Abteilung Algorithmik
Institut für Betriebssysteme
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WS 11/12

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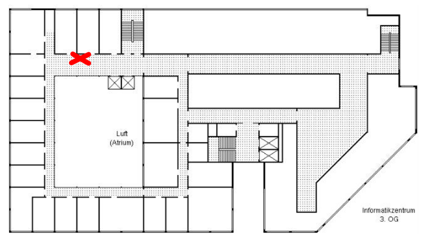
Computational Geometry
Homework Set 2, 21. 11. 2011

Solutions are due Wednesday, December 7th, 2011, until 11:25 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Monotonicity and Interior Cusps):
We define an *interior cusp* of a polygon as a reflex vertex v whose adjacent vertices either both have not larger or both have not smaller y-coordinates than v. Prove the following theorem:
If a polygon P has no interior cusp, then it is monotone with respect to the y-axis.
(Remember: A polygon is monotone if it can be partitioned into two chains monotone with respect to the same line. A chain $p_1, \ldots, p_k$ is called monotone with respect to a line L, if the projections of $p_1, \ldots, p_k$ onto L are ordered the same as in the chain.)

(15 points)

Exercise 2 (Triangulation of monotone Polygons):
Use the algorithm of Garey, Johnson, Preparata and Tarjan to triangulate the polygon P in Figure 1. For each iteration give the stack and diagonals that are drawn. Give the final triangulation of P.

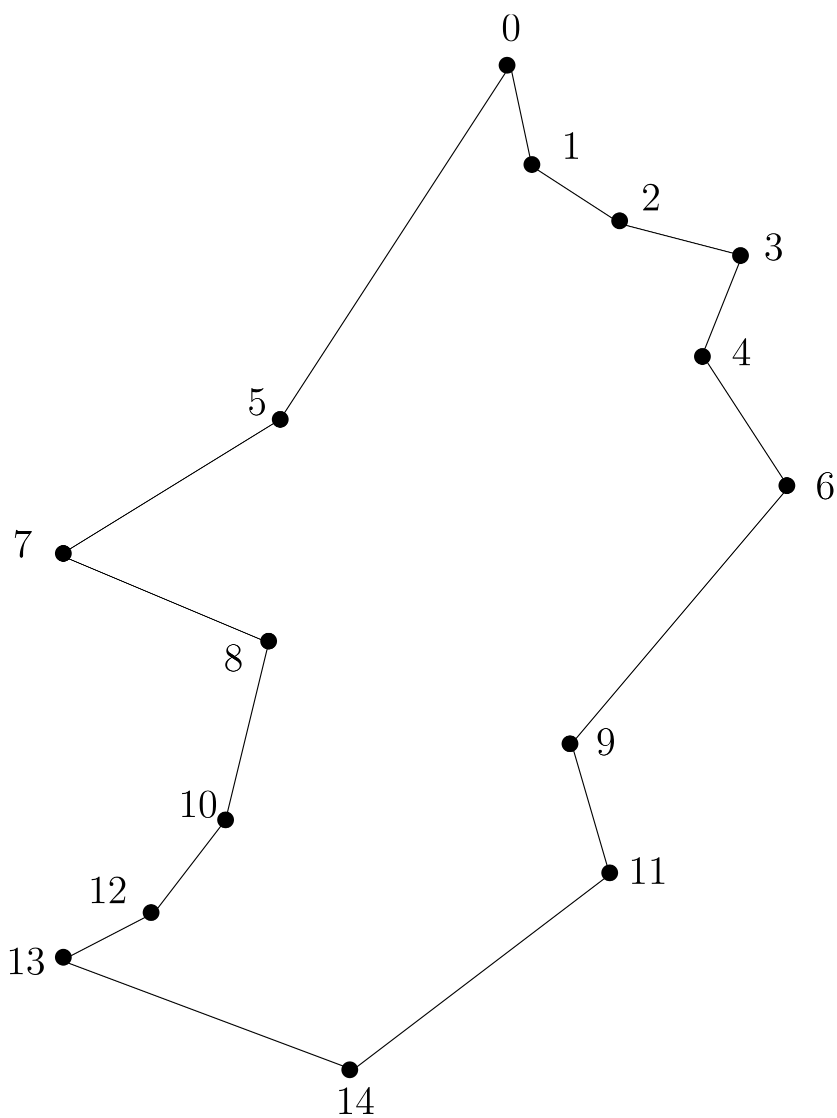


Figure 1: Polygon P.

(15 points)

Exercise 3 (Scissors Congruence):
A *dissection* of a polygon P cuts P into a finite number of smaller polygons. Triangulations can be viewed as a constrained form of dissections.
Given a dissection of a polygon P, we can rearrange its smaller polygonal pieces to create a new polygon Q of the same area. We say two polygons P and Q are *scissors congruent* if P can be cut into polygons $P_1, \ldots, P_n$ which then can be reassembled by rotations and translations to obtain Q.

a) Is the Greek cross from Figure 2 scissors congruent to a square?

b) Prove: Every triangle is scissors congruent with some rectangle.

c) Prove: Any two rectangles of the same area are scissors congruent.

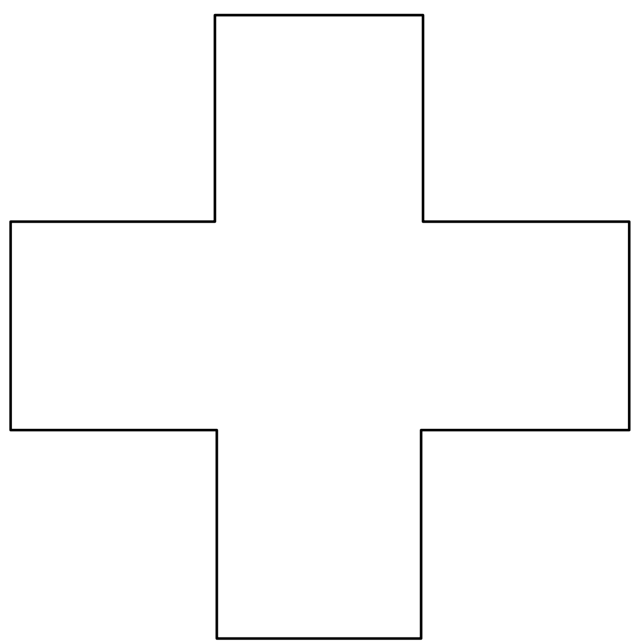


Figure 2: The Greek cross.

(10+10+10 points)