Linear Programming

[V. ch4]: Efficiency of the Simplex Method

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Performance Measures

Worst-Case Analysis of the Simplex Method

Empirical Average Performance

We saw that the simplex method (equipped with anti-cycling rules) will solve any linear programming problem for which an optimal solution exists. The question, now, is *how fast* it will solve such a problem.

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  looks at all problems of *a given "size"* and asks how much effort is needed to solve the *hardest*.
- average case
  looks at the average amount of effort needed to solve all.

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  looks at the average amount of effort needed to solve all.

⇝ Worst-case analyses are generally easier.
It needs to provide an upper bound on how much effort required + a specific example attaing this bound.

⇝ On an average case analysis, one needs a stochastic model of random problems and evaluate the amount of effort required to solve every problem in the sample space.

worst-case is more tractable, but less relevant while dealing with real problems

Before looking at the worst case, we must discuss:

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- $m \times n$
- number of non-zero elements
- the number of bits needed to store all data

Even with the last one, "size" of data might still be ambiguous. (why?)

we shall simply focus on m and n to characterize the size of a problem.

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Fortunately, a reasonable substitute:

Algorithms are generally iterative processes. So

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time to solve a problem $=$ *number of iterations* $\times$ *time required to do each iteration*

First factor is platform-independent and so a *resasonable suragate* for the acctual time (not when different algorthm are compared!)

We simply count the number of iterations (pivots).

Performance Measures

Worst-Case Analysis of the Simplex Method

Empirical Average Performance

Since the simplex method operates by moving from one dictionary to another (without cycling), an upper bound on the number of iterations is simply the number of possible dictionaries:

$$\binom{n+m}{m}$$

For a fixed value of the sum $n + m$, this expression is maximized when $m = n$.

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How big is it? Not hard to show:

$$\frac{1}{2n} 2^{2n} \leq \binom{2n}{n} \leq 2^{2n}$$

For $n = 25$:

$$2^{50} = 1.1259 \times 10^{15}$$

Our best chance for finding a bad example is to look at the case where $m = n$.

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lets look at the generic example:

Given constants $\quad 1 = \beta_1 \ll \beta_2 \ll \cdots \ll \beta_n$

$$\begin{aligned} \max_x \quad & \sum_{j=1}^{n} 2^{n-j} x_j - \frac{1}{2} \sum_{j=1}^{n} 2^{n-j} \beta_j \\ \text{s.t.} \quad & 2 \sum_{j=1}^{i-1} 2^{i-j} x_j + x_i \le \sum_{j=1}^{i-1} 2^{i-j} \beta_j + \beta_i \quad i = 1, 2, \ldots, n \\ & x_j \ge 0 \qquad i = 1, 2, \ldots, n \end{aligned}$$

For $n = 3$

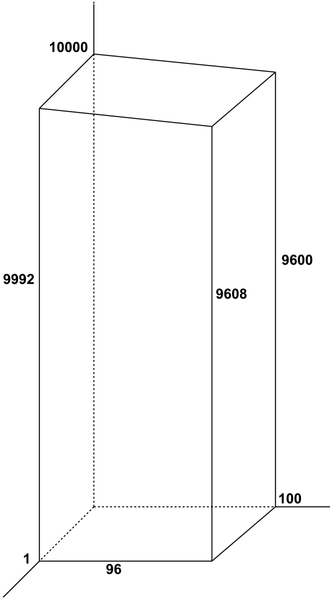
$$
\begin{array}{rlllll}
\max\limits_{x} & 4x_1 + & 2x_2 + & 1x_3 - & 2\beta_1 - \beta_2 - \frac{1}{2}\beta_3 \\
\text{s.t.} & 1x_1 + & + & & \leq \beta_1 \\
& 4x_1 + & 1x_2 + & & \leq 2\beta_1 + \beta_2 \\
& 8x_1 + & 4x_2 + & 1x_3 & \leq 4\beta_1 + 2\beta_2 + \beta_3 \\
& x_1, & x_2, & x_3 & \geq 0
\end{array}
$$

