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Due: 07.12.2022
Discussion: 07.12.2022

Homework 2

If you are interested in presenting your solution, please come to the tutorial on the discussion date. Please note that there may be other students who also want to present their solution; we will decide randomly between students, unless there is enough time in the tutorial session and the solutions are sufficiently different. If you feel like you have a good reason why you cannot attend the tutorial at all, please contact us via e-mail and we will find a way to deal with this situation so that you can still get your Studienleistung.

Exercise 1 (Performance comparison): 1 ✓
Compare the performance of the largest-coefficient and the smallest-index pivoting rules on the following linear program:

$$\begin{array}{lll} \max & 2x_1 & +x_2 \\ & 3x_1 & +x_2 \leq \quad 3 \\ & x_1, x_2 \geq 0 & \end{array}$$

Exercise 2 (Circulation Problem): 1 ✓
Consider the linear programming problems whose right-hand sides are identically zero:.

$$\begin{array}{llll} \max & \sum_{j=1}^{n} c_j x_j & & \\ & \sum_{j=1}^{n} a_{ij} x_j & \leq 0, & i = 1, 2, \ldots, m \\ & x_j & \geq 0, & j = 1, 2, \ldots, m \end{array}$$

Show that either $x_j = 0$ for all j is optimal or else the problem is unbounded.

Exercise 3 (Klee-Minty property): 2 ✓
Consider the dictionary

$$\zeta = -\sum_{j=1}^{n} \epsilon_j 10^{n-j} (\frac{1}{2}\beta_j - x_j)$$

$$w_i = \sum_{j=1}^{i-1} \epsilon_i \epsilon_j 10^{i-j} (b_j - 2x_j) \quad + \quad (\beta_i - x_i), \qquad i = 1, 2, \ldots, n$$

where the β_i 's are as in the Klee–Minty generic example, discussed in the fourth lecture, and where each ϵ_i is ± 1. Fix k and consider the pivot in which x_k enters the basis and w_k leaves the basis. Show that the resulting dictionary is of the same form as before. How are the new ϵ_i's related to the old ϵ_i's?

Exercise 4 (Klee-Minty exponential running time): 2 ✓
Use the result of the previous exercise to show that the generic Klee–Minty example, discussed in the fourth lecture, requires $2^n - 1$ iterations.