Mathematical Methods of Algorithms **Winter 2023/24**

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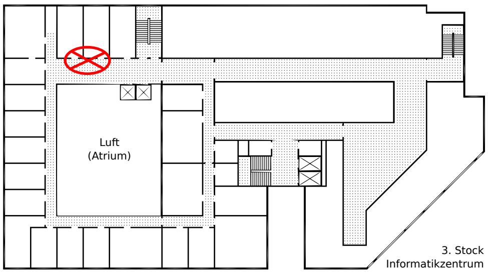
Due: 24.01.2024
Discussion: 24.01.2024

Homework 4

Solutions are to be left in the dedicated cupboard (see the pic) until 15:00 on the due date. Please put your name on all pages.



Exercise 1 (Dictionaries and Matrix Notation): 1 ✓
Consider the following linear programming problem:

$$\begin{array}{lrrrl} \max & -6x_1 & +40x_2 & -10x_3 & \\ & -2x_1 & +10x_2 & -2x_3 & \leq 10 \\ & +1x_1 & +9x_2 & +5x_3 & \leq 15 \end{array}$$

$$x_1, x_2, x_3 \geq 0$$

Suppose that, in solving this problem, you have arrived at the following dictionary:

$$\begin{array}{rrrrr} \zeta = & 40 + & 2x_1 - & 4x_4 - & 18x_3 \\ \hline x_2 = & 1 + & \frac{1}{5}x_1 - & \frac{1}{10}x_4 - & \frac{1}{5}x_3 \\ x_5 = & 6 - & \frac{14}{5}x_1 + & \frac{9}{10}x_4 - & \frac{16}{5}x_3 \end{array}$$

(a) Which variables are basic? Which are nonbasic?

(b) Write down the vector, $x^*_{\mathcal{B}}$, of current primal basic solution values.

(c) Write down the vector, $z^*_{\mathcal{N}}$, of current dual nonbasic solution values.

(d) Write down $B^{-1}N$.

(e) Is the primal solution associated with this dictionary feasible?

(f) Is it optimal?

(g) Is it degenerate?

Exercise 2 (Recovering Dictionaries): 1 ✓
Consider the following linear programming problem:

$$\begin{array}{lrrrrl} \max & +1x_1 & +2x_2 + 4x_3 & +8x_4 + 16x_5 & & \\ & +1x_1 & +2x_2 + 3x_3 & +4x_4 + 5x_5 & & \leq 2 \\ & +7x_1 & +5x_2 - 3x_3 & -2x_4 & & \leq 0 \end{array}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Consider the situation in which x_3 and x_5 are basic and all other variables are nonbasic. Write down:

(a) B

(b) N

(c) b

(d) $c_{\mathcal{B}}$

(e) $c_{\mathcal{N}}$

(f) $B^{-1}N$

(g) $x^*_{\mathcal{B}} = B^{-1}b$

(h) $\zeta^* = c^T_{\mathcal{B}} B^{-1} b$

(i) $z^*_{\mathcal{N}} = \left(B^{-1}N\right)^T c_{\mathcal{B}} - c_{\mathcal{N}}$

(j) The dictionary corresponding to this basis.

Exercise 3 (Primal Simplex): 2 ✓

Solve the following LP using the matrix form of the primal simplex algorithm.

$$\begin{array}{llllll} \max & +6x_1 & +8x_2 & +5x_3 & +9x_4 & \\ & +2x_1 & +1x_2 & +1x_3 & +3x_4 & \leq 5 \\ & +1x_1 & +3x_2 & +1x_3 & +2x_4 & \leq 3 \end{array}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Exercise 4 (Dual in Matrix Form): 2 ✓

Find the dual of the following linear program.

$$\begin{array}{lrcll} \max & & c^T x & & \\ & a \leq & Ax & \leq & b \\ & l \leq & x & \leq & u \end{array}$$