Computational Geometry
Chapter 1: Introduction

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1. Introduction
2. Complexity
3. Basic algorithmic tools
4. Geometric primitives
5. Notation and abbreviations

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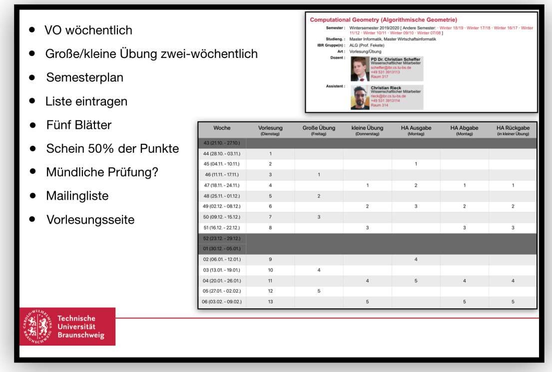
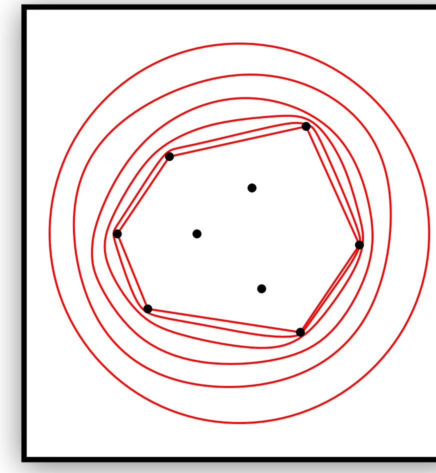
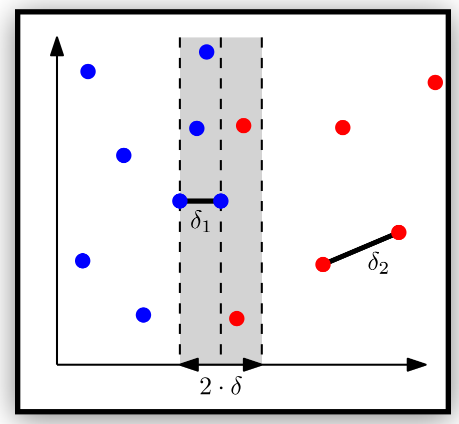
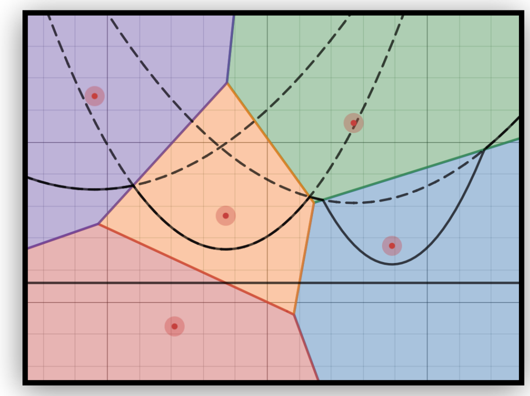
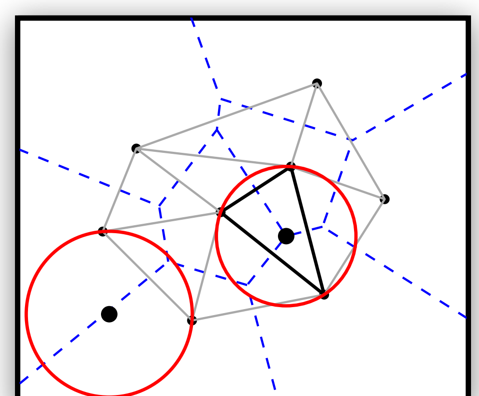
- Ch 1: Introduction
- Ch 2: Convex hulls
- Ch 3: Closest pairs
- Ch 4: Voronoi diagrams
- Ch 5: Delaunay triangulation
- Ch 6: Polygon triangulation

