Mergesort(A,p,r)

Input : Subarray von $A[1, ..., n]$, der bei p beginnt und bei r endet $(A[p, ..., r])$
Output : Den sortierten Subarray

```
if (p < r) then
  | q ← ⌊(p+r)/2⌋
  | Mergesort(A, p, q)
  | Mergesort(A, q + 1, r)
  | Merge(A, p, q, r)
end
```

Merge(A,p,q,r):

```
n1 ← q − p + 1
n2 ← r − q
create arrays L[1, ..., n1 + 1] and R[1, ..., n2 + 1]
for (i ← 1 to n1) do
  | L[i] ← A[p + i − 1]
end
for (j ← 1 to n2) do
  | R[j] ← A[q + j]
end
L[n1 + 1] ← ∞
R[n2 + 1] ← ∞
i ← 1
j ← 1
for k ← p to r do
  | if (L[i] ≤ R[j]) then
  |   | A[k] ← L[i] i = i + 1
  | end
  | else
  |   | A[k] ← R[j]
  |   | j ← j + 1
  | end
end
```

(ii) <u>Mergesort</u>

II

A: 2 8 6 5

2 8 | 6 5

2 | 8 | 6 5

2 8 | 6 | 5

2 8 | 5 6

2 5 6 8

A[1]=2, A[2]=8, A[3]=6, A[4]=5

Mergesort (A,1,4)

$q = \lfloor \frac{5}{2} \rfloor = 2$

Mergesort (A,1,2)

$q = \lfloor \frac{3}{2} \rfloor = 1$

Mergesort(A,1,1) ⟵ nichts passiert

Mergesort(A,2,2) ⟵ — " —

Merge (A,1,1,2) ⟶ 2 8 (*) ⟵ nicht, sondern (**)

Mergesort (A,3,4)

$q = \lfloor \frac{7}{2} \rfloor = 3$

Mergesort(A,3,3) ⟵ nichts passiert

Mergesort (A,4,4) ⟵ nichts passiert

Merge(A,3,3,4) ⟶ 5 6 (**)

Merge (A,1,2,4) ⟶ 2 5 6 7

(*):

$Merge(A, 1, 1, 2)$
$\uparrow \uparrow \uparrow$
$p \ q \ r$

$n_1 = q - p + 1 = 1 - 1 + 1 = 1$
$n_2 = r - q = 2 - 1 = 1$

$L[1,2] \quad R[1,2]$

$i=1$: $L[1] = A[1+1-1] = A[1] = 2$
$j=1$: $R[1] = A[1+1] = A[2] = 8$

$L[2] = \infty$
$R[2] = \infty$
$i = 1$
$j = 1$

$k=1$: $L[1] \leq R[1]$? $(2 \leq 8$? ja$)$

$\hookrightarrow A[1] = L[1] = 2$
$i = 2$

$k=2$: $L[2] \not\leq R[1]$ $(\infty \not\leq 8)$

$\hookrightarrow A[2] = R[1] = 8$

end

Somit: $A[1] = 2$, $A[2] = 8$

(**)

$Merge(A, 3, 3, 4)$
$\uparrow \uparrow \uparrow$
$p \ q \ r$

$n_1 = q - p + 1 = 3 - 3 + 1 = 1$
$n_2 = r - q = 4 - 3 = 1$

$L[1,2]$, $R[1,2]$

$i=1$: $L[1] = A[3+1-1] = A[3] = 6$
$j=1$: $R[1] = A[3+1] = A[4] = 5$

N

$L[2]=\infty$

$R[2]=\infty$

$i=1$

$j=1$

$k=3$: $L[1] \not\leq R[1]$

$\hookrightarrow A[3]=R[1]=5$

$j=2$

$k=4$: $L[1] \leq R[2]$ ✓ $(6 \leq \infty)$

$\hookrightarrow A[4]=L[1]=6$

$i=2$

Somit $A[3]=5$, $A[4]=6$

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V

6.Aufgabe: Sortieren

9 Punkte

Sortiere die folgenden Zahlen mit dem in der Vorlesung vorgestellten Mergesort. Kennzeichne in jedem Schritt, welche Teilfolgen gemischt werden.

$p=1,\ r=7$

$q=\left\lfloor \frac{p+r}{2} \right\rfloor = 4$

[7] [1] [8] [2] [5] [3] [4]

[1 7] [2 8] [3 5]

[1 2 7 8] [3 4 5]

[1 2 3 4 5 7 8]

VII

a) $T(n) = 256\, T\left(\frac{n}{4}\right) + n^3$

$= \sum_{i=1}^{256} T\left(\frac{1}{4} \cdot n\right) + \Theta(n^3)$

Also: $\alpha_i = \frac{1}{4}$, $i = 1, \ldots, 256$

$m = 256$

$k = 3$

$$\sum_{i=1}^{m} \alpha_i^{k} = \sum_{i=1}^{256} \left(\frac{1}{4}\right)^3 = 256 \cdot \frac{1}{64} > 1$$

$\hookrightarrow$ 3. Fall, wir suchen also c mit $\sum_{i=1}^{m} \alpha_i^{c} = 1$

$$\sum_{i=1}^{256} \left(\frac{1}{4}\right)^c = 1$$

$$\Leftrightarrow \quad 256 \left(\frac{1}{4}\right)^c = 1$$

$$\Leftrightarrow \quad \left(\frac{1}{4}\right)^c = \frac{1}{256}$$

$$\Leftrightarrow \quad 4^c = 256$$

$$\Leftrightarrow \quad c = \log_4 256$$

$$\Leftrightarrow \quad c = 4$$

$$\Rightarrow \quad T(n) \in \Theta(n^c) = \Theta(n^4)$$

VIII

b) $T(n) = 27 \cdot T\left(\frac{n}{3}\right) + n^3$

$\alpha_i = \cancel{27}\frac{1}{3}, \; i = 1, \ldots, 27$

$m = 27$

$k = 3$

$$\sum_{i=1}^{m} \alpha_i^{k} = \sum_{i=1}^{27} \left(\frac{1}{3}\right)^3 = 27 \cdot \frac{1}{27} = 1 \qquad \rightarrow \text{Fall 2}$$

$$\Rightarrow T(n) \in \Theta(n^k \log n) = \Theta(n^3 \log n)$$

c) $T(n) = 3\, T\left(\frac{n}{4}\right) + n^2$

$\alpha_i = \frac{1}{4}, \; i = 1, \ldots, 3$

$m = 3$

$k = 2$

$$\sum_{i=1}^{m} \alpha_i^{k} = \sum_{i=1}^{3} \left(\frac{1}{4}\right)^2 = 3 \cdot \frac{1}{16} < 1 \qquad \rightarrow \text{Fall 1}$$

$$\Rightarrow T(n) \in \Theta(n^k) = \Theta(n^2)$$