Abteilung Algorithmik
Institut für Betriebssysteme
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WS 12/13

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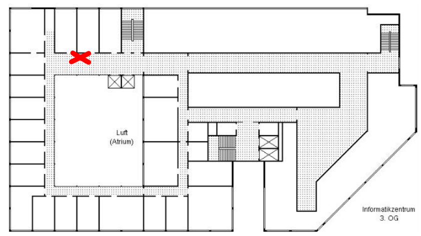
Computational Geometry
Homework Set 2, 21. 11. 2012

Solutions are due Wednesday, December 5th, 2012, until 11:25 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (The Fortress Problem):
For the *Fortress Problem* we are interested in the number of guards (vertex guards for the purposes of this exercise) that are needed to see the exterior of a polygon of n vertices. Here an exterior point y is seen by a guard at vertex z iff the segment zy does not intersect the interior of the polygon.
O'Rourke and Wood gave the following theorem:

Theorem 1. *$\lceil \frac{n}{2} \rceil$ vertex guards are sometimes necessary and always sufficient to see the exterior of a polygon of n vertices.*

Prove the necessity of $\lceil \frac{n}{2} \rceil$.

(10 Punkte)

Exercise 2 (Triangulation of monotone Polygons):
Use the algorithm of Garey, Johnson, Preparata and Tarjan to triangulate the polygon P in Figure 1. For each iteration give the stack and diagonals that are drawn. Give the final triangulation of P.

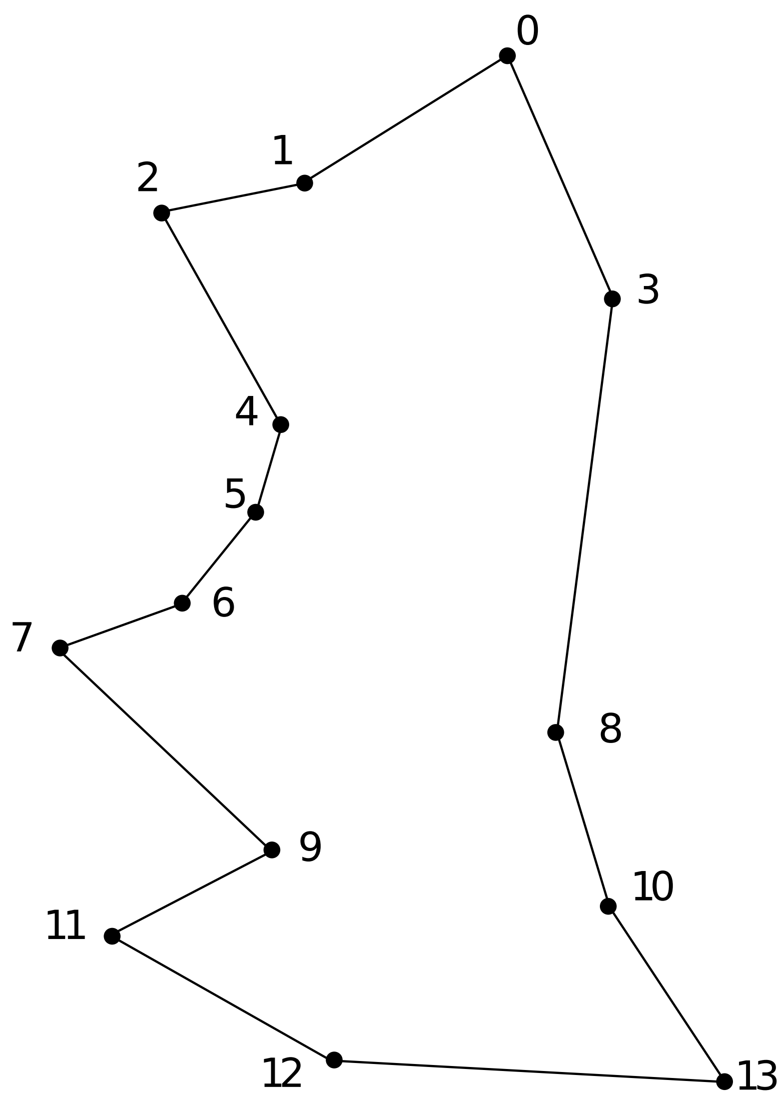


Figure 1: Polygon P.

(10 Punkte)

Exercise 3 (Scissors Congruence):
A *dissection* of a polygon P cuts P into a finite number of smaller polygons. Triangulations can be viewed as a constrained form of dissections.
Given a dissection of a polygon P, we can rearrange its smaller polygonal pieces to create a new polygon Q of the same area. We say two polygons P and Q are *scissors congruent* if P can be cut into polygons $P_1, \ldots, P_n$ which then can be reassembled by rotations and translations to obtain Q.

a) Is the Greek cross from Figure 2 scissors congruent to a square?

b) Prove: Every triangle is scissors congruent with some rectangle.

c) Prove: Any two rectangles of the same area are scissors congruent.

d) Let polygon P_1 be scissors congruent to polygon P_2, and let polygon P_2 be scissors congruent to polygon P_3. Show that polygon P_1 is scissors congruent to polygon P_3. In other words, show that scissors congruence is transitive.

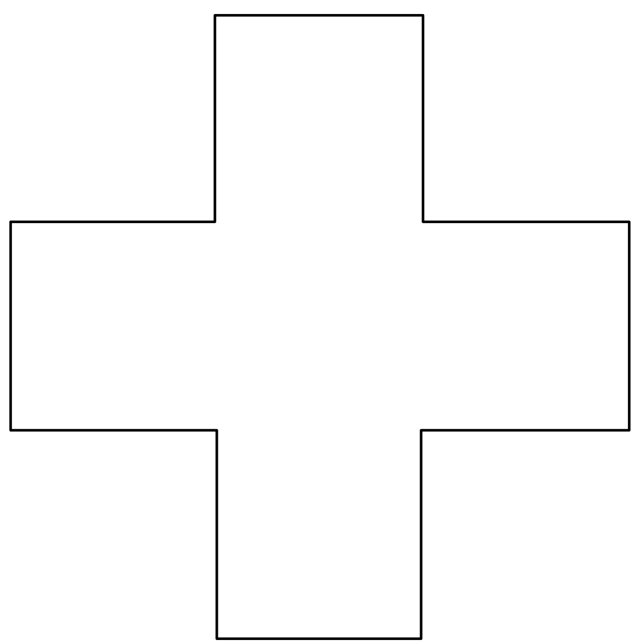


Figure 2: The Greek cross.

(10+10+10+10 Punkte)