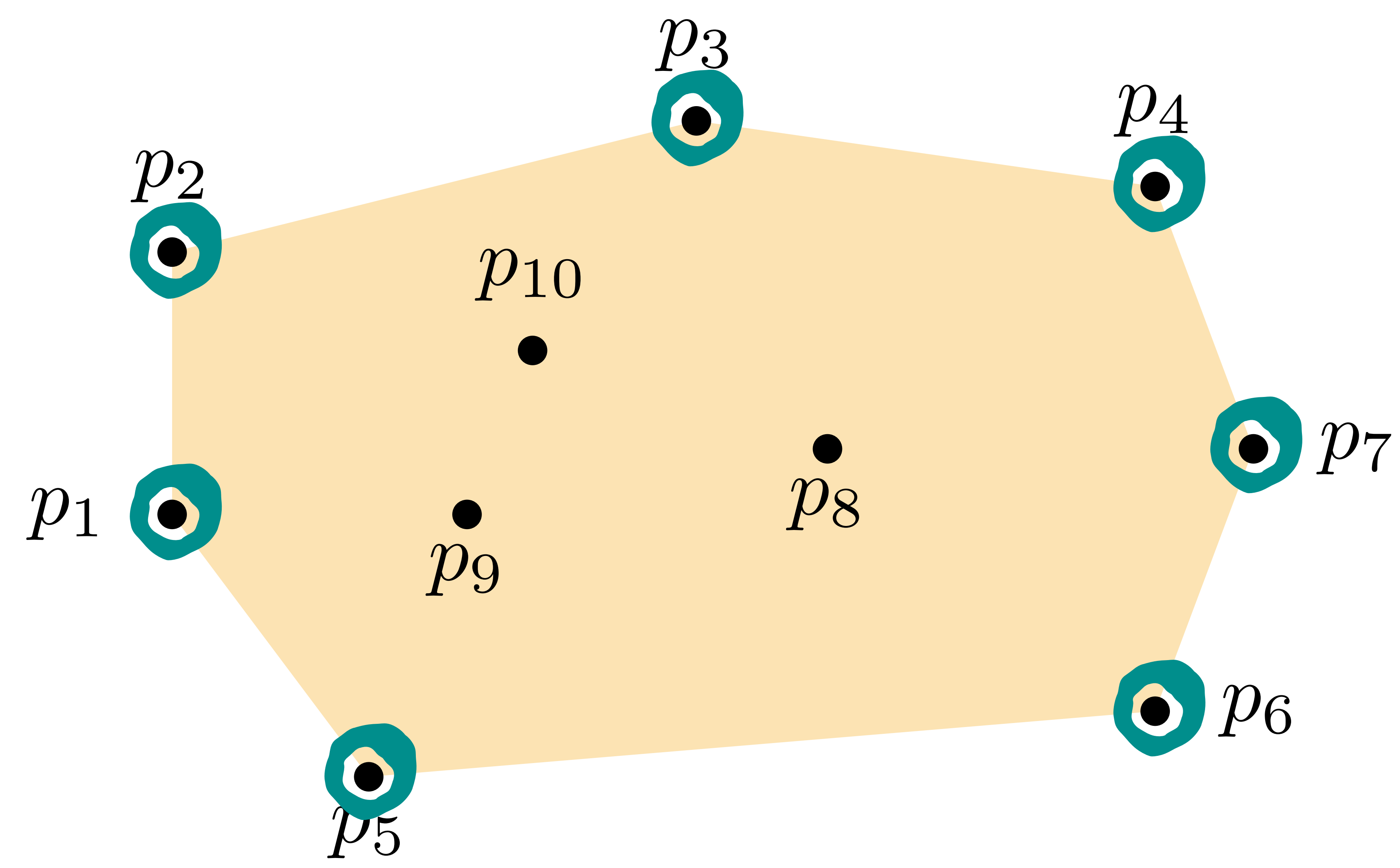
- *Definition 2.3:* $\mathrm{conv}(\mathscr{P}) = \{x \in \mathbb{R}^2 \mid x$ is a convex combination of $\mathscr{P}\}$.
- *This is an infinite set, but our algorithm(s) only give us points from $\mathscr{P}$!?*



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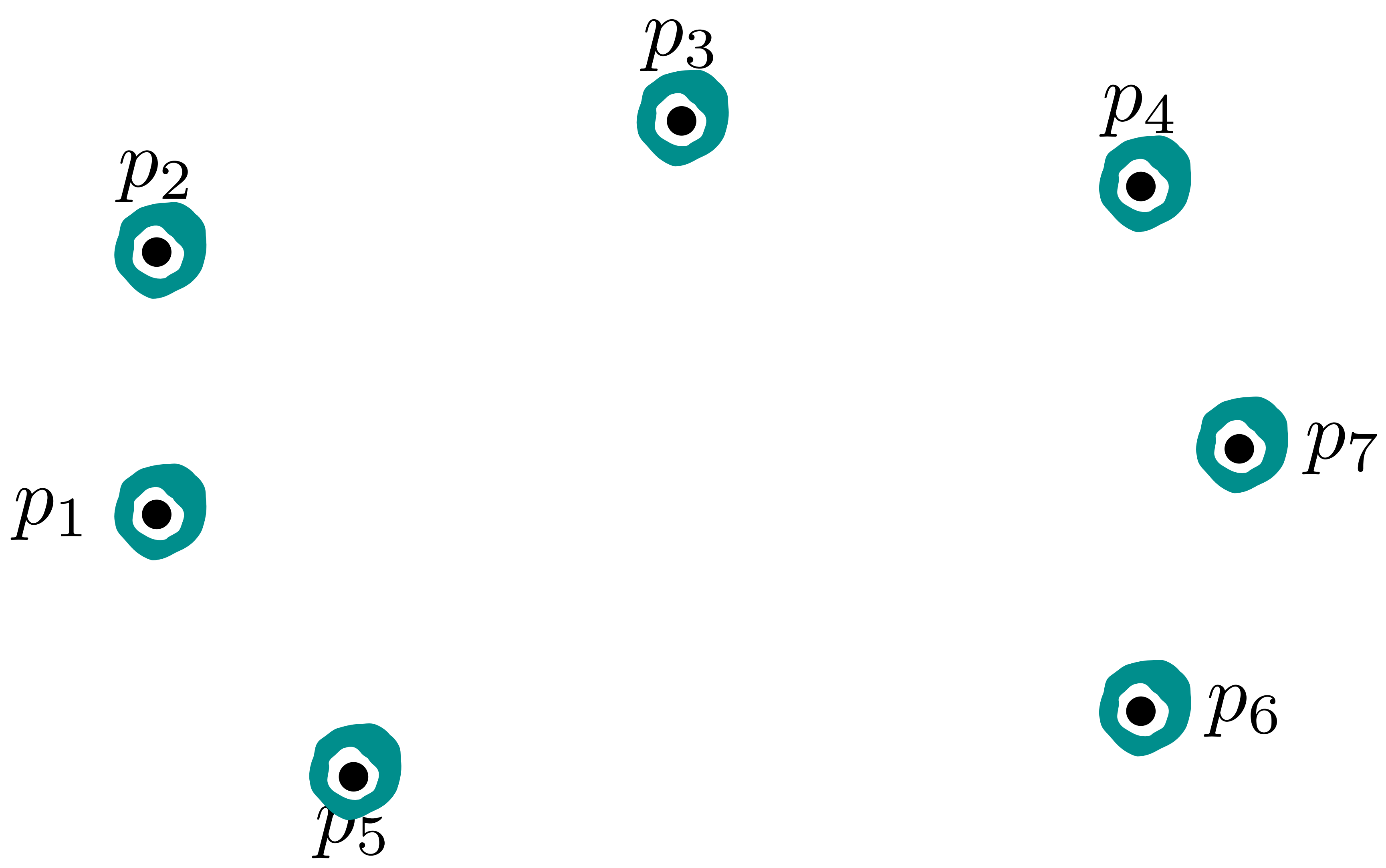
- *Definition 2.3:* $\mathrm{conv}(\mathscr{P}) = \{x \in \mathbb{R}^2 \mid x$ is a convex combination of $\mathscr{P}\}$.
- *This is an infinite set, but our algorithm(s) only give us points from* $\mathscr{P}$ *!?*
- $\mathrm{conv}(\mathscr{P}) = \mathscr{P}$ *???*

Point sets like this one are referred to as being in "convex position".



