Algorithms Group **Winter '19/20**
Departement of Computer Science - IBR
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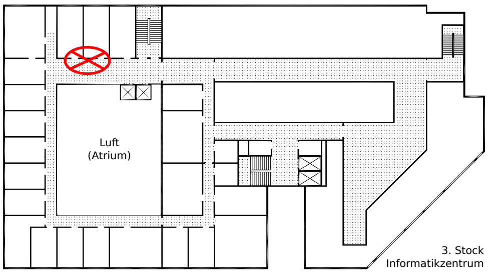
Approximation Algorithms
Exercise 1
30. Oktober 2019

Hand in your solutions until November 13, 11:30 am. You can hand in your solutions at the beginning of the tutorial or via the homework box in front of room IZ337. Please put your name on all pages.



Exercise 1 (Independent Set):
Let $G = (V, E)$ be a graph. A set of vertices $I \subseteq V$ is called *independent* if for all $u, v \in I : \{u, v\} \notin E$. The Independent Set Problem (IS) asks for an independent set of maximum cardinality.

a) Show that C is a vertex cover of G if and only if $I = V \setminus C$ is an independent set.

b) Prove that it is NP-complete to decide whether a given graph G has an independent set of a given size $k \in \mathbb{N}$.

(5+5 P.)

Exercise 2 (Vertex Cover):

We have seen in the lecture that the (minimum) Vertex Cover Problem (VC) is NP-complete in general. Show that, when the input graph is a tree, VC can be solved in polynomial time. **(10 P.)**

Exercise 3 (Vertex Cover):

We consider two greedy algorithms (Greedy 1 and Greedy 2) for the Vertex Cover Problem in a graph $G = (V, E)$. Show that neither algorithm guarantees a constant approximation factor, not even for bipartite graphs.

(5+10 P.)

Algorithm 1: Greedy 1

Data: Graph $G = (V, E)$
Result: Vertex cover $C \subseteq V$
$C \leftarrow \emptyset$;
while $E \neq \emptyset$ **do**
  Choose an edge $e \in E$ and choose a vertex v of e;
  $C \leftarrow C \cup \{v\}$;
  $E \leftarrow E \setminus \{e \in E \mid v \in e\}$;
return C;

Algorithm 2: Greedy 2

Data: Graph $G = (V, E)$
Result: Vertex cover $C \subseteq V$
$C \leftarrow \emptyset$;
while $E \neq \emptyset$ **do**
  Choose a vertex v with maximal degree in the *current* graph (V, E);
  $C \leftarrow C \cup \{v\}$;
  $E \leftarrow E \setminus \{e \in E \mid v \in e\}$;
return C;

Exercise 4 (Diameter of Sets of Points):
Let P be a set of n points in $\mathbb{R}^d$ (assume d is constant). The *diameter* Λ of P is a pair of points $p, q \in P$ that realizes the maximum distance between any two points of P. Assuming that the distance between points can be computed in $O(1)$, the diameter of P can trivially be computed in $O(|P|^2)$ time. However, show that in $O(|P|)$ time, we can compute a 2-approximation Λ' of the diameter with $\Lambda' \leq \Lambda \leq 2 \cdot \Lambda'$.
Provide a better factor for the special case of $d = 2$. **(10+5 P.)**