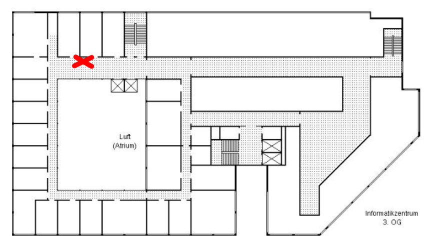
Abteilung Algorithmik
Institut für Betriebssysteme
und Rechnerverbund
TU Braunschweig

Prof. Dr. Sándor Fekete
Dr. Christiane Schmidt

WS 11/12

Computational Geometry
Homework Set 4, 19. 12. 2011

Solutions are due Wednesday, January 18th, 2012, until 11:25 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Convex Objects): Show:

a) The intersection of any (not necessarily finite) set of convex objects is convex.

b) All Voronoi regions are convex.

(5+3 points)

Exercise 2 (Voronoi Diagrams):

a) Construct a point set with three sites whose Voronoi vertex is exterior to the triangle determined by the sites.

b) Without invoking Euclid, provide a simple proof that Figure 1 is impossible.

c) For each $n \geq 3$ is it possible to construct an example of a point set with n sites having exactly one Voronoi vertex?

d) Show: A Voronoi diagram $Vor(S)$ for a point set S has infinite-line edges if and only if all sites of S are collinear.

(3+6+3+10 points)

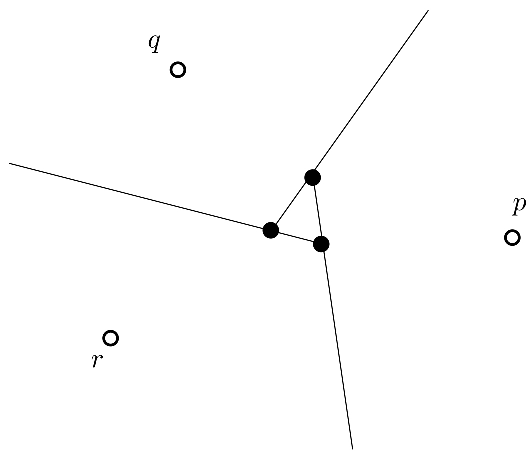


Figure 1:

Exercise 3 (Christmas Exercise):

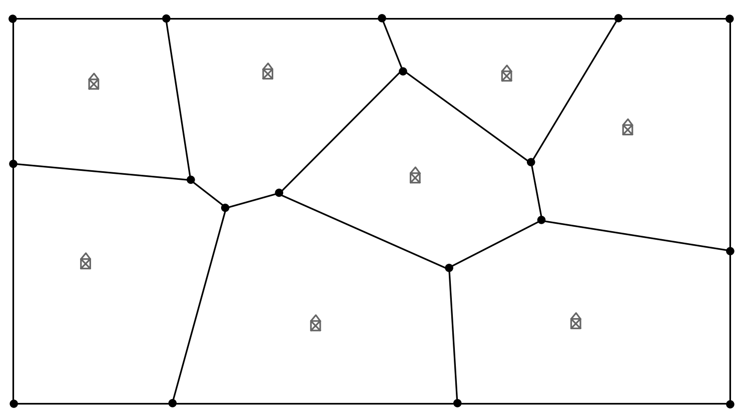


Figure 2: Sketch of one of Castiel's Voronoi Diagrams.

In order to distribute the presents efficiently Santa Claus (S_C) established hidden interim storages all over the world. The positions of these storages are $S_1, \ldots, S_n \in \mathbb{R}^2$ with $n \in \mathbb{N}$.
The remaining tuition fees are sufficient to build one new interim storage. The potential area for its location is bounded by a rectangle. The storage should be placed such that it has the largest possible distance to the next storage—improving the coverage. S_C is left with time for $O(n \log n)$ steps, only, until the new storage must be constructed.

a) Give an algorithm that determines the position of the new storage according to the given criteria. Explain the correctness of your algorithm and that it meets the required running time.

b) Assume the children that need to be supplied are uniformly and dense distributed in the area. Is S_C's strategy adequate to minimize the maximal walking distance between a storage and a child within the storage's area of influence?

   If not, give a counterexample with a short explanation. If yes, prove that there can be no position for the new storage resulting in a shorter longest walking distance.

(Hint: Angel Castiel is able to construct a DCEL that contains the Voronoi Diagram of the storages including the surrounding rectangle in $O(n \log n)$. The faces of the DCEL include references to their sites.)

(15+15 points)

Merry Christmas and a Happy New Year!