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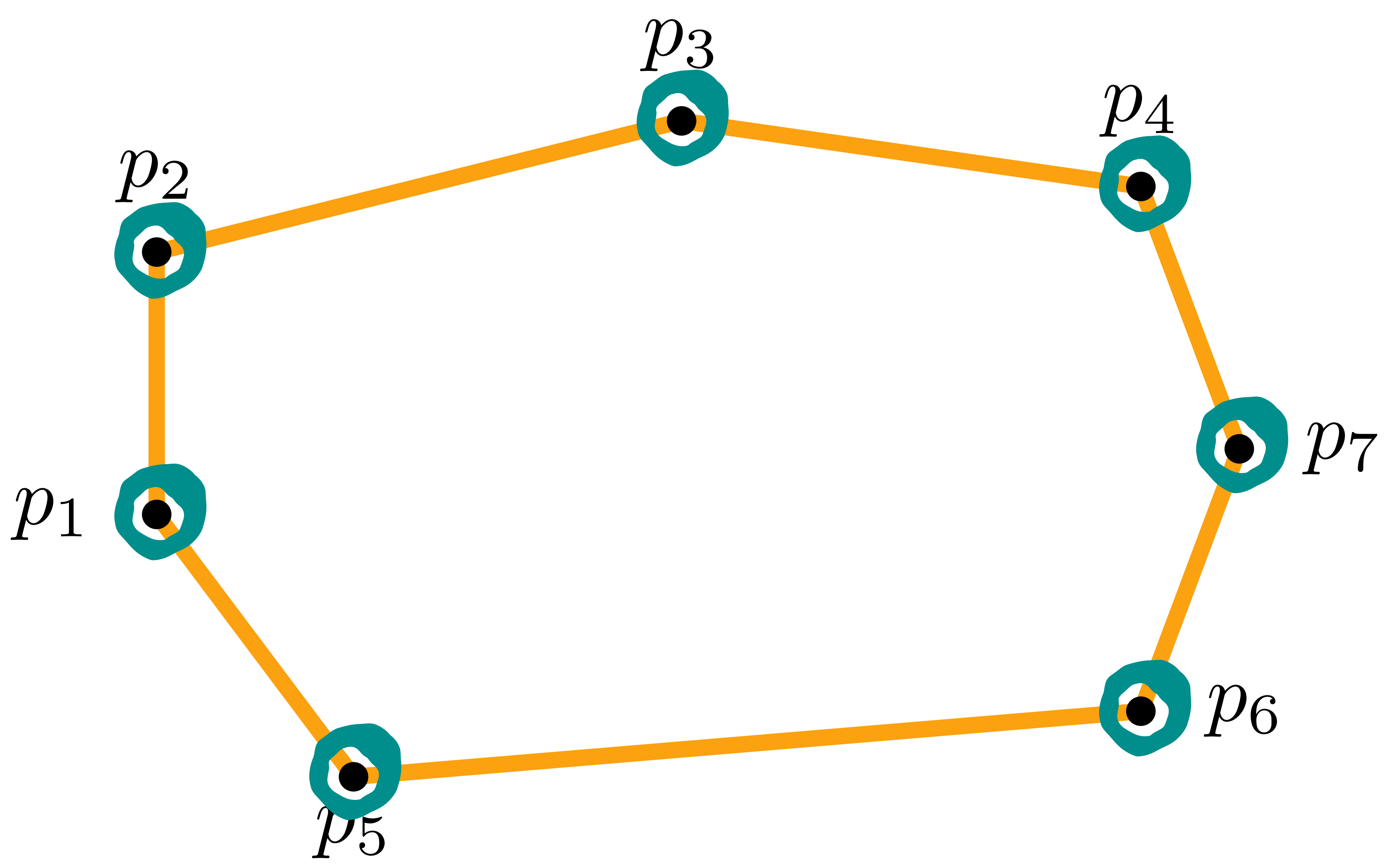
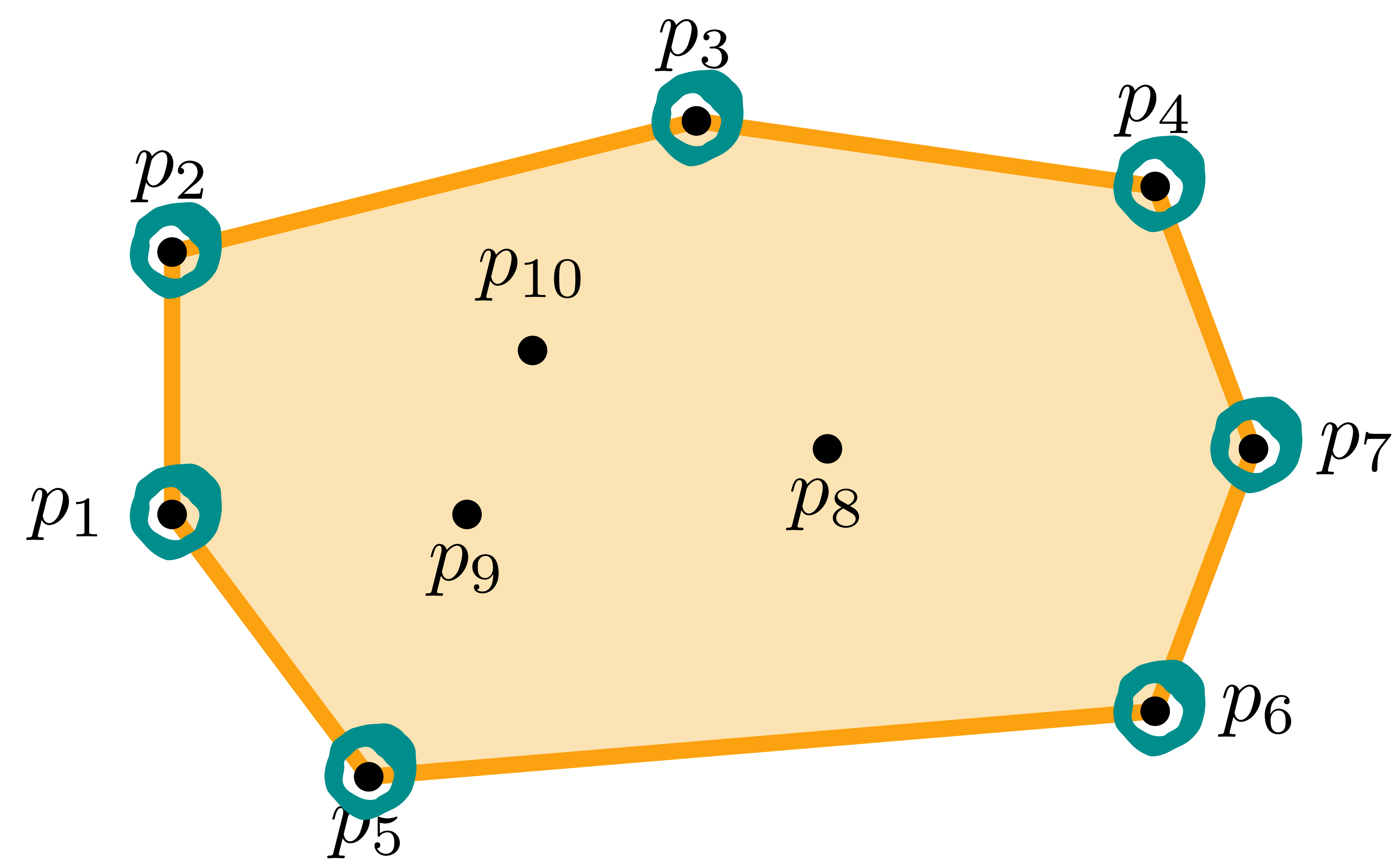
- $\cancel{\mathrm{conv}(\mathscr{P}) = \mathscr{P}}$ *???*



The convex hull

… is a (usually infinite) point set, described by a convex polygon.