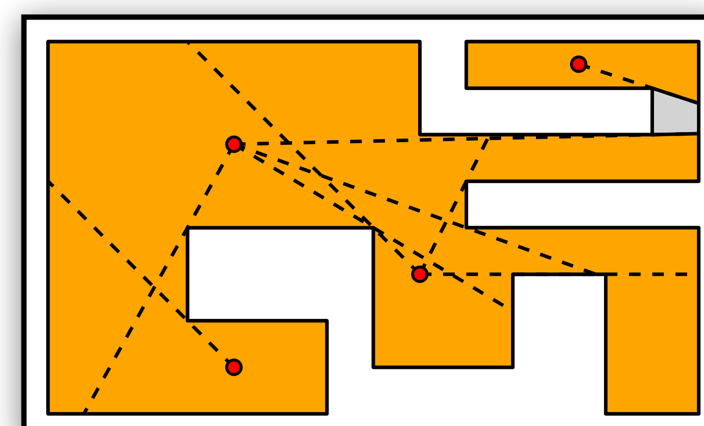












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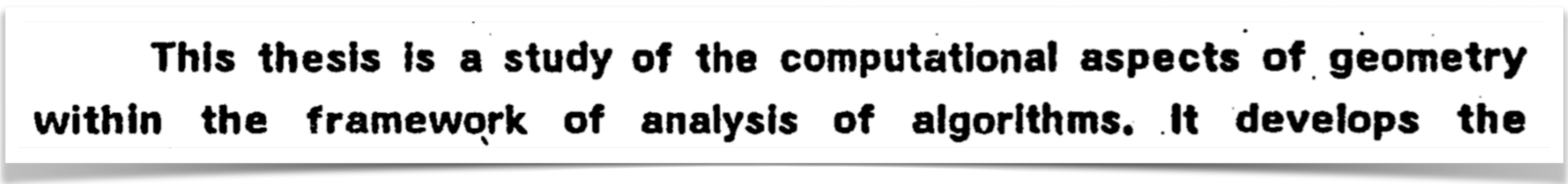
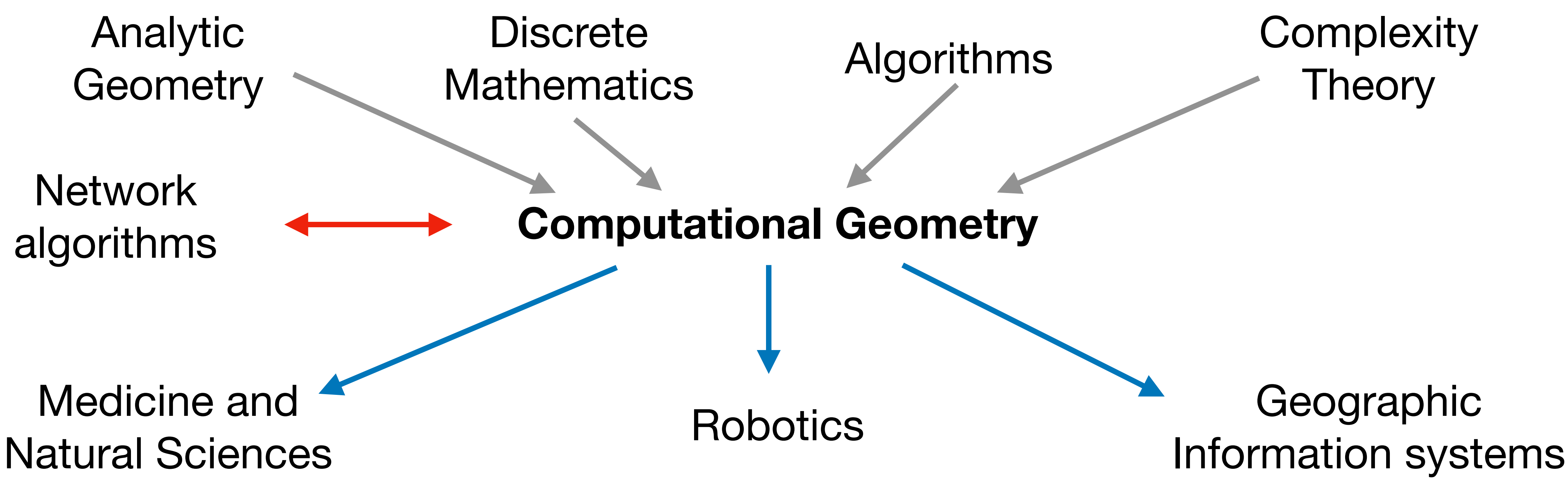
- Computational Geometry:

