



Please submit your handwritten answers in groups of two or three, using the box in front of IZ338 before the exercise timeslot on the due date above. Make sure to include your full names, matriculation numbers, and the programmes that you are enrolled in.

In accordance with the *guidelines* of the TU Braunschweig, using AI tools such as LLMs to solve any part of the exercises is **not permitted**.

**Exercise 1.****(5 points)**

Refer to *Definitions 2.2 and 2.3* from the lecture. Prove or disprove: The intersection of two convex hulls according to *Definition 2.3* is itself convex for all finite sets  $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}^2$ , i.e.,

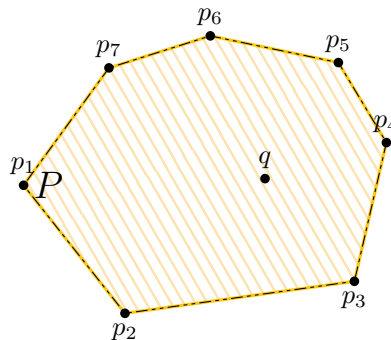
$$\text{conv}(\text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q})) = \text{conv}(\mathcal{P}) \cap \text{conv}(\mathcal{Q}).$$

(Hint: Review the properties of convex hulls and combinations carefully.)

**Definition E1 (General position).** When investigating a problem, we often assume that the input data obeys *general position*, which excludes specific edge cases and simplifies analysis. For this sheet, a point set obeys general position if and only if no three points in it are collinear.

**Exercise 2 (Point in Convex Polygon Problem).****(15 points)**

Using only the known predicates, design an  $\mathcal{O}(\log n)$  algorithm for the following problem. Given a convex polygon  $P = (p_1, p_2, \dots, p_n) \in (\mathbb{R} \times \mathbb{R})^n$  and a point  $q \in \mathbb{R}^2$ , does  $P$  contain  $q$ ? You may assume that the point set  $\{p_1, p_2, \dots, p_n, q\}$  obeys general position.



**Figure 1:** Example: the point  $q$  is located inside the convex polygon  $P$ .

**Exercise 3 (Farthest Point Pairs).****(5+10 points)**

Given a set  $\mathcal{P}$  of  $n$  points in the Euclidean plane, two points  $p, q \in \mathcal{P}$  are a *farthest pair* in  $\mathcal{P}$  if

$$\forall u, v \in \mathcal{P} : \|p - q\|_2 \geq \|u - v\|_2.$$

The Euclidean distance between  $p$  and  $q$  is then also called the *diameter* of  $\mathcal{P}$ .

- Prove that all farthest pairs in  $\mathcal{P}$  are vertices of the convex hull  $\text{conv}(\mathcal{P})$ .
- Design an  $\mathcal{O}(n)$  algorithm that approximates the diameter of  $\mathcal{P}$  up to a constant factor. Briefly explain approximation factor and runtime!