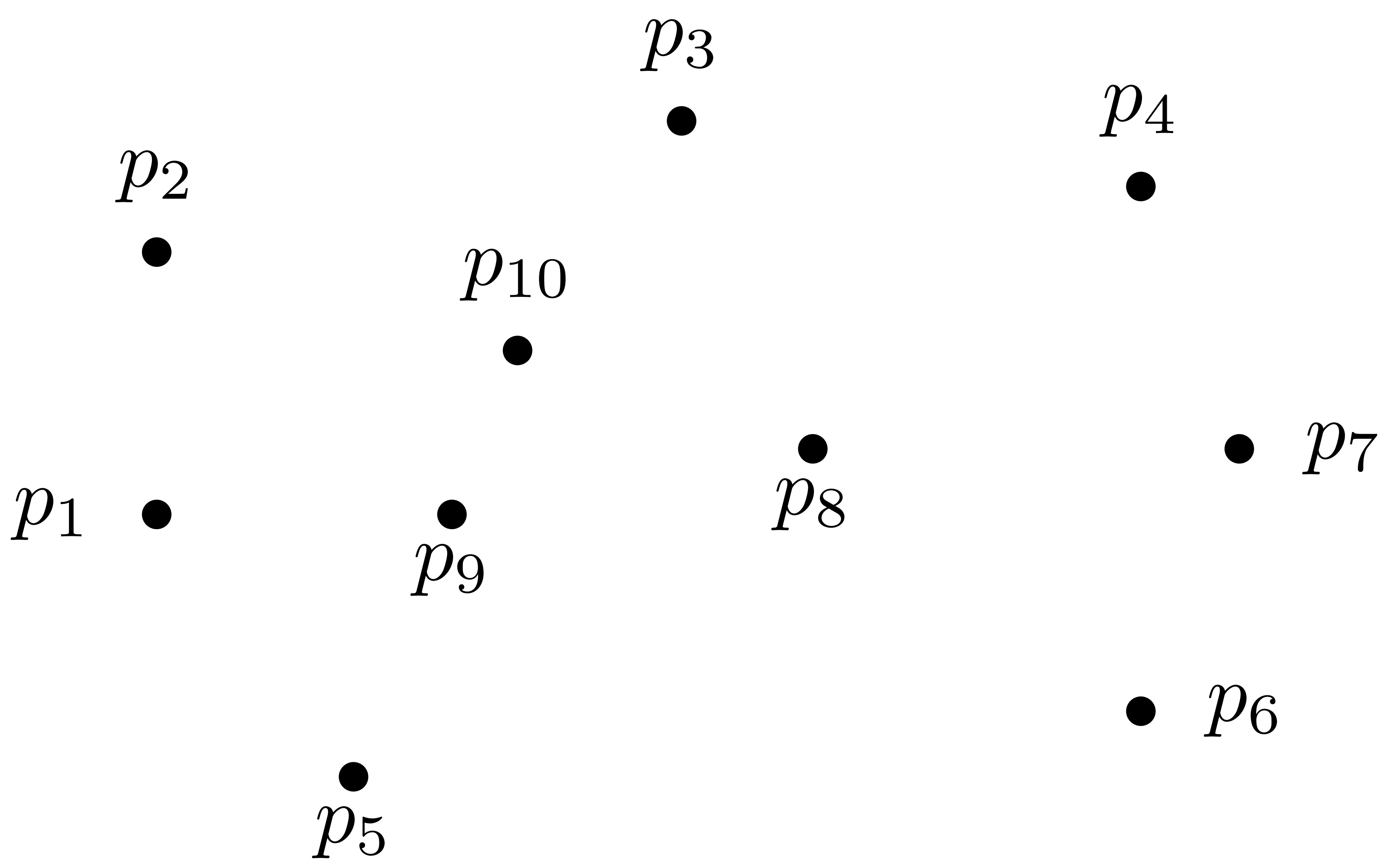
- Given a finite point set $\mathscr{P} := \{p_1, p_2, \dots, p_n\} \subset \mathbb{R}^2$.
- *Definition 2.3:* $\operatorname{conv}(\mathscr{P}) = \{x \in \mathbb{R}^2 \mid x$ is a convex combination of $\mathscr{P}\}$
- *Our algorithms compute the <u>boundary set</u>* $\partial \operatorname{conv}(\mathscr{P})$ *as a convex polygon!*



Detour: Point sets & Polygons

Point sets & Polygons
What's the difference?

- Consider a finite point set $\mathscr{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$.
- What's a polygon P using these points?



Point sets & Polygons

What's the difference?

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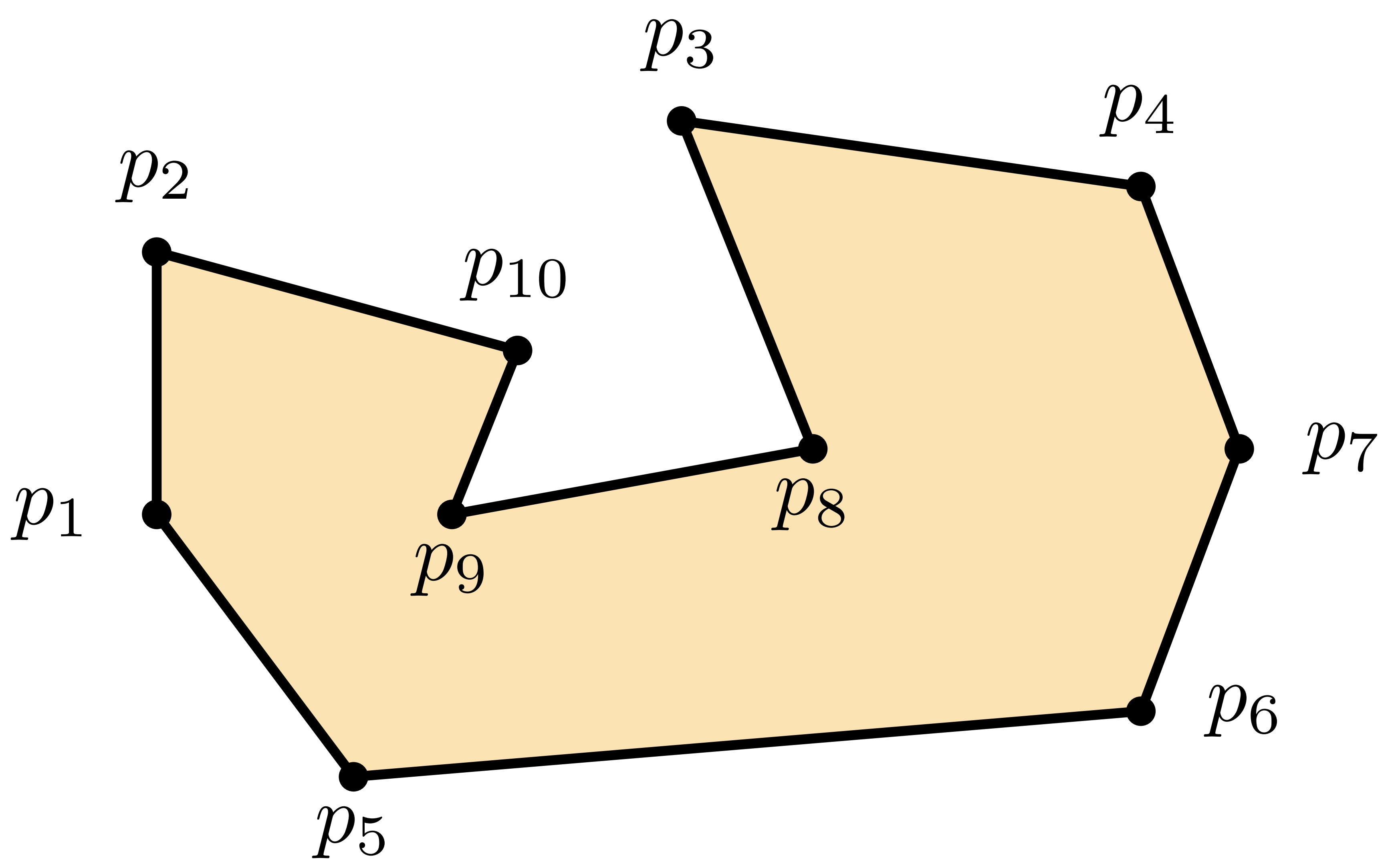
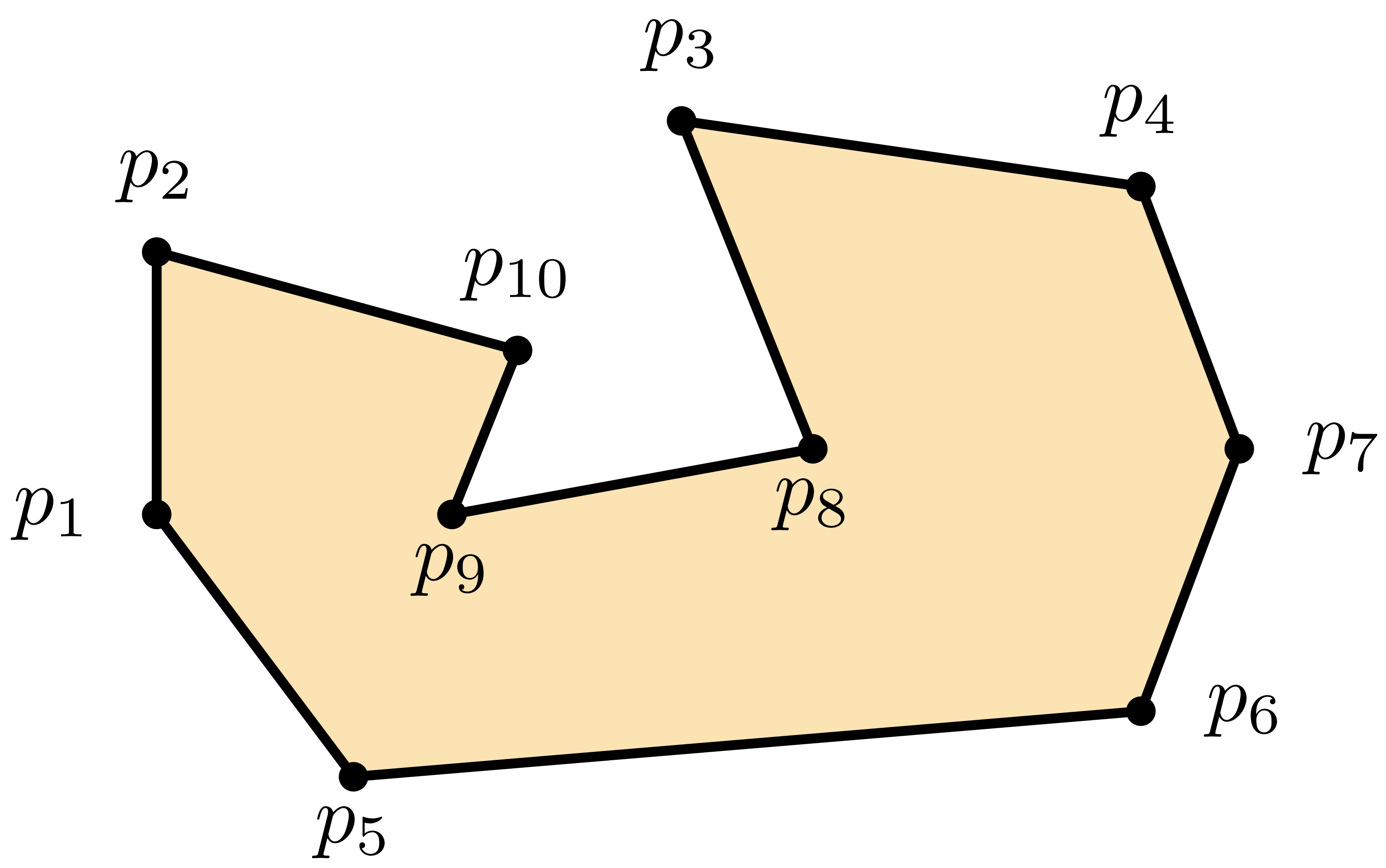
- What's a polygon P using these points?



