Prof. Dr. Sándor P. Fekete
Peter Kramer
Due 12.12.2024
Discussion 19.12.2024

Please submit your handwritten answers in pairs, *using the box in front of* `IZ338` before *the exercise timeslot on the due date above. Make sure to include your* full names, matriculation numbers, *and the* programmes *that you are enrolled in. In accordance with the* `guidelines` *of the TU Braunschweig, using AI tools to solve any part of the exercises is* ***not permitted****.*

Exercise 1 (Safehouse Problem). **(5+15 points)**

You've successfully pulled off your biggest heist yet – you've stolen the golden Leibniz cookie! However, agents of `baked goods manufacturing co.` *are now on your tail, and all roads out of the city have been locked down in search for the famous symbol of delicacy: You cannot leave. You decide that your best bet is to find a safehouse in town as far as possible from all cookie outles, hide the cookie there, and lie low.*

Given an axis-aligned rectangle R in the Euclidean plane $\mathbb{R}^2$ that defines the city limits and the locations of nearby (both inside and outside the city) cookie outlets $c_1, c_2, \ldots, c_n$, you need a location inside R that maximizes the distance to the closest cookie outlet, see Fig. 1. (Hint: Start by thinking about a geometric suitable structure from the lecture to discretize the problem.)

a) Identify a *(finite!)* set of candidate locations in R to choose from. Argue why an optimal solution is contained in this set.

b) Design an $\mathcal{O}(n \log n)$ time algorithm based on your candidates and prove its correctness.

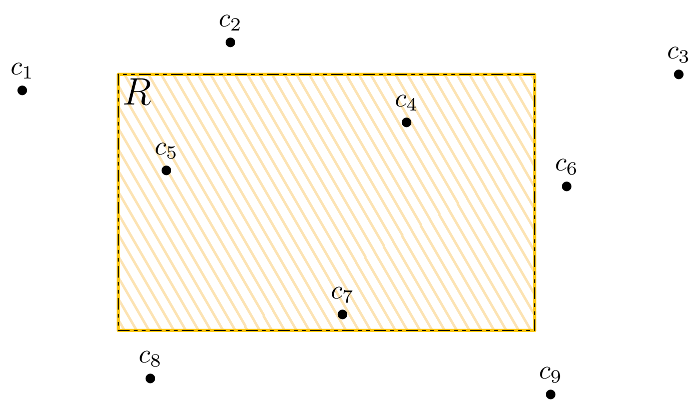

Figure 1: You require a location inside R that maximizes the distance to the nearest cookie outlet c_i.

Exercise 2 (Minimal enclosing disk). **(5+15 points)**

Let $\mathcal{P}$ be a set of n points in the Euclidean plane in *general position*, such that no four points in $\mathcal{P}$ are concyclic: No four points lie on a common circle. A *minimal enclosing disk* $md(\mathcal{P})$ is a disk with minimal radius that contains $\mathcal{P}$. Let $c \in \mathbb{R}^2$ and $r \in \mathbb{R}$ be its center and radius.

Using concepts from the lecture and the tutorial, design an algorithm that determines $md(\mathcal{P})$ in less than $\mathcal{O}(n^2)$ time, or better. (Hint: Review Tutorial 4)