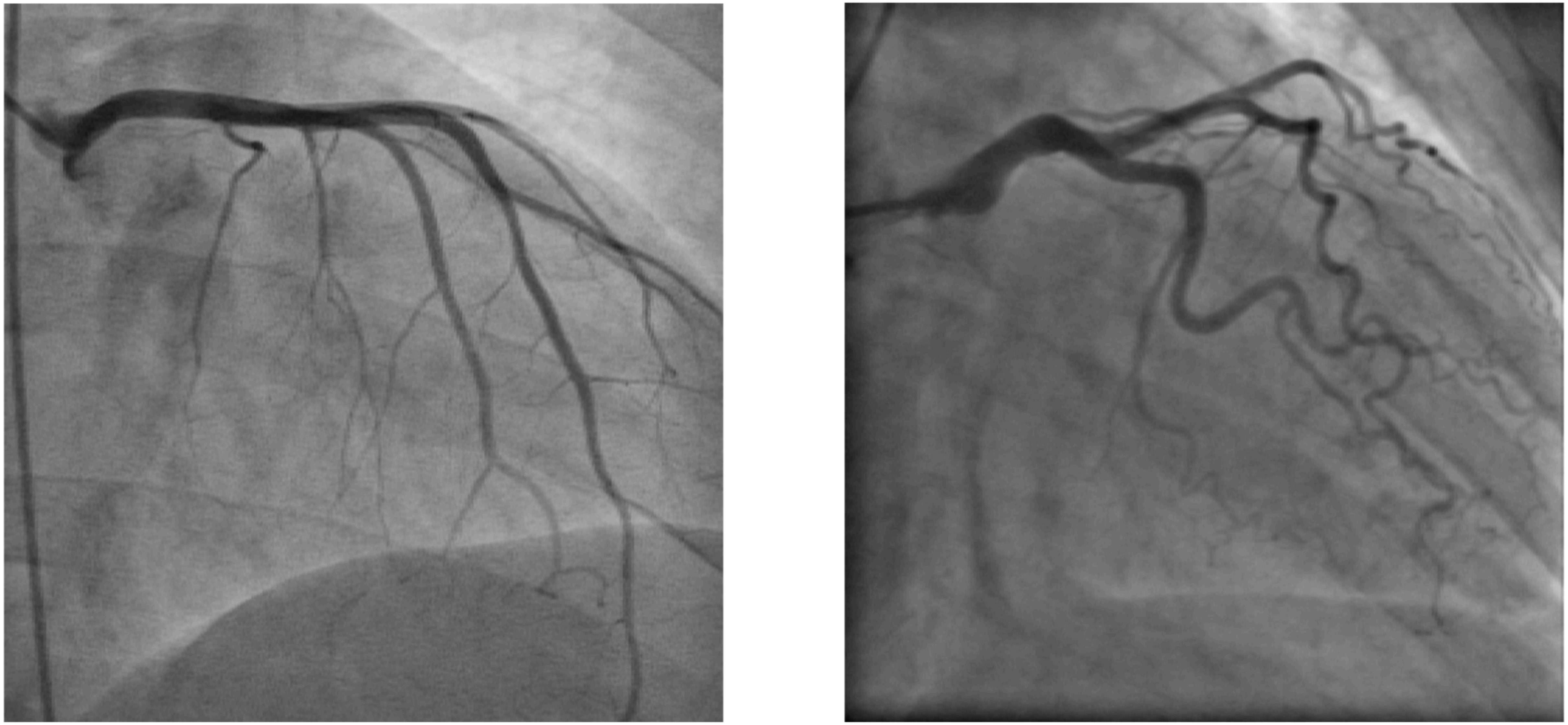
  „Dealing with geometric problem with methods of Computer Science“

- Michael Ian Shamos: Computational Geometry, Diss. Yale, 1987.



This thesis is a study of the computational aspects of geometry within the framework of analysis of algorithms. It develops the





Medicine and
Natural Sciences

Eine der Folgewirkungen anhaltend zu hohen Blutdrucks ist die so genannten „hypertensive Herzkrankheit“, die sich (laienhaft formuliert) in einer Ausdehnung gewisser Herzkranzgefäße äußert. Angiographisch läßt sich diese Symptomatik daran erkennen, dass die entsprechenden Gefäße im systolischen Zustand eine relativ hohe „Kurvigkeit“ aufweisen—vgl. die untenstehende Abbildung.