Point sets & Polygons

What's the difference?

- Consider a finite point set $\mathscr{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$.
- An ordered sequence $P := (p_{\pi(1)}, \ \ldots \ , p_{\pi(n)}) \in \mathbb{R}^{2\times n}$ describes a polygon.



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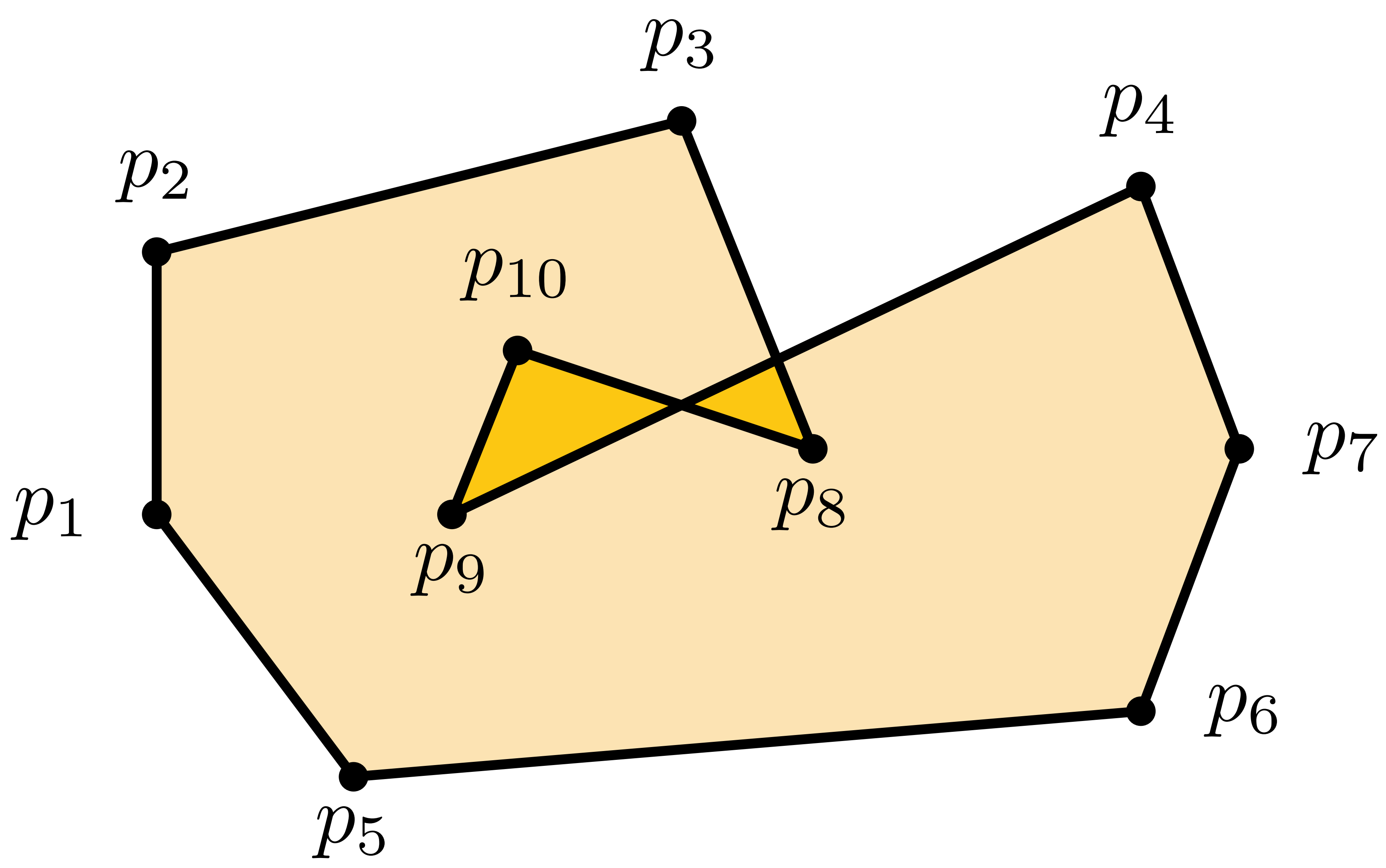