For $n = 3$

$$\begin{aligned}
\max_x \quad & 4x_1 + 2x_2 + 1x_3 - 2\beta_1 - \beta_2 - \frac{1}{2}\beta_3 \\
\text{s.t.} \quad & 1x_1 + \quad\quad + \quad\quad \leq \beta_1 \\
& 4x_1 + 1x_2 + \quad\quad \leq 2\beta_1 + \beta_2 \\
& 8x_1 + 4x_2 + 1x_3 \leq 4\beta_1 + 2\beta_2 + \beta_3 \\
& x_1, \quad x_2, \quad x_3 \geq 0
\end{aligned}$$

Assuming $\beta_2 = 98, \beta_3 = 9800$, the feasible region looks like :

$$\begin{aligned}
& 1x_1 \leq 1 \\
& 4x_1 + 1x_2 \leq 100 \\
& 8x_1 + 4x_2 + 1x_3 \leq 10000 \\
& x_1, \quad x_2, \quad x_3 \geq 0
\end{aligned}$$


10000
9992
9608
9600
100
1
96

Constraints represent a “minor” distortion to an 3-dimensional hypercube:

$$0 \leq x1 \leq 1$$
$$0 \leq x2 \leq 100$$
$$0 \leq x3 \leq 10000$$

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This region has $2^{n=3}$ vertices and the idea is to trick the pivot rule so as to visit all of them.

Back to the generic Klee-Minty example ($n = 3$), the first dictionary is

$$\begin{array}{rcl} \zeta &=& -2\beta_1 - 1\beta_2 - 0.5\beta_3 + 4x_1 + 2x_2 + 1x_3 \\ \hline w_1 &=& +1\beta_1 - 1x_1 \\ w_2 &=& +2\beta_1 + 1\beta_2 - 4x_1 - 1x_2 \\ w_3 &=& +4\beta_1 + 2\beta_2 + 1\beta_3 - 8x_1 - 4x_2 - 1x_3 \end{array}$$

which is feasible.

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which is feasible.

Using the largest-coefficient rule: (x_1, w_1) are the entering, leaving pair.

Corresponding vertex on the Klee-Minty cube is: $(0, 0, 0)$

After the first pivot

$$
\begin{array}{rrrrrrr}
\zeta = + & 2\beta_1 - & 1\beta_2 - & 0.5\beta_3 - & 4w_1 + & \boxed{2}\, x_2 + & \boxed{1}\, x_3 \\
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x_1 = + & 1\beta_1 & & - & 1w_1 & & \\
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for the next pivot: (x_2, w_2)

After the first pivot

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for the next pivot: (x_2, w_2)

Corresponding vertex on the Klee-Minty cube is: $(\beta_1, 0, 0)$

After the second pivot

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\begin{array}{rcrrrrrr}
\zeta & = & - & 2\beta_1 + & 1\beta_2 - & 0.5\beta_3 + & 4\,w_1 - & 2w_2 + & 1\,x_3 \\
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Corresponding vertex on the Klee-Minty cube is: $(\beta_1, -2\beta_1 + 1\beta_2, 0)$

After the third pivot

$$
\begin{array}{rrrrrrr}
\zeta = + & 2\beta_1 + & 1\beta_2 - & 0.5\beta_3 - & 4x_1 - & 2w_2 + & \colorbox{pink}{1}\, x_3 \\
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w_1 = + & 1\beta_1 & & - & 1x_1 & & \\
x_2 = + & 2\beta_1 + & 1\beta_2 & - & 4x_1 - & 1w_2 & \\
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for the next pivot: (x_3, w_3)

After the third pivot

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\begin{array}{rrrrrrr}
\zeta = + & 2\beta_1 + & 1\beta_2 - & 0.5\beta_3 - & 4x_1 - & 2w_2 + & 1\,x_3 \\
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w_1 = + & 1\beta_1 & & - & 1x_1 & & \\
x_2 = + & 2\beta_1 + & 1\beta_2 & - & 4x_1 - & 1w_2 & \\
w_3 = - & 4\beta_1 - & 2\beta_2 + & 1\beta_3 + & 8x_1 + & 4w_2 - & 1x_3
\end{array}
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for the next pivot: (x_3, w_3)

