Mathematical Methods of Algorithms **Winter 2023/24**

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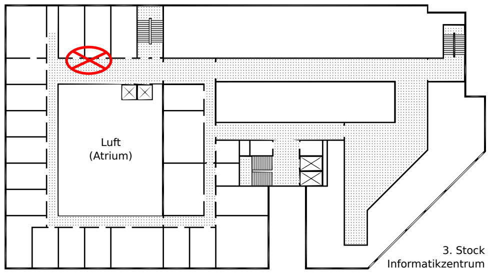
Due: 10.01.2024
Discussion: 10.01.2024

Homework 3

Solutions are to be left in the dedicated cupboard (see the pic) until 15:00 on the due date. Please put your name on all pages.



Exercise 1 (Dual-Primal 2 Phase): 1 ✓
Consider the following linear programming problem:

$$\begin{array}{lrrl} \max & +1x_1 & -1x_2 & \\ & +1x_1 & +1x_2 & \leq +2 \\ & +2x_1 & +2x_2 & \geq +2 \end{array}$$

$$x_1, x_2 \geq 0$$

Solve the problem in two phases where in the first phase you use dual simplex algorithm to get to a feasible primal dictionary and the primal simplex to continue to optimality.

Exercise 2 (Number of Dictionaries): 1 ✓
Since the simplex method operates by moving from one dictionary to another (without cycling), an upper bound on the number of (simplex) iterations is simply the number of possible dictionaries, $\binom{n+m}{m}$. Consider the case where $n = m$ and show

$$\frac{1}{2n}2^{2n} \leq \binom{2n}{n} \leq 2^{2n},$$

as a result the number of dictionaries could be exponentially large.

Exercise 3 (Klee-Minty property): 2 ✓
Consider the dictionary

$$\zeta = -\sum_{j=1}^{n} \epsilon_j 10^{n-j}(\frac{1}{2}\beta_j - x_j)$$

$$w_i = \sum_{j=1}^{i-1} \epsilon_i \epsilon_j 10^{i-j}(\beta_j - 2x_j) \quad + \quad (\beta_i - x_i), \qquad i = 1, 2, \dots, n$$

where the β_i 's are as in the Klee–Minty generic example, discussed in the fourth lecture, and where each ϵ_i is ± 1. Fix k and consider the pivot in which x_k enters the basis and w_k leaves the basis. Show that the resulting dictionary is of the same form as before. How are the new ϵ_i's related to the old ϵ_i's?

Exercise 4 (Klee-Minty exponential running time): 2 ✓

Use the result of the previous exercise to show that the generic Klee-Minty example, discussed in the sixth lecture, requires $2^n - 1$ iterations.