Abteilung Algorithmik
Institut für Betriebssysteme und Rechnerverbund
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winter 10/11

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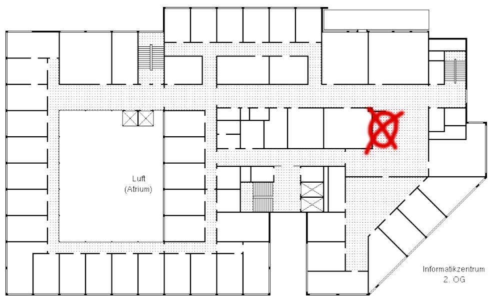
Combinatorial Algorithms
homework set #4, 16. 12. 2010

Solutions are due thursday, January 06, 2011, either

- at the beginning of the tutorial in room IZ161 or
- until 16:40 in the cupboard for handing in practice sheets.



Please put your name on all pages!

Exercise 1: Let M be the matric matroid of A (A is a matrix over $\mathbb{R}$):

$$A = \begin{bmatrix} 1 & 4 & 1 & 2 & 0 \\ 3 & 1 & 0 & 3 & 1 \\ 2 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Give a matrix B, such that M* is the matric matroid of B.

(2 P.)

Exercise 2: We call a matroid M *self-dual* if $M \simeq M*$. Let M be the matric matroid of the following matrix :

$$\left[\begin{array}{c|rrrr} & 0 & -1 & -1 & -1 \\ & 1 & 0 & -1 & 1 \\ I_4 & 1 & 1 & 0 & -1 \\ & 1 & -1 & 1 & 0 \end{array}\right]$$

Is M self-dual?

(2 P.)

Exercise 3 (The Shannon Switching Game): Consider the following game—the Shannon Switching Game: We are given an undirected graph $G = (V, E)$ and two vertices $s, t \in V$. We have to players: Join and Cut. The players choose edges from E alternatively.
On Cuts turn, Cut deletes an edge from G. On Joins turn, Join fixes an edge so that it cannot be deleted. Join wins the game if Join can fix edges connecting s to t. Cut wins

the game if Join does not. Notice that in this game, it is never disadvantageous to move. (Cut can be no worse off if the graph has one fewer edge, and Join can be no worse off if an additional edge is fixed.) Therefore, any game falls into one of three categories:

- Join game: Join wins, even if Join plays second.
- Cut game: Cut wins, even if Cut plays second.
- Neutral game: Whichever player moves first wins.

This game can be generalized to matroids: The two players Join and Cut play in a matroid. In the graphical case, an edge e is inserted into G which connects s and t. Neither player is allowed to move on this edge. With this new framework, Join wants to create a circuit containing e in M_1, and Cut wants to create a circuit in the dual M* containing e: $M = (S' = E + e, \mathcal{I}), M^* = (E + e, \mathcal{I}^*)$. In this more general matroid setting, join wants to pick $A \subset E$ such that $A + e$ contains a circuit C of M with $e \in C$. Cut wants to pick $B \subset E$ such that $B + e$ contains a circuit C of M^* with $e \in C$.
Note that the cocircuits of a matric matroid are exactly the minimal cuts.

a) Prove the following lemma:

Lemma 1 *Let* $C \in \mathcal{C}(M), D \in \mathcal{C}(M^*), then |D \cap C| \neq 1.$

b) Prove that not both of the players can win.

(Note: it is possible to prove that exactly one of the players wins!)

(3+3 P.)

...Frohe Weihnachten und einen guten Rutsch!