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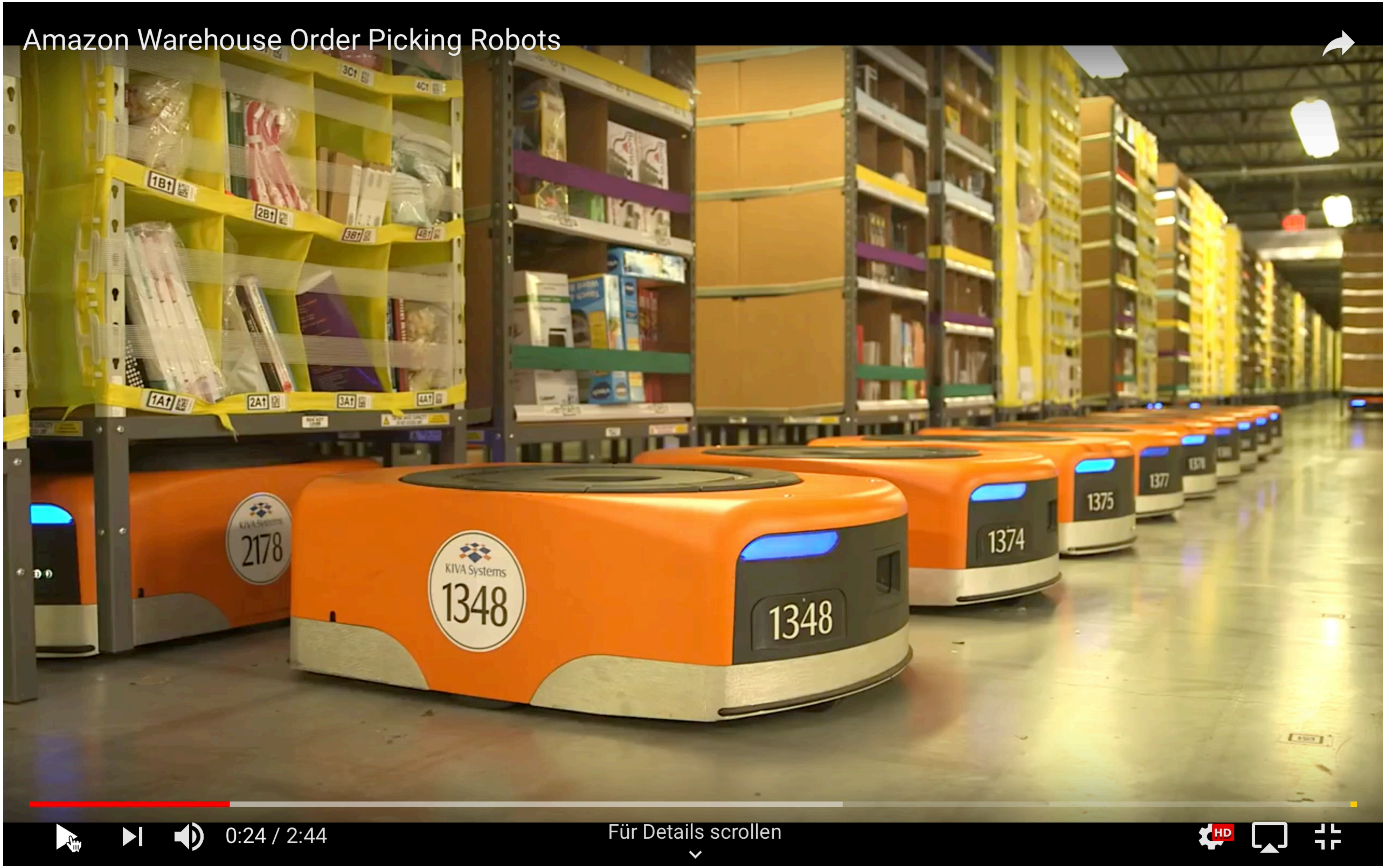

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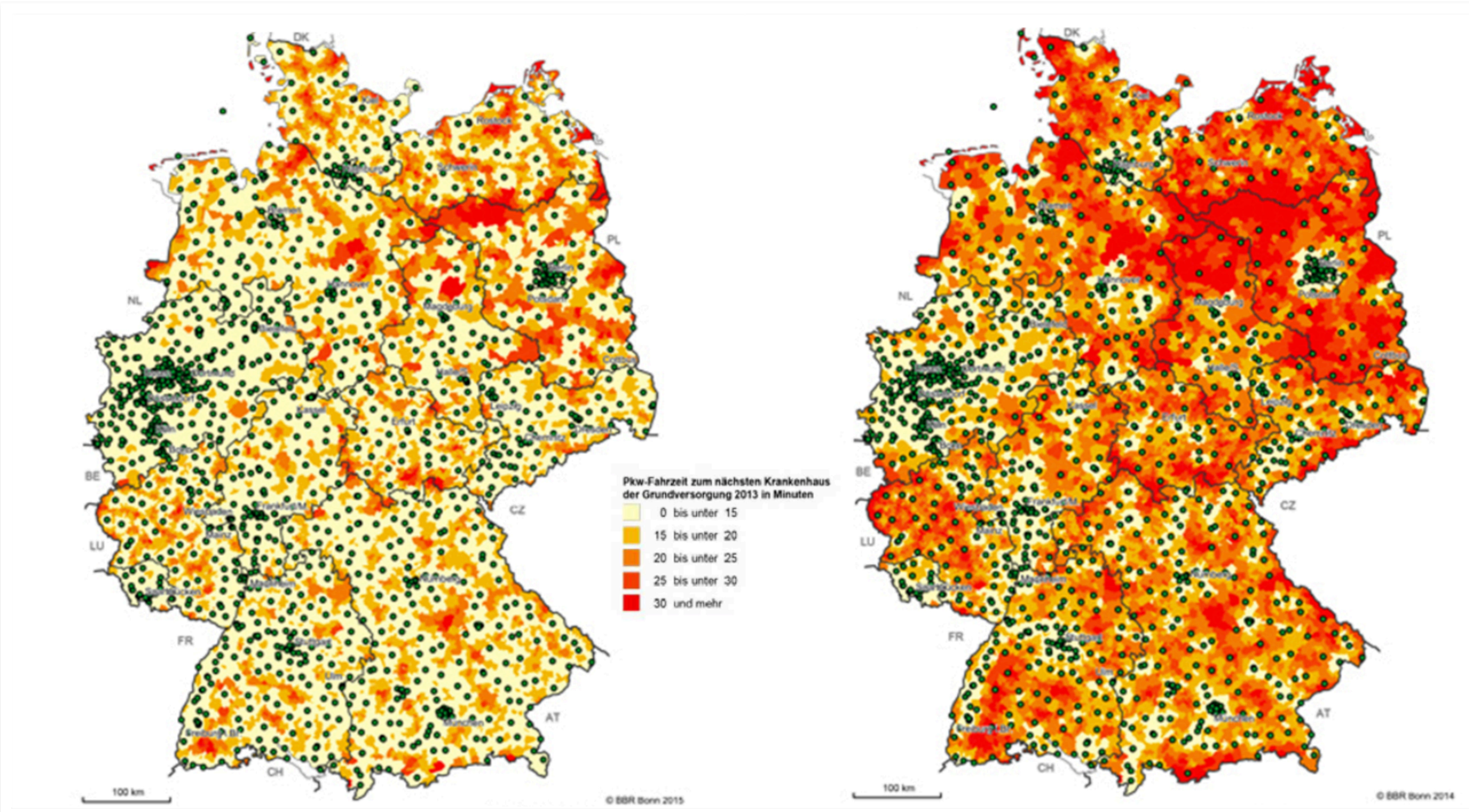

