Abteilung Algorithmik **Winter 2017/2018**
Institut für Betriebssysteme und Rechnerverbund
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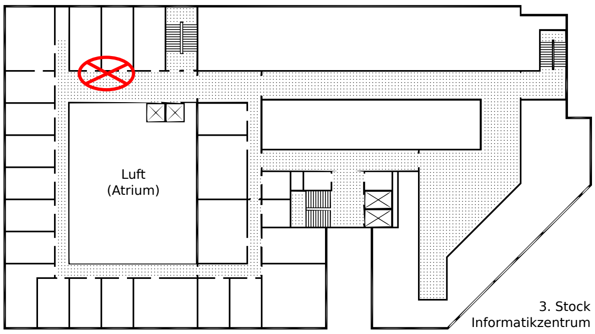
Algorithmische Geometrie
Übung 5 vom 12. 01. 2017

Abgabe der Lösungen bis zum Freitag, den 26. 01. 2018 um 11:30 im Hausaufgabenrückgabeschrank.



Bitte die Blätter vorne deutlich mit eigenem Namen sowie Matrikel- und Gruppennummer versehen!

Aufgabe 1 (Triangulations): Let S be a point set with $n = k + h$ points, not all collinear. Show that a triangulation $T(S)$ with h points on the convex hull and k interior points has

(i) $2k + h - 2$ resp. $2n - h - 2$ triangles.

(ii) $3k + 2h - 3$ resp. $3n - h - 3$ edges.

(iii) At least one vertex of degree at most five.

(5 Punkte)

Aufgabe 2 (Flip graphs of convex n-gons):

a) Give the flip graph of a convex 6-gon. Denote the triangulations represented in each node (either in the nodes itself, or in a seperate drawing).

b) Show: all nodes of the flip graph of a convex n-gon have degree $n - 3$.

(5 Punkte)

Aufgabe 3 (Art Gallery): When we consider the Art Gallery Problem, we ask for the minimum number of guards that are sufficient to monitor a specific given polygon P.

Minimum guards vs. Minimal guards A set of guards is *minimal* if we cannot delete one of these guards without loosing the complete coverage property (the set is minimal w.r.t. inclusion).

a) Give an example of a simple polygon P and a set of 5 guards that cover it such that deletion of any one guard causes part of the gallery P to be unseen (i.e., the set of 5 guards is minimal), but the guard number, $g(P)$, for P is less than 5: ($g(P) < 5$).

b) Give an example that the ratio between number of guards in a minimal set and number of guards in a minimum set cannot be bounded by a constant.

Guarding the boundary vs. guarding the polygon

Prove or disprove the following statement:

c) Given a simple polygon P and a placement of some number of guards in P. If the guards see every point of the boundary, δP, they also see every point in the interior of P.

(5 Punkte)

Aufgabe 4 (The Fortress Problem): For the Fortress Problem we are interested in the number of guards (vertex guards for the purposes of this exercise) that are needed to see the exterior of a polygon of n vertices. Here an exterior point y is seen by a guard at vertex z iff the segment zy does not intersect the interior of the polygon.

a) Prove that $\lceil n/2 \rceil$ vertex guards are sometimes necessary to see the exterior of a simple polygon of n vertices.

b) Give an example that it is not sufficient to place a guard at every second vertex.

(5 Punkte)