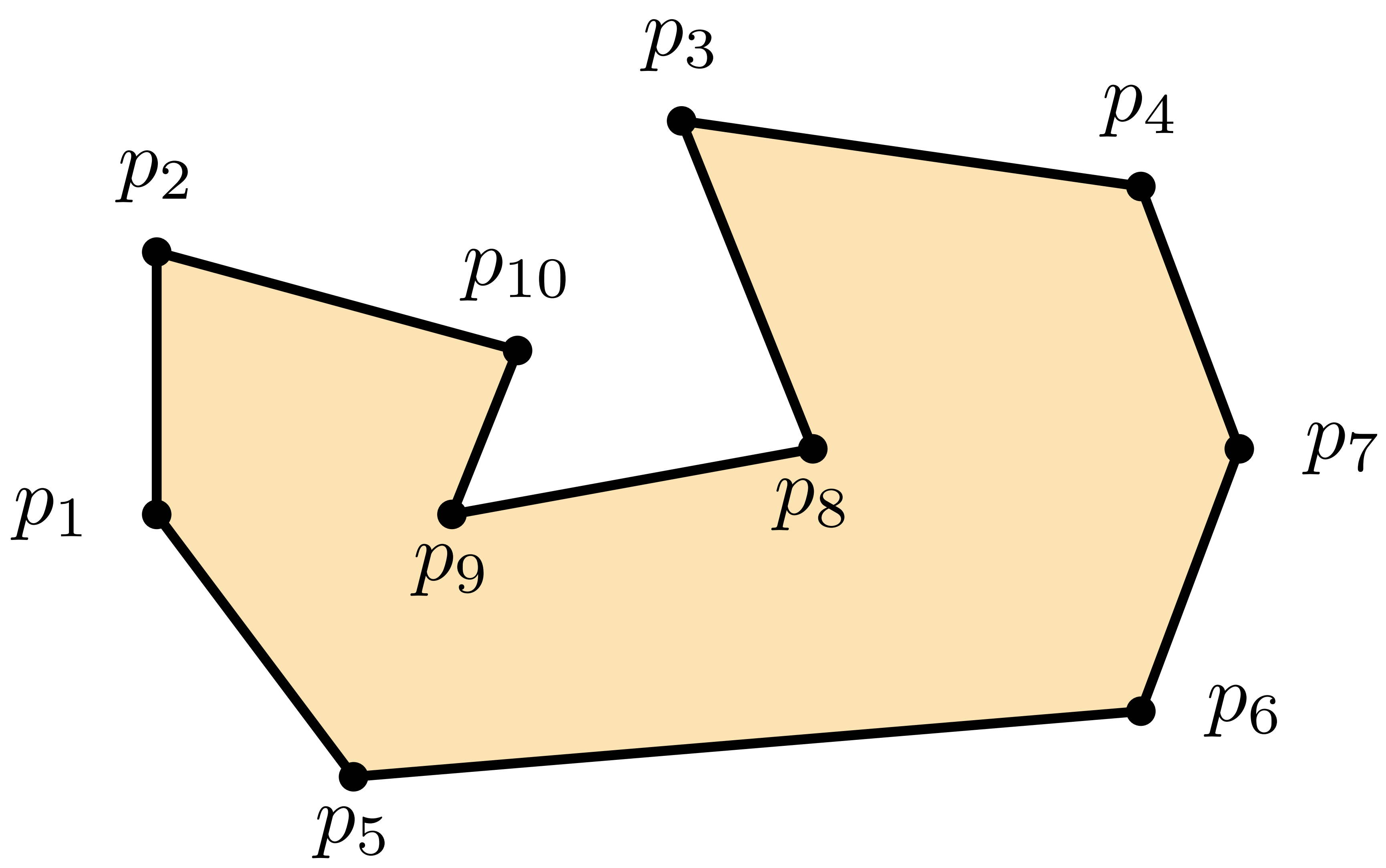
Point sets & Polygons

What's the difference?

- Consider a finite point set $\mathscr{P} := \{p_1, p_2, \ldots, p_n\} \subset \mathbb{R}^2$.
- A *simple* polygon P does not self-intersect.

Note: We will encounter further types of polygons later on :)

Typically, we describe polygons in CCW order:
$(p_1, p_5, p_6, p_7, p_4, p_3, p_8, p_9, p_10, p_2)$



Homework Sheet #1

Peter Kramer | November 6th, 2025

Computational Geometry – Sheet 1 **Winter 2025/2026**

Prof. Dr. Sándor P. Fekete **Due** 20.11.2025

Peter Kramer **Discussion** 27.11.2025

Please submit your handwritten answers in groups of two or three, *using the box in front of* *IZ338* *before the exercise timeslot on the due date above. Make sure to include your* full names, matriculation numbers, *and the* programmes *that you are enrolled in.*

In accordance with the **guidelines** *of the TU Braunschweig, using AI tools such as LLMs to solve any part of the exercises is* ***not permitted***.

Exercise 1. **(5 points)**

Refer to *Definitions 2.2 and 2.3* from the lecture. Prove or disprove: The intersection of two convex hulls according to *Definition 2.3* is itself convex for all finite sets $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}^2$, i.e.,

$$\operatorname{conv}(\operatorname{conv}(\mathcal{P}) \cap \operatorname{conv}(\mathcal{Q})) = \operatorname{conv}(\mathcal{P}) \cap \operatorname{conv}(\mathcal{Q}).$$

(Hint: Review the properties of convex hulls and combinations carefully.)

Definition E1 (General position). When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edges cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

Exercise 2 (Point in Convex Polygon Problem). **(15 points)**

Using only the known predicates, design an $\mathcal{O}(\log n)$ algorithm for the following problem. Given a convex polygon $P = (p_1, p_2, \dots, p_n) \in (\mathbb{R} \times \mathbb{R})^n$ and a point $q \in \mathbb{R}^2$, does P contain q? You may assume that the point set $\{p_1, p_2, \dots, p_n, q\}$ obeys general position.

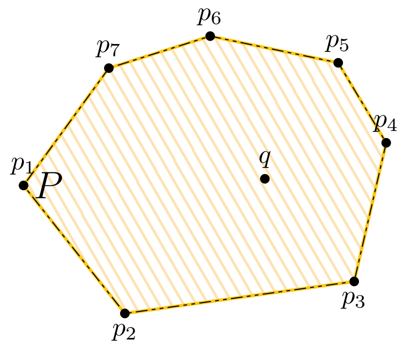


Figure 1: Example: the point q is located inside the convex polygon P.

Exercise 3 (Farthest Point Pairs). **(5+10 points)**

Given a set $\mathcal{P}$ of n points in the Euclidean plane, two points $p, q \in \mathcal{P}$ are a *farthest pair* in $\mathcal{P}$ if

$$\forall\, u, v \in \mathcal{P}: \quad |p - q| \geq |u - v|.$$

The Euclidean distance between p and q is then also called the *diameter* of $\mathcal{P}$.

a) Prove that all farthest pairs in $\mathcal{P}$ are vertices of the convex hull $\operatorname{conv}(\mathcal{P})$.

b) Design an $\mathcal{O}(n)$ algorithm that approximates the diameter of $\mathcal{P}$ up to a constant factor. Briefly explain approximation factor and runtime!

1/1

Two weeks!

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These sheets are about logical deduction, so LLMs will most likely just generate plausible-sounding garbage.

Please save us the headache :)

Translation services such as DeepL are allowed, of course. If you so prefer, you may write your solutions in german.

Prof. Dr. Sándor P. Fekete
Peter Kramer

Due 20.11.2025
Discussion 27.11.2025

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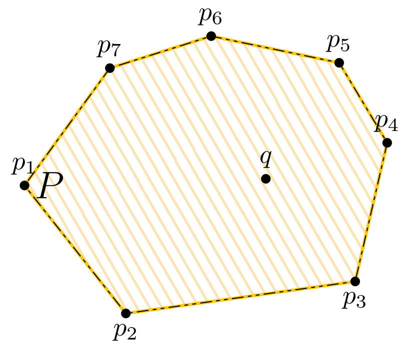

Figure 1: Example: the point q is located inside the convex polygon P.

