Abteilung Algorithmik
Institut für Betriebssysteme
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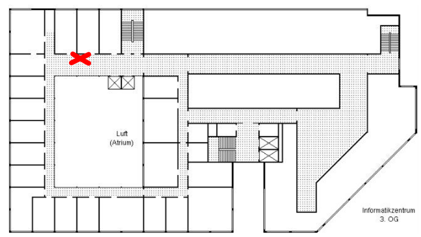
WS 11/12

Computational Geometry
Homework Set 5, 16. 01. 2012

Solutions are due Wednesday, February 1st, 2012, until 11:25 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Delaunay Triangulations and MSTs):
Show: For a point set S, a minimum spanning tree of S is a subset of the Delaunay triangulation of S.

(15 points)

Exercise 2 (Delaunay Triangulations and Edge Flips):
Show for the edge-flip algorithm (Algorithm 1) from the tutorial that for every n, there is a triangulation of n points that requires $\Omega(n^2)$ flips to transform it into a Delaunay triangulation.

(15 points)

Exercise 3 (Lower Bound for Voronoi):
Assume that an algorithm ALG is able to compute the Voronoi diagram of a point set P with $|P| = n$ in
$O(f(n))$ with $f(n) \in o(n \log n)$ (that is, faster than $O(n \log n)$).
Prove that this is not possible by showing that you could use ALG to sort n numbers in $O(n + f(n))$.

(15 points)

Exercise 4 (Voronoi Lookup):
Given: A point set P with $n \in \mathbb{N}$ points and its Voronoi diagram $Vor(P)$.

Give a method for the construction of a data structure that allows to find the nearest site for an arbitrary point $q \in \mathbb{R}^2$ (that is, to determine in which cell q lies).

Generating the data structure can be arbitrarily complex; once it is established, it should allow finding the next site for arbitrary q in time $O(\log n)$. Explain why your method complies with a lookup time of $O(\log n)$.

Hint: In case q is located on a Voronoi edge or a Voronoi vertex the next site is not uniquely defined. Your lookup should simply give one of those next sites.

(15 points)