Abteilung Algorithmik **winter 10/11**
Institut für Betriebssysteme und Rechnerverbund
TU Braunschweig

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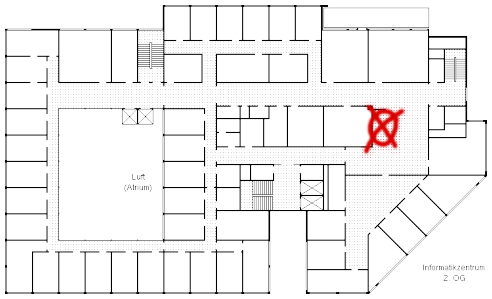
Combinatorial Algorithms
homework set #6, 20. 01. 2011

Solutions are due thursday, February 3, 2011, either

- at the beginning of the tutorial in room IZ161 or
- until 16:40 in the cupboard for handing in practice sheets.

Please put your name on all pages!



Exercise 1 (union of uniform matroids): Let $m = m_1 + m_2 + \ldots + m_n$. Determine $U_{m_1,n} \vee U_{m_2,n} \vee \ldots \vee U_{m_k,n}$. Is this a uniform matroid? **(2 P.)**

Exercise 2 (union of graphic matroids): Consider the graphs G_1 and G_2. Is $M(G_1) \vee M(G_2)$ graphic? **(3 P.)**

Figure 1: G_1 and G_2.

Exercise 3 (Painting a graph - matroid intersection): We define problem RWB as follows: We want to paint each edge of a given graph G either red, white or blue, subject to the constraint that not all the edges of any cycle are painted the same color. Depending on the graph, it may or may not be possible to do so.

a) Give a formulation of this problem as an intersection $M = M_1 \cap \ldots \cap M_k$ of matroids $M_1, \ldots, M_k$, where every solution for RWB corresponds with a basis of M.

b) Characterize which bases of M solve RWB.

Bonus Points: Earn 2 yummy extra points with a solution with $k = 2$! **(2+1 P.)**

Exercise 4: Consider the following graph G^* and the graphic matroid $M(G^*) = (E, \mathcal{I})$:

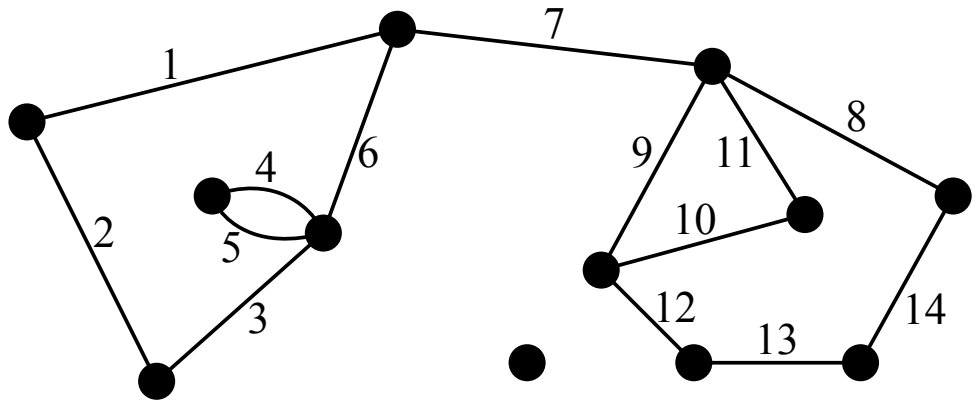


Figure 2: G^*

Provide a minimal (i.e., irredundant) description of the polytope

$$\mathrm{IND}(\mathcal{I}) := \mathrm{conv}\{\chi^F : F \in \mathcal{I}\} .$$

(2 P.)