Computational Geometry – Sheet 2 | **Winter 2025/2026**

Prof. Dr. Sándor P. Fekete | **Due** 04.12.2024

Peter Kramer | **Discussion** 11.12.2024

Please submit your handwritten answers in groups of two or three*, using the box in front of IZ338* before *15:00 on the due date. Make sure to include your* full names, matriculation numbers*, and the* programmes *that you are enrolled in.*

In accordance with the ***guidelines*** *of the TU Braunschweig, using AI tools such as LLMs to solve any part of the exercises is* ***not permitted****.*

Exercise 1 (Geometric Predicates). **(5 points)**

Using only the leftTurn and rightTurn predicates from Lecture 1, design a geometric predicate for the Euclidean plane that decides whether a line segment $\overline{pq}$ intersects a triangle $\triangle(u, v, w)$:

$$\operatorname{conv}(p, q) \cap \operatorname{conv}(u, v, w) = \varnothing\ ?$$

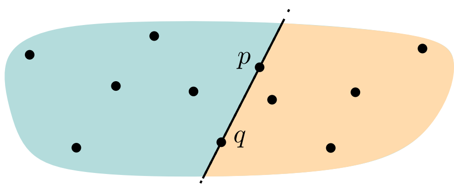
You may assume that (u, v, w) are in counterclockwise order and that no three points are collinear. Please explain your solution and briefly argue its correctness.

Exercise 2 (Partitioning Points). **(15 points)**

Consider a set $\mathcal{P}$ in the Euclidean plane $\mathbb{R}^2$ in general position according to *Definition E1*.

a) Prove that there exist points $p, q \in \mathcal{P}$ that divide $\mathcal{P}$ evenly based on left-/rightTurn:

$$|\{\, r \in \mathcal{P} \quad | \quad \operatorname{leftTurn}(p, q, r) = \texttt{true} \,\}| = {}^{|\mathcal{P}|}/_2 \pm 1.$$



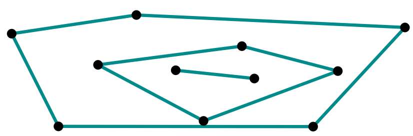
b) Design an algorithm that finds p and q in $\mathcal{O}(n)$ time for $n = |\mathcal{P}|$.

(Hint: Start with **b)**, a good correctness proof can also give you a constructive proof of existence.)

Exercise 3 (Convex layers). **(10 points)**

The *convex layers* of a finite point set $\mathcal{P}$ in the plane correspond to a decomposition of $\mathcal{P}$ into nested, convex polygons (*layers*). The outermost layer L_0 consists exactly of the extremal points defining $\operatorname{conv}(P)$. The next layer is recursively defined as points defining $\operatorname{conv}(P \setminus L_0)$, meaning

$$L_i = \mathcal{P} \cap \delta \operatorname{conv}\Big(\mathcal{P} \setminus \bigcup_{j \in [0,i]} L_j\Big).$$



Design an algorithm which computes the convex layers of n points in the Euclidean plane, in $\mathcal{O}(n^2)$ time. Briefly argue its runtime and correctness.