Abteilung Algorithmik **winter 10/11**
Institut für Betriebssysteme und Rechnerverbund
TU Braunschweig

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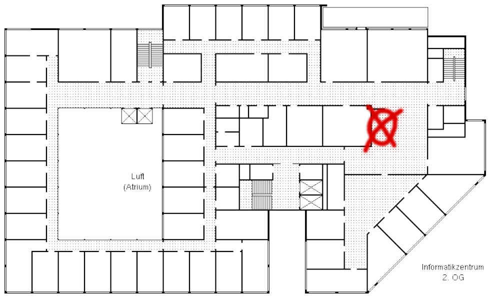
Combinatorial Algorithms
homework set #5, 06. 01. 2011

Solutions are due thursday, January 20, 2011, either

- at the beginning of the tutorial in room IZ161 or
- until 16:40 in the cupboard for handing in practice sheets.



Please put your name on all pages!

Exercise 1 (The Shannon Switching Game II): Recall the Shannon Switching Game from homework set #4 (exercise 3).
Prove that exactly one of the players win.
(Given a set $A \subset S$ in a matroid $M = (S, \mathcal{I})$, define the span of A as $span(A) := \{e \in S | r(A + e) = r(A)\}$.) **(3 P.)**

Exercise 2: Derive and prove a closed formula for $r(\cdot)$ in graphic matroids. **(2 P.)**

Exercise 3 (Euler's formula): Euler's formula for the number of faces in any planar connected graph G (with any embedding) is:

$$|F(G)| = |E(G)| - |V(G)| + 2 \tag{1}$$

(with $F(G)$, $E(G)$ and $V(G)$ denoting the faces, edges and vertices of G, respectively).
Deduce this formula using Theorem 3 from the tutorial (January 06).
(For a connected planar graph G with an arbitrary embedding: $\mathcal{M}(G^*) = (\mathcal{M}(G)^*)$.)
(3 P.)

Exercise 4: Lemma 2.11 states for an independence system $(E, \mathcal{I})$

$$q(\mathcal{I}) \leq \frac{c(G)}{OPT} \tag{2}$$

Show that the bound is sharp, that is, that there is a cost function where the lower bound is attained.

(2 P.)