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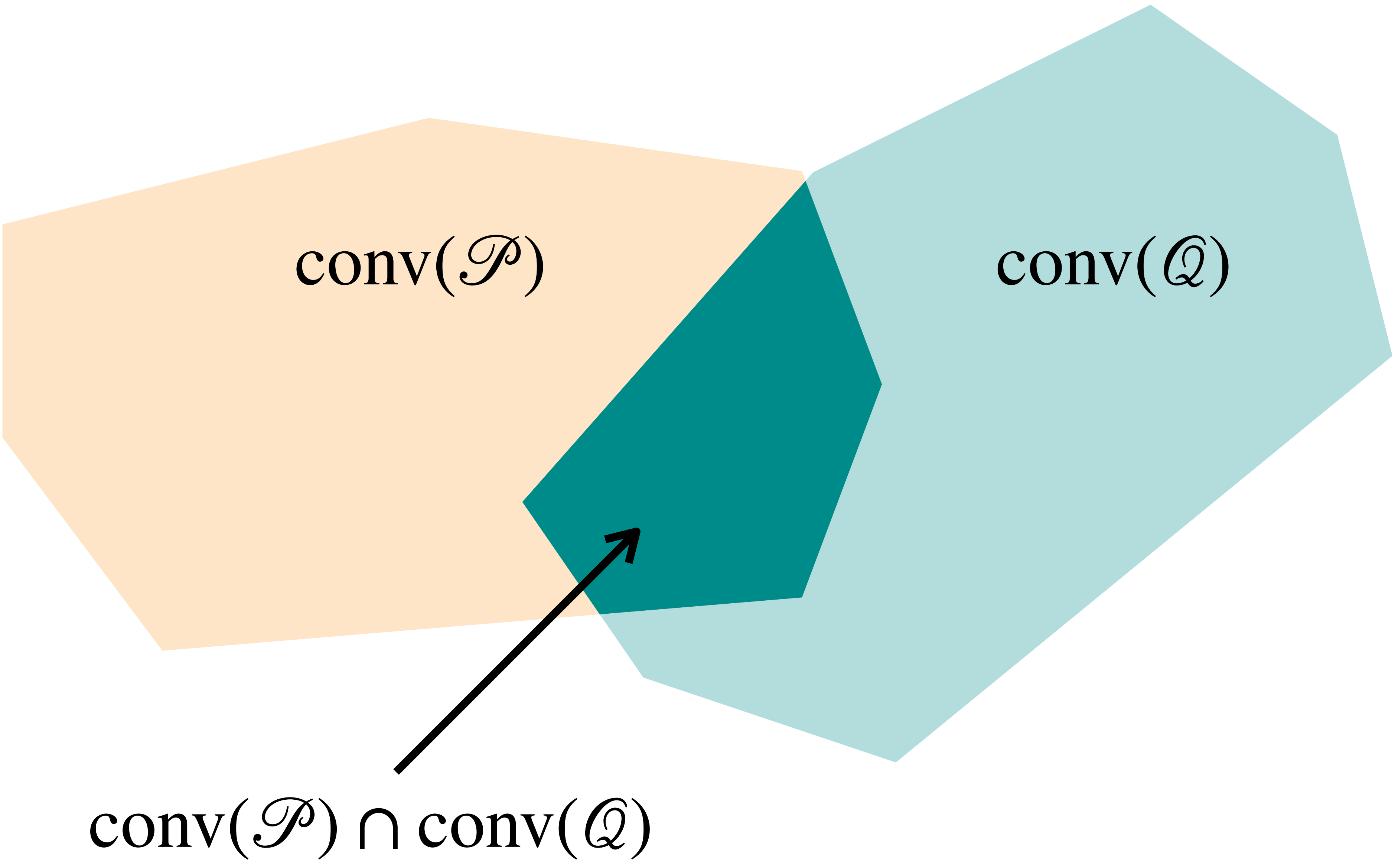
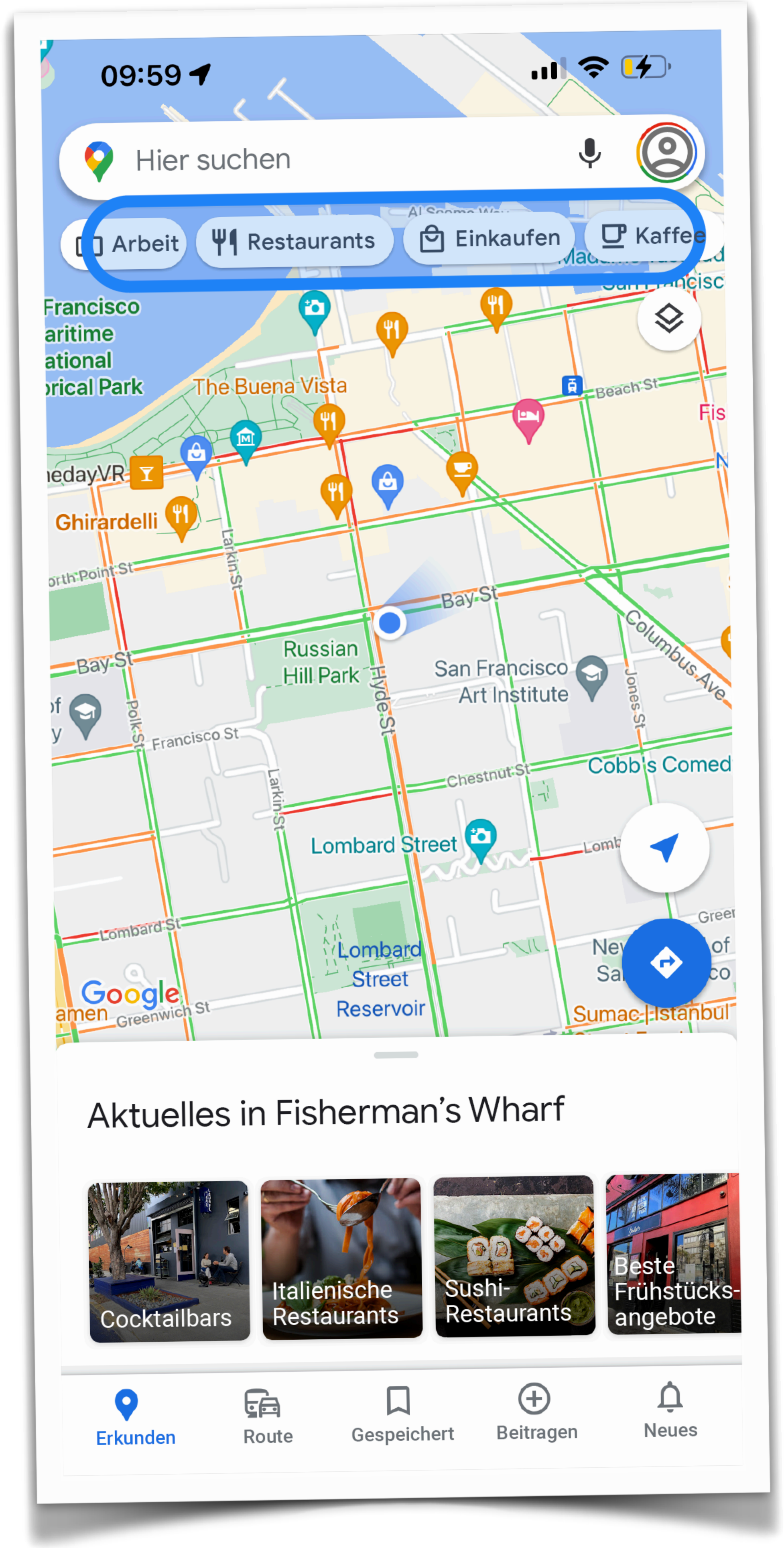


Point Location Problems

“Where am I?”

- Given geometric information such as a map in the plane, how can we decide **where** we are?



