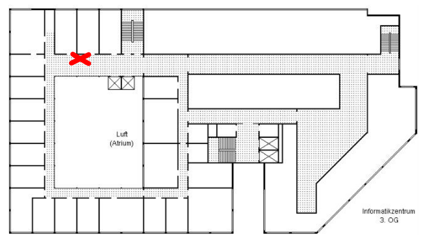
Abteilung Algorithmik
Institut für Betriebssysteme
und Rechnerverbund
TU Braunschweig

Prof. Dr. Sándor Fekete
Dr. Christiane Schmidt

WS 12/13

Computational Geometry
Homework Set 4, 19. 12. 2012

Solutions are due Wednesday, January 09th, 2013, until 11:25 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Convex Objects): Show:

a) The intersection of any (not necessarily finite) set of convex objects is convex.

b) All Voronoi regions are convex.

(5+3,5 Punkte)

Exercise 2 (Convex Hull):
Let P be a point set of n points in $\mathbb{R}^d$. $P = \{p_1, p_2, \ldots, p_n\}$. Show:
$x \in conv(P) \Leftrightarrow x = \lambda_1 p_1 + \lambda_2 p_2 + \ldots + \lambda_n p_n$ with only $(d+1)$ $\lambda_i \neq 0$ (i.e., each point can be expressed as a convex combination of $d+1$ "original" points).

(10 Punkte)

Exercise 3 (Voronoi Diagrams):

a) Construct a point set with three sites whose Voronoi vertex is exterior to the triangle determined by the sites.

b) Without invoking Euclid, provide a simple proof that Figure 1 is impossible.

c) Show: A Voronoi diagram $Vor(S)$ for a point set S has infinite-line edges if and only if all sites of S are collinear.

d) Describe the structure of the Voronoi Diagram for the vertices of a regular polygon.

(3+6+10+4 Punkte)

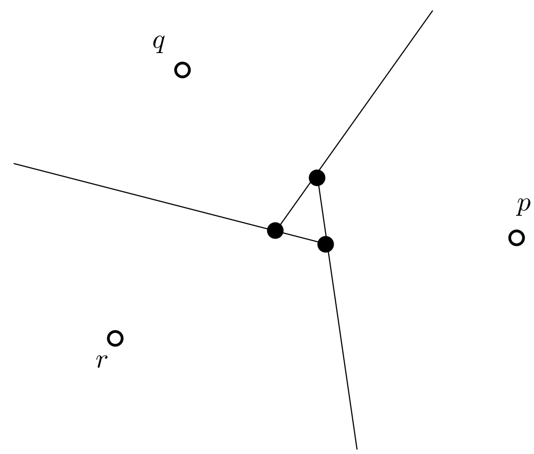


Figure 1:

Exercise 4 (Voronoi Diagrams II):
Which of the following statements are true?

a) The only way in which an existing arc can disappear from the beach line is through a circle event.

b) The only way in which an existing arc can disappear from the beach line is through a site event.

c) $O(n^3)$ circle events are processed.

d) $O(n)$ circle events are processed.

e) The beach line consists of at most $2n$ parabolic arcs.

(1,5+1,5+1,5+1,5+1,5 Punkte)

Exercise 5 (Christmas Exercise):

In order to distribute the presents efficiently, Santa Claus (S_C) established hidden interim storages all over the world. The positions of these storages are $S_1, \ldots, S_n \in \mathbb{R}^2$ with $n \in \mathbb{N}$.
The remaining tuition fees are sufficient to build one new interim storage. The potential area for its location is bounded by a rectangle. The storage should be placed such that it has the largest possible distance to the next storage—improving the coverage.
Assume the children that need to be supplied are uniformly and densely distributed in the area. Does S_C's strategy for adding a new storage guarantee

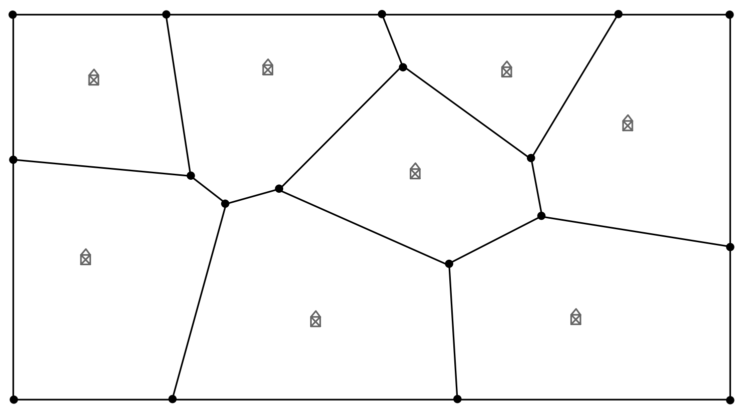


Figure 2: Sketch of one of Castiel's Voronoi Diagrams.

that the maximal walking distance between a storage and a child within the storage's area of influence is minimized?
If not, give a counterexample with a short explanation. If yes, prove that there can be no position for the new storage resulting in a shorter longest walking distance.

(15 Punkte)

Merry Christmas and a Happy New Year!