Mathematical Methods of Algorithms **Winter 2022/23**

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Due: 21.12.2022
Discussion: 21.12.2022

Homework 3

If you are interested in presenting your solution, please come to the tutorial on the discussion date. Please note that there may be other students who also want to present their solution; we will decide randomly between students, unless there is enough time in the tutorial session and the solutions are sufficiently different. If you feel like you have a good reason why you cannot attend the tutorial at all, please contact us via e-mail and we will find a way to deal with this situation so that you can still get your Studienleistung.

Exercise 1 (Primal-Dual computations): 1 ✓
Consider the following linear programming problem:

$$\begin{array}{lllll} \max & +2x_1 & +8x_2 - 1x_3 & -2x_4 & \\ & +2x_1 & +3x_2 & +6x_4 & \leq 6 \\ & -2x_1 & +4x_2 + 3x_3 & & \leq 1.5 \\ & +3x_1 & +2x_2 - 2x_3 & -4x_4 & \leq 4 \end{array}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Suppose that, in solving this problem, you have arrived at the following dictionary:

$$\begin{array}{rrrrrr} \zeta = & 3.5 - & 0.25w_1 + & 6.25x_2 - & 0.5w_3 - & 1.5x_4 \\ \hline x_1 = & 3 - & 0.5w_1 - & 1.5x_2 & - & 3x_4 \\ w_2 = & 0 + & 1.25w_1 - & 3.25x_2 - & 1.5w_3 + & 13.5x_4 \\ x_3 = & 2.5 - & 0.75w_1 - & 1.25x_2 + & 0.5w_3 - & 6.5x_4 \end{array}$$

(a) Write down the dual problem.

(b) In the dictionary shown above, which variables are basic? Which are nonbasic?

(c) Write down the primal solution corresponding to the given dictionary. Is it feasible? Is it degenerate?

(d) Write down the corresponding dual dictionary.

(e) Write down the dual solution. Is it feasible?

(f) Do the primal/dual solutions you wrote above satisfy the complementary slackness property?

(g) Is the current primal solution optimal?

(h) For the next (primal) pivot, which variable will enter if the largest-coefficient rule is used? Which will leave? Will the pivot be degenerate?

Exercise 2 (Primal-Dual relationship): 1 ✓
The CEO of a large corporation needs the solution to the following linear programming problem:

$$\begin{array}{lllll} \max & -2x_1 & -2x_2 - 6x_3 & & \\ & +x_1 & -2x_2 - 2x_3 & \leq -15 \\ & -2x_1 & +x_2 - x_3 & \leq -30 \\ & & x_1, x_2, x_3 \geq 0 & & \end{array}$$

Because the numbers represent millions of dollars, an error would be very costly. Hence, he's independently delegated the task of computing the solution to two subordinates: Alice and Brad. Alice came back with the following values for the primal and, by the way, for the associated dual values:

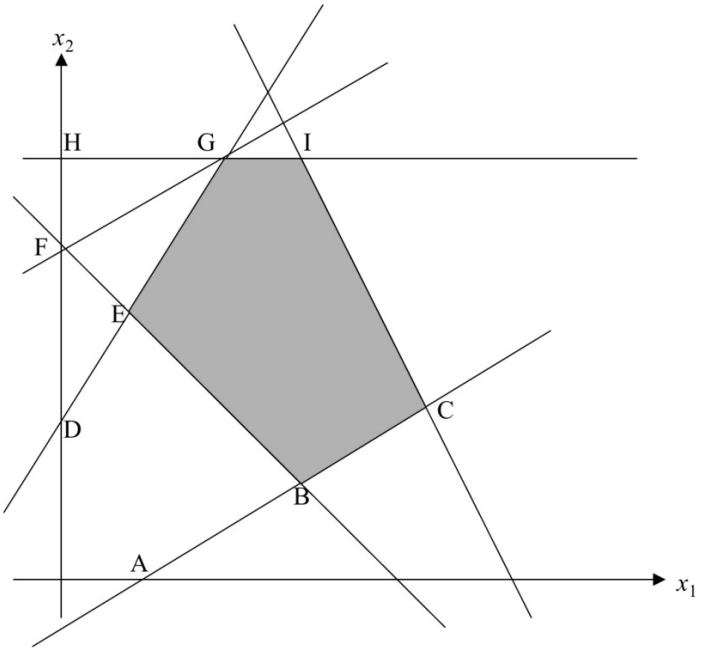
$$x_1^* = 25, x_2^* = 20, x_3^* = 0 \text{ and } y_1^* = y_2^* = 2$$

Brad provided the following values, which are different from Alice's:

$$x_1^* = 9, x_2^* = 0, x_3^* = 12 \text{ and } y_1^* = y_2^* = 2$$

Since the two solutions disagree, you're being asked to determine who, if anyone, is correct. So, who is? Explain.

Exercise 3 (Dual Simplex): 1 ✓
Consider a problem whose constraints are as shown here:



Suppose that the objective is to minimize x_1 . Clearly, the optimal solution is at vertex E. It is not possible for the dual simplex method starting at vertex H to visit vertices G then E? Explain why.

Exercise 4 (Dual-Primal two phase algorithm): 1 ✓
Solve the following linear program using the dual–primal two-phase algorithm.

$$\begin{array}{lrll} \max & +2x_1 & -6x_2 & \\ & -1x_1 & -1x_2 - 1x_3 & \leq -2 \\ & +2x_1 & -1x_2 + 1x_3 & \leq +1 \end{array}$$

$$x_1, x_2, x_3 \geq 0$$