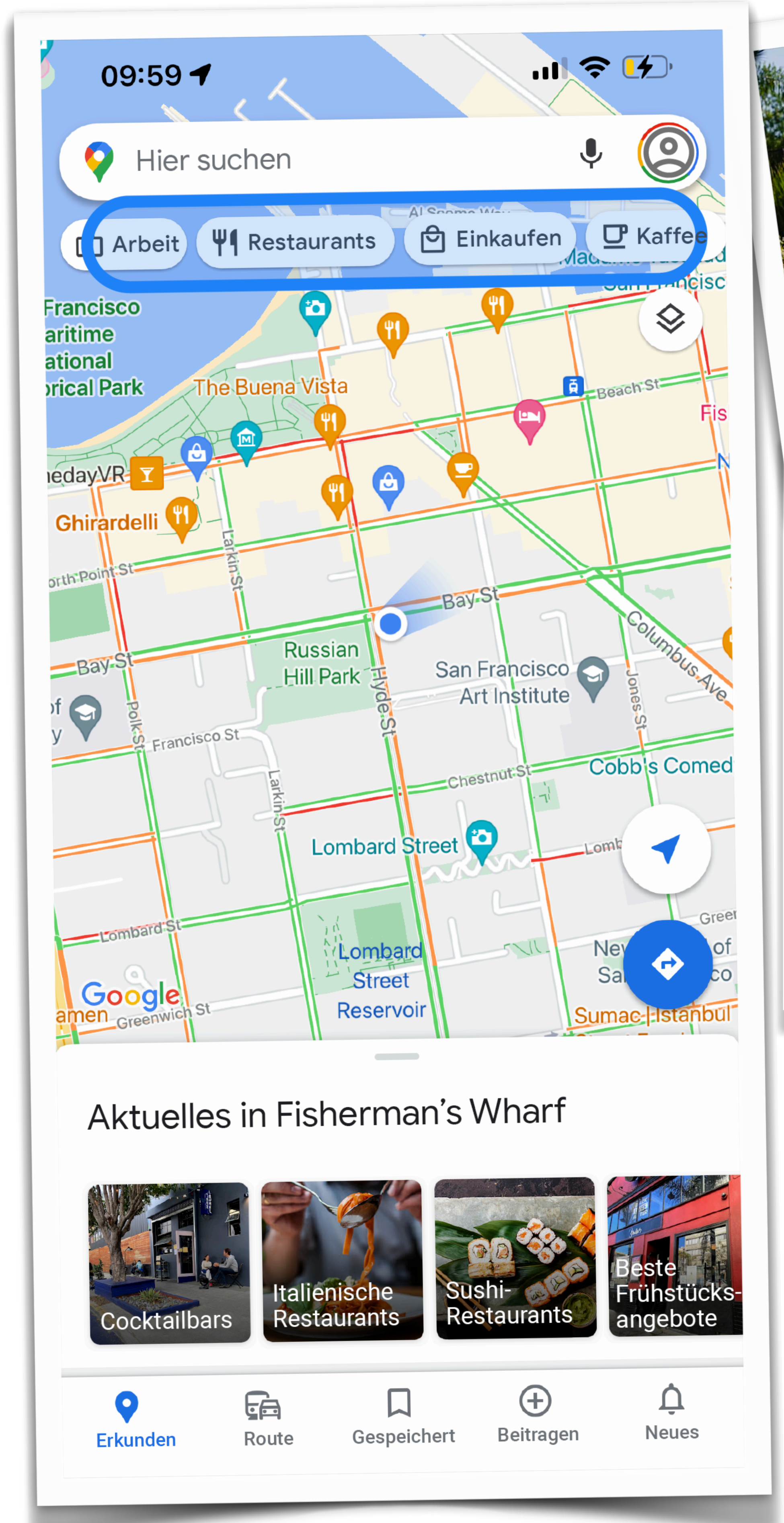
Point Location Problems

"Where am I?"

- Given geometric information such as a map in the plane, how can we decide **where** we are?

- *"In which country am I right now?"*
  "Can I leave this rental bike here?"
  "Do these virtual objects collide?"

- ***"Is point p inside of region P?"***



Applications: Geofencing, Navigation, Simulation Software, Outlier Detection, …

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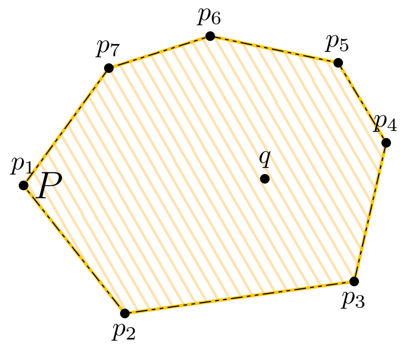

Figure 1: Example: the point q is located inside the convex polygon P.

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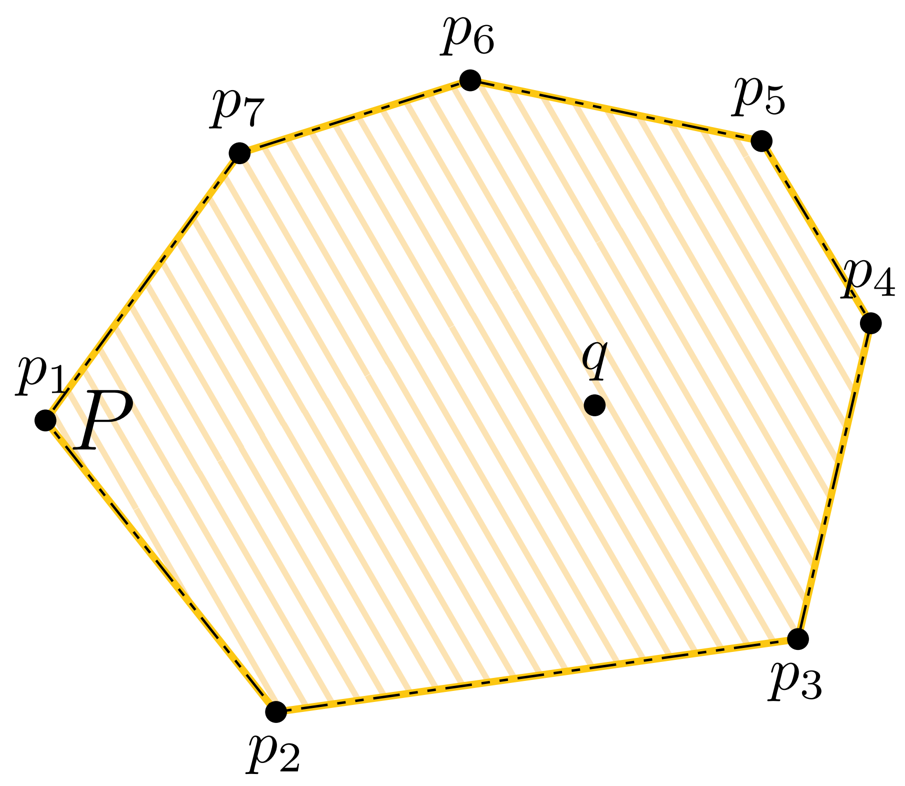

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Please submit your handwritten answers in groups of two or three, *using the box in front of* *IZ338* before *the exercise timeslot on the due date above. Make sure to include your* full names, matriculation numbers, *and the* programmes *that you are enrolled in.*

In accordance with the ***guidelines*** *of the TU Braunschweig, using AI tools such as LLMs to solve any part of the exercises is* ***not permitted.***

Exercise 1. **(5 points)**

Refer to *Definitions 2.2 and 2.3* from the lecture. Prove or disprove: The intersection of two convex hulls according to *Definition 2.3* is itself convex for all finite sets $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}^2$, i.e.,

$$\operatorname{conv}(\operatorname{conv}(\mathcal{P}) \cap \operatorname{conv}(\mathcal{Q})) = \operatorname{conv}(\mathcal{P}) \cap \operatorname{conv}(\mathcal{Q}).$$

(Hint: Review the properties of convex hulls and combinations carefully.)

Definition E1 (General position). When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edges cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

Exercise 2 (Point in Convex Polygon Problem). **(15 points)**

Using only the known predicates, design an $\mathcal{O}(\log n)$ algorithm for the following problem. Given a convex polygon $P = (p_1, p_2, \ldots, p_n) \in (\mathbb{R} \times \mathbb{R})^n$ and a point $q \in \mathbb{R}^2$, does P contain q? You may assume that the point set $\{p_1, p_2, \ldots, p_n, q\}$ obeys general position.

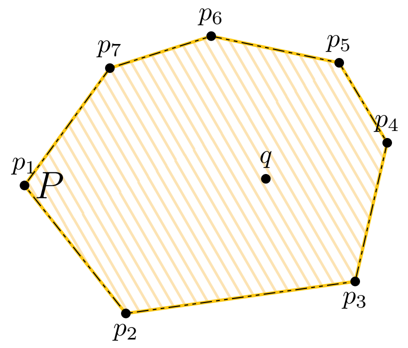

Figure 1: Example: the point q is located inside the convex polygon P.

Exercise 3 (Farthest Point Pairs). **(5+10 points)**

Given a set $\mathcal{P}$ of n points in the Euclidean plane, two points $p, q \in \mathcal{P}$ are a *farthest pair* in $\mathcal{P}$ if

$$\forall\, u, v \in \mathcal{P}: \quad |p - q| \;\geq\; |u - v|.$$

The Euclidean distance between p and q is then also called the *diameter* of $\mathcal{P}$.

a) Prove that all farthest pairs in $\mathcal{P}$ are vertices of the convex hull $\operatorname{conv}(\mathcal{P})$.

b) Design an $\mathcal{O}(n)$ algorithm that approximates the diameter of $\mathcal{P}$ up to a constant factor. Briefly explain approximation factor and runtime!