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Status quo:

- Polygonial-time algorithms for 2D problems

  „Let P be a set of points in the plane …“

Trends:

- Interaction with application fields

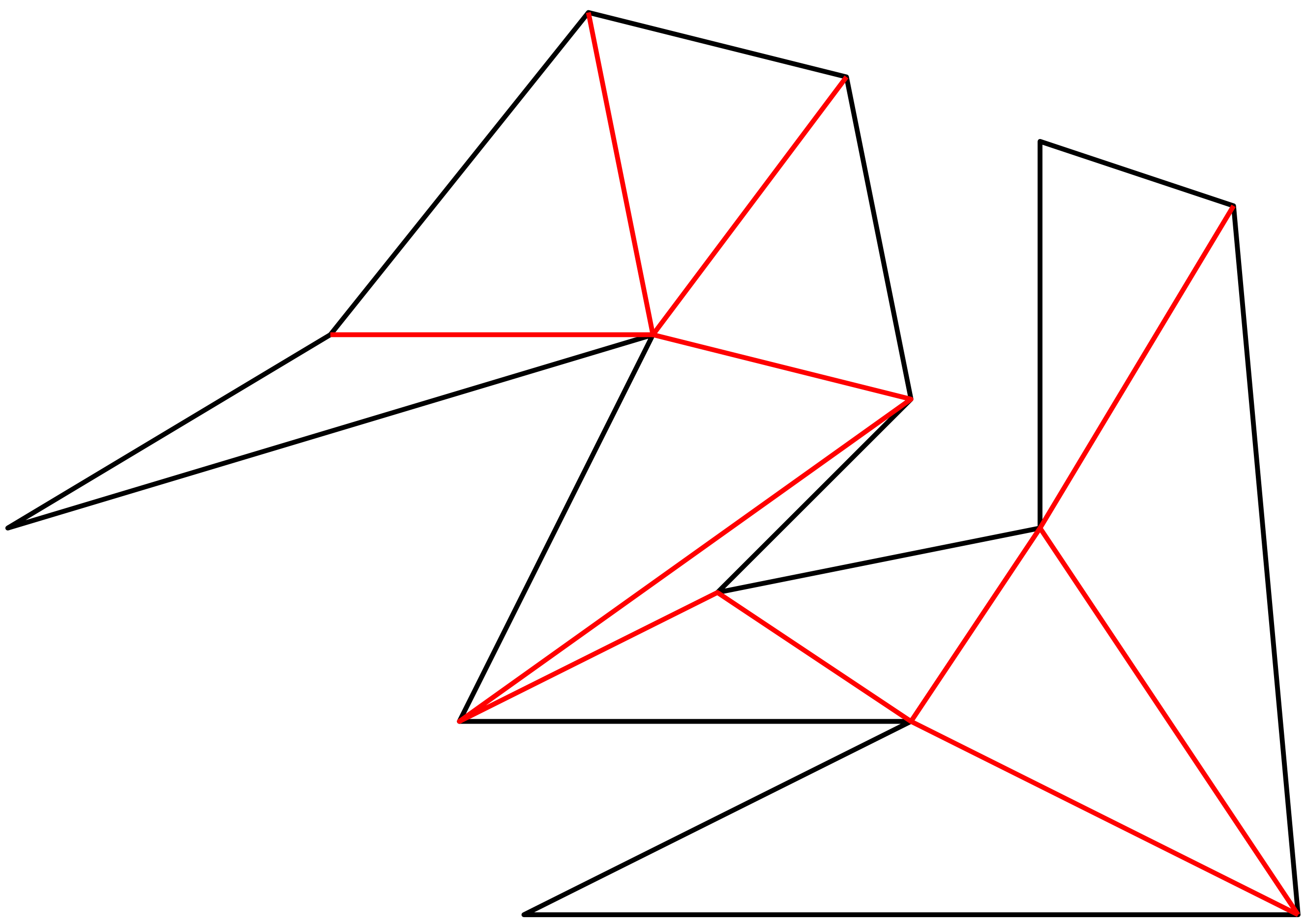
  -> Potential: new challenges.

