Mathematical Methods of Algorithms **Winter 2022/23**

Dr. Phillip Keldenich
Dr. Ahmad Moradi

Due: 18.01.2023
Discussion: 18.01.2023

Homework 4

If you are interested in presenting your solution, please come to the tutorial on the discussion date. Please note that there may be other students who also want to present their solution; we will decide randomly between students, unless there is enough time in the tutorial session and the solutions are sufficiently different. If you feel like you have a good reason why you cannot attend the tutorial at all, please contact us via e-mail and we will find a way to deal with this situation so that you can still get your Studienleistung.

Exercise 1 (Dictionary components in matrix notation): 1 ✓
Consider the following linear programming problem:

$$\begin{array}{lrrrl} \max & -6x_1 & +32x_2 & -9x_3 & \\ & -2x_1 & +10x_2 & -3x_3 & \leq -6 \\ & +1x_1 & -7x_2 & +2x_3 & \leq +4 \end{array}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Suppose that, in solving this problem, you have arrived at the following dictionary:

$$\begin{array}{rrrrr} \zeta = & -18 - & 3x_4 + & 2x_2 & \\ \hline x_3 = & +2 - & 1x_4 + & 4x_2 - & 2x_5 \\ x_1 = & 0 + & 2x_4 - & 1x_2 + & 3x_5 \end{array}$$

(a) Which variables are basic? Which are nonbasic?

(b) Write down the vector, $x^*_{\mathcal{B}}$, of current primal basic solution values.

(c) Write down the vector, $z^*_{\mathcal{N}}$, of current dual nonbasic solution values.

(d) Write down $B^{-1}N$

(e) Is the primal solution associated with this dictionary feasible?

(f) Is it optimal?

(g) Is it degenerate?

Exercise 2 (Dictionary components in matrix notation): 1 ✓
Consider the following linear programming problem:

$$\begin{array}{lrrrrl} \max & +1x_1 & +2x_2 + 4x_3 & +8x_4 + 16x_5 & & \\ & +1x_1 & +2x_2 + 3x_3 & +4x_4 + 5x_5 & & \leq 2 \\ & +7x_1 & +5x_2 - 3x_3 & -2x_4 & & \leq 0 \end{array}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Consider the situation in which x_3 and x_5 are basic and all other variables are nonbasic. Write down:

(a) B

(b) N

(c) b

(d) $c_{\mathcal{B}}$

(e) $c_{\mathcal{N}}$

(f) $B^{-1}N$

(g) $x^*_{\mathcal{B}} = B^{-1}b$

(h) $\zeta^* = c^T_{\mathcal{B}} B^{-1} b$

(i) $z^*_{\mathcal{N}} = \left(B^{-1}N\right)^T c_{\mathcal{B}} - c_{\mathcal{N}}$

(j) The dictionary corresponding to this basis.

Exercise 3 (Primal Simplex): 2 ✓
Solve the following LP with matrix form of the primal simplex algorithm.

$$\begin{array}{llll} \max & +6x_1 & +8x_2 + 5x_3 & +9x_4 \\ & +2x_1 & +1x_2 + 1x_3 & +3x_4 \leq 5 \\ & +1x_1 & +3x_2 + 1x_3 & +2x_4 \leq 3 \end{array}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Exercise 4 (Dual problem in matrix form): 2 ✓
Find the dual of the following linear program:

$$\begin{array}{lrcl} \max & & c^T x & \\ & a \leq & Ax & \leq b \\ & l \leq & x & \leq u \end{array}$$

Exercise 5 (Lagrangian Dual in matrix form): 2 ✓
Let A be a given $m \times n$ matrix, c a given n-vector, and b a given m-vector. Consider the following max–min problem:

$$\max_{x \geq 0} \min_{y \geq 0} \left(c^T x - y^T A x + b^T y\right)$$

(a) By noting that the inner optimization can be carried out explicitly, show that this problem can be reduced to a linear programming problem. Write it explicitly.

(b) What linear programming problem do you get if the min and max are interchanged?