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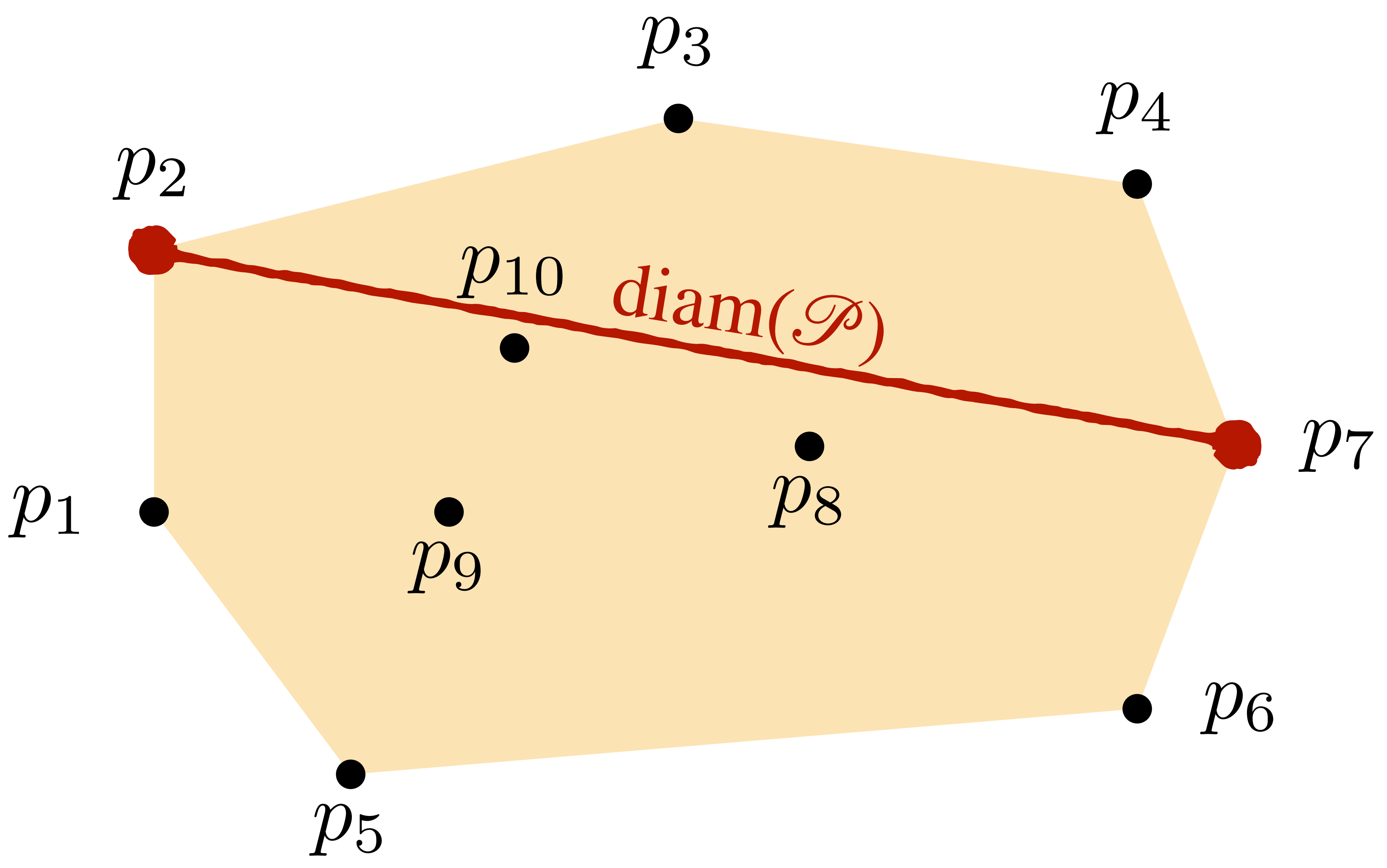


For a c-approximation, argue that your algorithm's output is

- at most $c \cdot \operatorname{diam}(\mathscr{P})$, and
- at least $\frac{1}{c} \cdot \operatorname{diam}(\mathscr{P})$.

Homework Sheet #0

Peter Kramer | November 6th, 2025

Computational Geometry – Sheet 0 **Winter 2025/2026**

Prof. Dr. Sándor P. Fekete
Peter Kramer

These are tasks for the first small tutorial on 13.11.2025, which will be a live exercise. Although they do not need to be handed it, you are encouraged to study the tasks ahead of time!

Optional Exercise 1 (Big-O Notation).

a) Consider the functions $f_1, f_2, g_1, g_2 : \mathbb{N} \to \mathbb{R}^+$. Prove that the following is true:

(i) $f_1 \in \mathcal{O}(g_1),\ f_2 \in \mathcal{O}(g_2) \Rightarrow f_1 + f_2 \in \mathcal{O}(g_1 + g_2)$.

(ii) $f_1 \in \mathcal{O}(g_1),\ f_2 \in \mathcal{O}(g_2) \Rightarrow f_1 + f_2 \in \mathcal{O}(\max\{g_1 + g_2\})$.

b) Order the following complexity classes by inclusion ($\subseteq$). Mark equality where appropriate.

$$\mathcal{O}(15),\quad \mathcal{O}(n\log n),\quad \mathcal{O}\binom{n}{5},\quad \mathcal{O}(\log n),\quad \mathcal{O}(2^n),\quad \mathcal{O}(n^2),\quad \mathcal{O}(n-\log n),\quad \mathcal{O}(3^n)$$

Optional Exercise 2 (Convex Combinations).

Describe the set of all convex combinations of the points $(0,1)^T$ and $(1,0)^T$ in your own words. In other words, find

$$\left\{\begin{pmatrix}x\\y\end{pmatrix} \mid \begin{pmatrix}x\\y\end{pmatrix} = \lambda\begin{pmatrix}0\\1\end{pmatrix} + (1-\lambda)\begin{pmatrix}1\\0\end{pmatrix}, \text{where } 0 \le \lambda \le 1\right\}.$$

Find another two points that determine the same set or determine that this is impossible.

Optional Exercise 3 (Convex Hull of Line Segments).

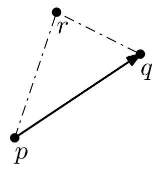
Let S be a set of n line segments in the plane. Prove that the convex hull of S is exactly the same as the convex hull of the $2n$ endpoints of the segments.

Optional Exercise 4 (Signed Area of a Triangle).

For the convex hull algorithms we have seen, we test whether a point r lies left or right of the directed line $\overline{pq}$ through two points p and q. Let $p = (p_x, p_y)^T$, $q = (q_x, q_y)^T$, $r = (r_x, r_y)^T$.

Show that the sign of the determinant

$$D = \begin{vmatrix}1 & p_x & p_y\\ 1 & q_x & q_y\\ 1 & r_x & r_y\end{vmatrix}$$



determines whether r lies to the left or right of the line $\overline{pq}$.

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Computational Geometry – Sheet 0 **Winter 2025/2026**

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Peter Kramer

These are tasks for the first small tutorial on 13.11.2025, which will be a live exercise. Although they do not need to be handed it, you are encouraged to study the tasks ahead of time!

Exercises for the first small tutorial:

Next Thursday, this room.

(Not to be handed in)

Big O Notation

Catching up, if necessary.

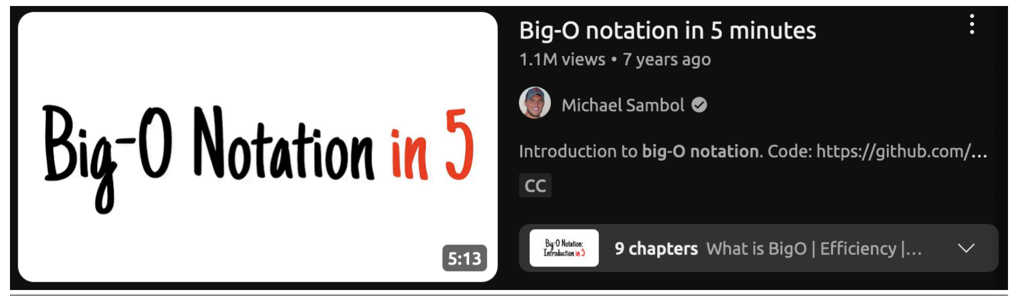
- If you speak German: There is plenty of material on the course website of “Algorithmen und Datenstrukturen”

Institut für Betriebssysteme und Rechnerverbund
Algorithmik

- If you don’t: There is plenty of other courseware (e.g., from MIT) available!

MIT OpenCourseWare

- There also exist a variety of short-form explanation videos, such as:



TL;DL

The important parts

- **Sign up for the mailing list!**
- One tutorial a week (except holidays).
- Biweekly homework: **Find groups!**
- First **sheet is out now**, due in 2 weeks.
- Live exercise next week.
- Material can be found on the website.
  - Slides, videos, and homework sheets.

November						
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

- Any questions?
  - Ask Peter kramer@ibr.cs.tu-bs.de or or Lisa glowczew@ibr.cs.tu-bs.de