Geometric problems:

- Easily stated
- Easy to visualize

Example: Polygon triangulation

- Given: Polygonal floorplan
- Wanted: Partition into triangles



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Reminder:

- Runtime as a function of „input size" n
- n „sufficiently large"

Details:

- Input size: #basic components (points, lines, circles, …)
- Runtime := #basic operations (comparing numbers, computing distances, ….)

Asymptotic bounds

- $\mathcal{O}(\cdot) \leftrightarrow$ Upper bound (Runtime of algorithms, complexity of objects, …).
- $\Omega(\cdot) \leftrightarrow$ Lower bound (Solvability of problems, runtime of algorithms, …).
- Master theorem

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3. **Basic algorithmic tools**
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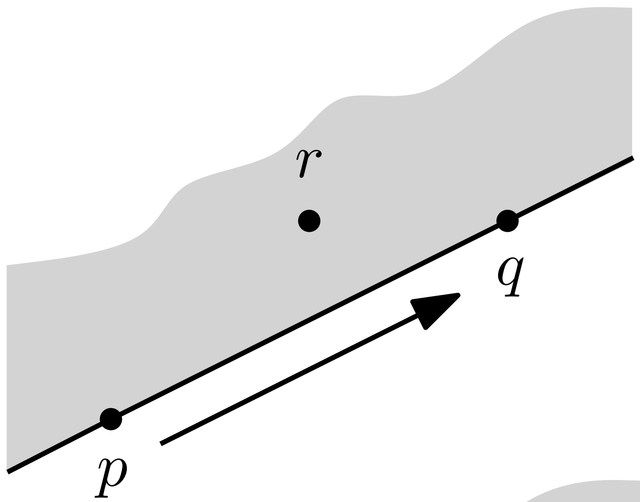
- Binary search in $\mathcal{O}(\log n)$, sorting in $\mathcal{O}(n \log n)$
- Median and Rank-k in $\mathcal{O}(n)$
- AVL trees and priority queues
    - $\mathcal{O}(\log n)$ query time
    - $\mathcal{O}(\log n)$ update time
    - $\mathcal{O}(n)$ memory consumption
- Hashing
- Perfect dynamic Hashing [Dietzfelbinger et al., 1994, Pagh und Rodler, 2004]
    - $\mathcal{O}(1)$ query time
    - $\mathcal{O}(1)$ expected update time
    - $\mathcal{O}(n)$ expected memory usage

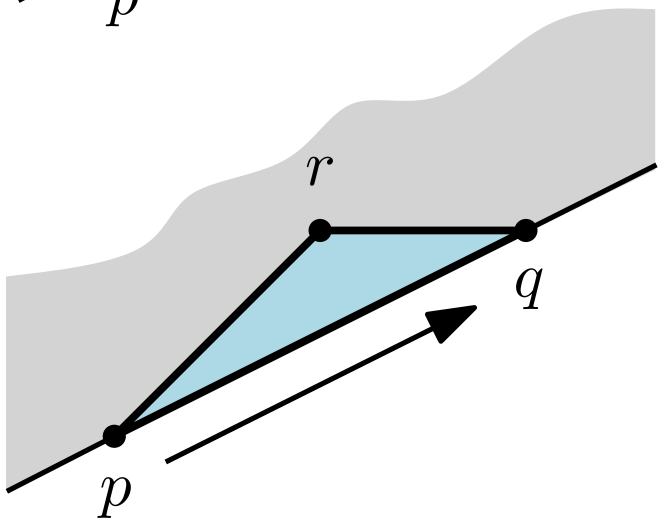
Point-line test:

- Does point $r \in \mathbb{R}^2$ lie on directed line through $p, q \in \mathbb{R}^2$?
  $\rightarrow$ If not: Does it lie left or right of the line?



