Prof. Dr. Sándor P. Fekete
Peter Kramer

Due 22.01.2026
Discussion 29.01.2026

Please submit your handwritten answers in groups of two or three*, using the box in front of* `IZ338` before *15:00 on the due date. Make sure to include your* full names, matriculation numbers*, and the* programmes *that you are enrolled in.*

In accordance with the `guidelines` *of the TU Braunschweig, using AI tools such as LLMs to solve any part of the exercises is* ***not permitted****.*

A sum of 75 *points (roughly* 50%*) across all sheets suffice to pass the coursework.*

You may assume general position for all tasks on this sheet, i.e., that no three vertices of the polygons are collinear.

Exercise 1 (Convex vertices). **(7 points)**

Let P be a simple polygon of $n \geq 3$ vertices. Prove that P has at least three convex vertices.

Exercise 2 (Reflex vertices). **(5 points)**

Let P be a simple polygon with k reflex vertices. What is the *minimum* number of subpolygons into which P must be divided by cutting along diagonals such that each subpolygon is convex?

Exercise 3 (Triangulations of polygons with holes). **(3+5 points)**

Let P be a simple polygon with h holes, and let n be the total number of its vertices (including vertices of the holes). Find a formula for the number of triangles in a triangulation of P and prove its correctness.

Exercise 4 (Dual graphs of triangulations). **(5 points)**

The *weak dual graph* of a triangulation contains one vertex per triangle, and an edge between a pair of vertices exactly if the corresponding triangles share an edge.

Let P be a convex polygon. Prove or disprove: There exists a triangulation of P such that the weak dual graph is a path, i.e., every vertex has degree at most two.

Exercise 5 (Algorithmic paradigms). **(2+2+2 points)**

We have seen the algorithmic paradigms *divide and conquer*, *randomized incremental construction*, and *sweep line*. Give examples and briefly explain each paradigm in your own words.

Exercise 6 (Bonus: Colorful triangulations). **(8+4 points)**

Let P be a convex polygon with n vertices $p_1, \ldots, p_n$. We assign each vertex p_i one of two colors such that $c(p_i) \in \{\texttt{red}, \texttt{blue}\}$. A triangulation of P is then *colorful* exactly if every triangle contains at least one vertex of each color.

a) Prove that there exists a colorful triangulation for any coloring of P that uses both colors.

b) Show that this does not extend to general (simple) polygons.