Abteilung Algorithmik **Winter 2018/2019**
Institut für Betriebssysteme und Rechnerverbund
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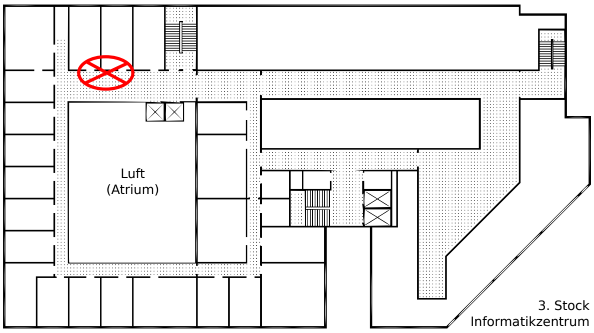
Algorithmische Geometrie
Übung 5 vom 12. 11. 2019

Abgabe der Lösungen bis zum Freitag, den 25. 01. 2019 um 11:30 im Hausaufgabenrückgabeschrank.

Bitte die Blätter vorne deutlich mit eigenem Namen sowie Matrikel- und Gruppennummer versehen!



Aufgabe 1 (Delaunay vs. Minimum-Weight Triangulation): The *weight* of a triangulation of a set of points in the Euclidean plane is defined as the sum of lengths of its edges. The *minimum-weight triangulation* (MWT) is the triangulation with minimal weight among all triangulations of the point set.

a) Give an example of a point set with as few points as possible that shows that the Delaunay triangulation is not equal to the MWT.

b) Show that the weight of the Delaunay triangulation can exceed the weight of the MWT by a factor of $\Omega(n)$.

(5 Punkte)

Aufgabe 2 (Arrangement Bounding Box): Let L be a set of lines in the plane. Describe an $O(n \log n)$ time algorithm to compute an axis-parallel rectangle that contains all the vertices of the arrangement $\mathcal{A}(L)$ that is induced by L. **(5 Punkte)**

Aufgabe 3 (The Art Gallery Problem): When we consider the Art Gallery Problem, we ask for the minimum number of guards that are sufficient to monitor a specific given polygon P.

Minimum guards vs. Minimal guards A set of guards is *minimal* if we cannot delete one of these guards without loosing the complete coverage property (the set is minimal w.r.t. inclusion).

a) Give an example of a simple polygon P and a set of 5 guards that cover it such that deletion of any one guard causes part of the gallery P to be unseen (i.e., the set of 5 guards is minimal), but the guard number, $g(P)$, for P is less than 5: $(g(P) < 5)$.

b) Give an example that the ratio between number of guards in a minimal set and number of guards in a minimum set cannot be bounded by a constant.

Guarding the boundary vs. guarding the polygon

Prove or disprove the following statement:

c) Given a simple polygon P and a placement of some number of guards in P. If the guards see every point of the boundary, δP, they also see every point in the interior of P.

(5 Punkte)

Aufgabe 4 (The Fortress Problem): For the Fortress Problem we are interested in the number of guards (vertex guards for the purposes of this exercise) that are needed to see the exterior of a polygon of n vertices. Here an exterior point y is seen by a guard at vertex z iff the segment zy does not intersect the interior of the polygon.

a) Prove that $\lceil n/2 \rceil$ vertex guards are sometimes necessary to see the exterior of a simple polygon of n vertices.

b) Give an example that it is not sufficient to place a guard at every second vertex.

(5 Punkte)