Computational Geometry – Sheet 0 **Winter 2025/2026**

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These are tasks for the first small tutorial on 13.11.2025, which will be a live exercise. Although they do not need to be handed it, you are encouraged to study the tasks ahead of time!

Optional Exercise 1 (Big-O Notation).

a) Consider the functions f_1, f_2, g_1, $g_2 : \mathbb{N} \to \mathbb{R}^+$. Prove that the following is true:

(i) $f_1 \in \mathcal{O}(g_1),\ f_2 \in \mathcal{O}(g_2) \Rightarrow f_1 + f_2 \in \mathcal{O}(g_1 + g_2)$.

(ii) $f_1 \in \mathcal{O}(g_1),\ f_2 \in \mathcal{O}(g_2) \Rightarrow f_1 + f_2 \in \mathcal{O}(\max\{g_1 + g_2\})$.

b) Order the following complexity classes by inclusion ($\subseteq$). Mark equality where appropriate.

$$\mathcal{O}(15), \quad \mathcal{O}(n \log n), \quad \mathcal{O}\left(\frac{n}{5}\right), \quad \mathcal{O}(\log n), \quad \mathcal{O}(2^n), \quad \mathcal{O}(n^2), \quad \mathcal{O}(n - \log n), \quad \mathcal{O}(3^n)$$

Optional Exercise 2 (Convex Combinations).

Describe the set of all convex combinations of the points $(0, 1)^T$ and $(1, 0)^T$ in your own words. In other words, find

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} + (1 - \lambda) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{where } 0 \leq \lambda \leq 1 \right\}.$$

Find another two points that determine the same set or determine that this is impossible.

Optional Exercise 3 (Convex Hull of Line Segments).

Let S be a set of n line segments in the plane. Prove that the convex hull of S is exactly the same as the convex hull of the $2n$ endpoints of the segments.

Optional Exercise 4 (Signed Area of a Triangle).

For the convex hull algorithms we have seen, we test whether a point r lies left or right of the directed line $\overline{pq}$ through two points p and q. Let $p = (p_x, p_y)^T$, $q = (q_x, q_y)^T$, $r = (r_x, r_y)^T$.

Show that the sign of the determinant

$$D = \begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$$

determines whether r lies to the left or right of the line $\overline{pq}$.