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After the fourth pivot

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\begin{array}{rrrrrrr}
\zeta = - & 2\beta_1 - & 1\beta_2 + & 0.5\beta_3 + & \boxed{4}\, x_1 + & \boxed{2}\, w_2 - & 1w_3 \\
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for the next pivot: (x_1, w_1)

Corresponding vertex on the Klee-Minty cube is: $(0, 2\beta_1 + 1\beta_2, -4\beta_1 - 2\beta_2 + 1\beta_3)$

After the fifth pivot

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\begin{array}{rcccccc}
\zeta = + & 2\beta_1 - & 1\beta_2 + & 0.5\beta_3 - & 4w_1 + & 2\,w_2 - & 1w_3 \\
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x_1 = + & 1\beta_1 & & - & 1w_1 & & \\
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\zeta & = & + & 2\beta_1 - & 1\beta_2 + & 0.5\beta_3 - & 4w_1 + & \boxed{2}\, w_2 - \quad 1w_3 \\
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for the next pivot: (w_2, x_2)

After the fifth pivot

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\begin{array}{rcrrrrrr}
\zeta & = & + & 2\beta_1 - & 1\beta_2 + & 0.5\beta_3 - & 4w_1 + & \boxed{2}\, w_2 - & 1w_3 \\
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x_1 & = & + & 1\beta_1 & & - & 1w_1 & & \\
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Corresponding vertex on the Klee-Minty cube is: $(1\beta_1, -2\beta_1 + 1\beta_2, +4\beta_1 - 2\beta_2 + 1\beta_3)$

After the sixth pivot

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\begin{array}{rcrrrrrr}
\zeta = & - & 2\beta_1 + & 1\beta_2 + & 0.5\beta_3 + & 4\, w_1 - & 2x_2 - & 1w_3 \\
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x_1 = & + & 1\beta_1 & & - & 1w_1 & & \\
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And finally after the seventh pivot

$$\begin{array}{rrrrrrr}
\zeta = + & 2\beta_1 + & 1\beta_2 + & 0.5\beta_3 - & 4x_1 - & 2x_2 - & 1w_3 \\
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w_1 = + & 1\beta_1 & & - & 1x_1 & & \\
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The optimal dictionary.

And finally after the seventh pivot

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\end{array}
$$

The optimal dictionary.

Corresponding vertex on the Klee-Minty cube is: $(0, 0, +4\beta_1 + 2\beta_2 + 1\beta_3)$

Onservations:

- It took $7 = 2^3 - 1$ iterations.

  Largest-coefficient rule can take $2^n - 1$ pivots to solve a problem in n variables and constraints.

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  (The age of the universe is estimated to be 13.7 billion years)

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  On a machin with 1000 ips. It will take 40 billion years to solve!
  (The age of the universe is estimated to be 13.7 billion years)

  Yet, problems with 10,000 to 100,000 variables/constraints are solved routinely every day.
  (Worst case analysis!)

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- **Note** that the final dictionary could have been reached from the initial dictionary in just one pivot if we had selected x_3 to be the entering variable. But the largest-coefficient rule dictated selecting x_1 and the exponential worst case running time.

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  Does there exist a varient of the Simplex method with polynomial worst case performance?

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- Indeed this is an open question :

  Does there exist a varient of the Simplex method with polynomial worst case performance?

- However, For linear programs, we have other algorithm (called *interior point* methods) with *polynomial worstcase performance*. In contrast to the simplex algorithm, which finds an optimal solution by traversing the edges between vertices on a polyhedral set, interior-point methods move through the interior of the feasible region.

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These are the necessary steps needed to take toward a proof of worst case exaponantial time of the generic Klee-Minty example. (Left as part of to the second HW)

Performance Measures

Worst-Case Analysis of the Simplex Method

Empirical Average Performance

Lets again consider the simple simplex implementation discussed previous week.