- Oriented area A of $\triangle(p, q, r)$

$$A(\triangle(p,q,r)) = \frac{1}{2}\begin{vmatrix} 1 & p.x & p.y \\ 1 & q.x & q.y \\ 1 & r.x & r.y \end{vmatrix}$$



Observe:

$A(\triangle(p,q,r)) > 0 \Leftrightarrow p, q, r$ oriented in counterclockwise (CCW) order

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Point-line test (II)

Observe:

$A(\triangle(p,q,r)) > 0 \Leftrightarrow \; p,q,r$ oriented in counterclockwise (CCW) order

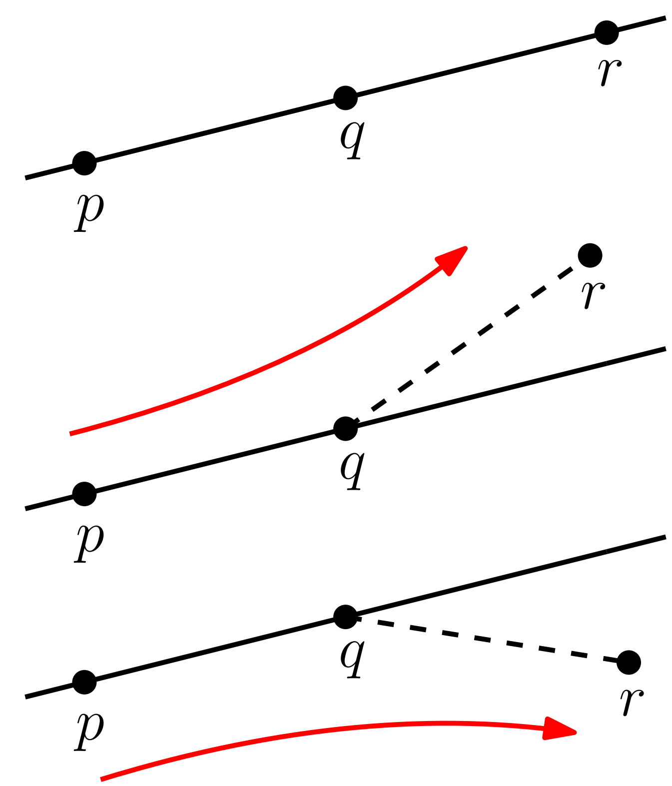
- $$A(\triangle(p,q,r)) \begin{Bmatrix} > 0 \\ = 0 \\ < 0 \end{Bmatrix} \Leftrightarrow p,q,r \begin{Bmatrix} \text{Left turn} \\ \text{collinear} \\ \text{Right turn} \end{Bmatrix}$$

- Three predicates:

$$\begin{aligned} \mathit{collinear} &: \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{B} \\ &\quad (p,q,r) \mapsto (A(\triangle(p,q,r)) = 0) \end{aligned}$$

$$\begin{aligned} \mathit{leftTurn} &: \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{B} \\ &\quad (p,q,r) \mapsto (A(\triangle(p,q,r)) > 0) \end{aligned}$$

$$\begin{aligned} \mathit{rightTurn} &: \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{B} \\ &\quad (p,q,r) \mapsto (A(\triangle(p,q,r)) < 0) \end{aligned}$$



Point-line test:

- Does point $r \in \mathbb{R}^2$ lie on directed line through $p, q \in \mathbb{R}^2$?
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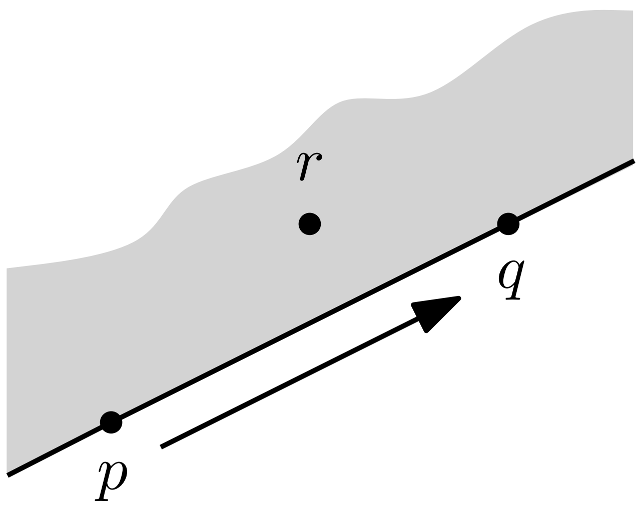
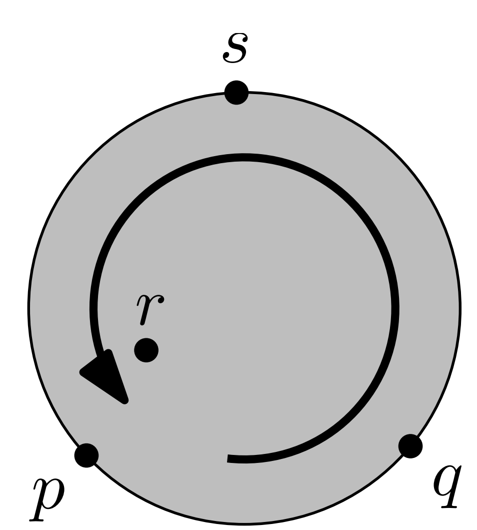
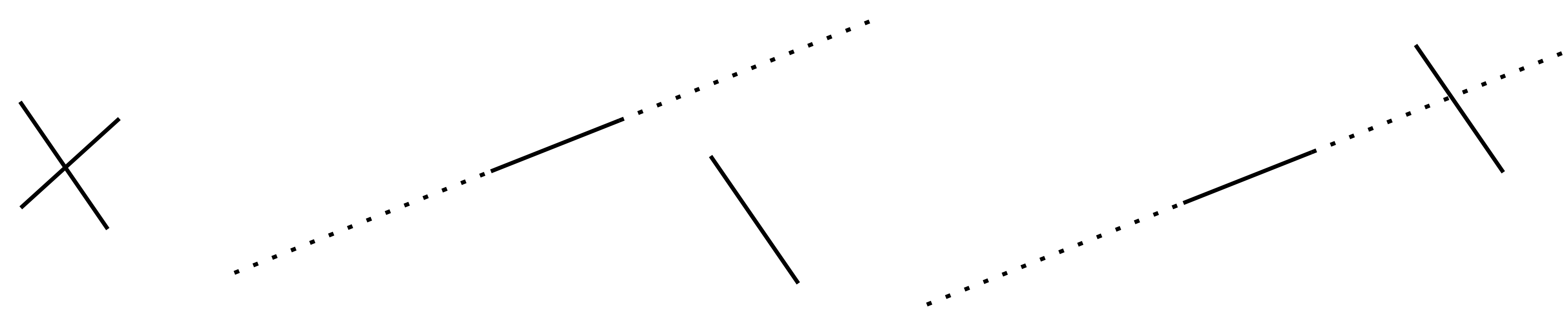


- Does point $r \in \mathbb{R}^2$ lie on (CCW) oriented circle?
  (induced by $s, q, p \in \mathbb{R}^2$)



- Intersection test (given pairs of segments, etc…)



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Abbreviations:

- # := cardinality (e.g., #vertices of a polygon)
- w.l.o.g.: Without loss of generality

Notation:

- Unless stated otherwise
- $\overline{ab}$:= segment between a, b
- $|A|$:= cardinality (but also, e.g., $|P|$ = #vertices of polygon P)
- ∂A := Boundary of set A
- A° := Interior of set A

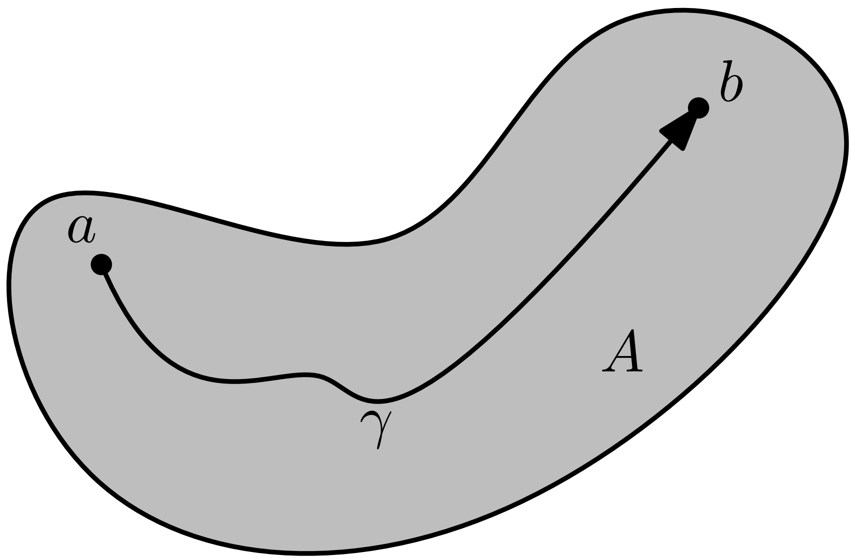
Path connectivity:

- $A \subset \mathbb{R}^d$ (path-) connected

  $:\Leftrightarrow$

  $\forall a, b \in A : \exists$ continuous curve $\gamma : [0,1] \to A$

  with $\gamma(0) = a, \gamma(1) = b$



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- Ch 7+: Further topics