Abteilung Algorithmik **winter 10/11**
Institut für Betriebssysteme und Rechnerverbund
TU Braunschweig

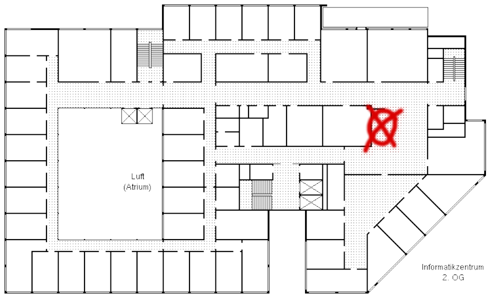
Dr. Alexander Kröller
Christiane Schmidt

Combinatorial Algorithms
homework set #3, 02. 12. 2010

Solutions are due thursday, December 16, 2010, either

- at the beginning of the tutorial in room IZ161 or
- until 16:40 in the cupboard for handing in practice sheets.

Please put your name on all pages!



Exercise 1: Consider the graph $G = (V, E)$ from Figure 1 with $E = \{1, 2, \ldots, 13\}$. Give the circuits and the cocircuits for the graphic matroid on E.

(3 P.)

Exercise 2: Suppose that a certain set of one-worker jobs has been arranged in order of importance and that we want to fill the jobs from a pool of workers each of whom is qualified to perform some subset of the jobs. We also assume that the jobs are to be done simultaneously so that no worker can be assigned to more than one job. In general, it will not be possible to fill all the jobs, so we seek a way of choosing the set of jobs to be filled that is optimal relative to the priority order on the jobs.
We reformulate the problem in terms of matroids: Let S be the set of jobs and Y the set of workers. For each $y \in Y$, let A_y be the set of jobs that worker y is qualified to perform. Let $\mathcal{A} = (A_y : y \in Y)$.
Then, the maximum number of jobs that can be done simultaneously is the size of a largest partial transversal of $\mathcal{A}$.
Let $p : S \to \mathbb{R}$ be the function that corresponds to the priority order on the jobs, a higher p-value corresponding to a higher priority for the job. A *job assignment* is a basis of the matroid on $\mathcal{A}$.
Let B be the job assignment $\{x_1, x_2, \ldots, x_r\}$ where $p(x_1) \geq p(x_2) \geq \ldots \geq p(x_r)$. Then B is called *optimal* if, for any other job assignment $\{z_1, z_2, \ldots, z_r\}$ where $p(z_1) \geq p(z_2) \geq \ldots \geq p(z_r)$, we have $p(x_i) \geq p(z_i) \forall i \in \{1, 2, \ldots\}$.

a) Does the greedy algorithm find an optimal job assignment? Justify your claim.

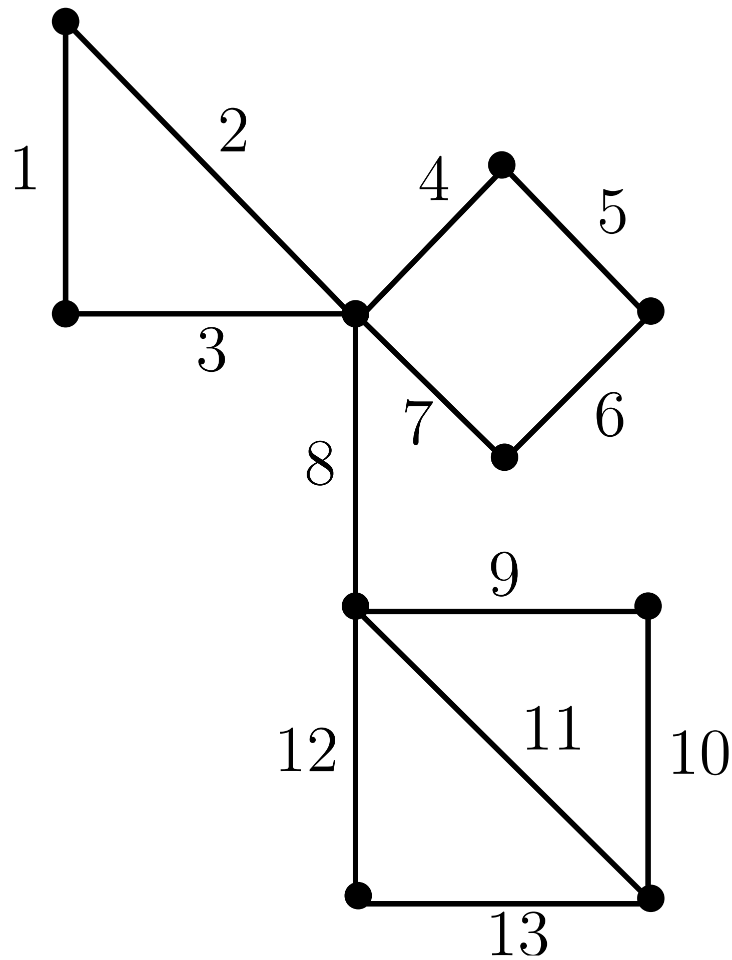


Figure 1: The graph G for exercise 1.

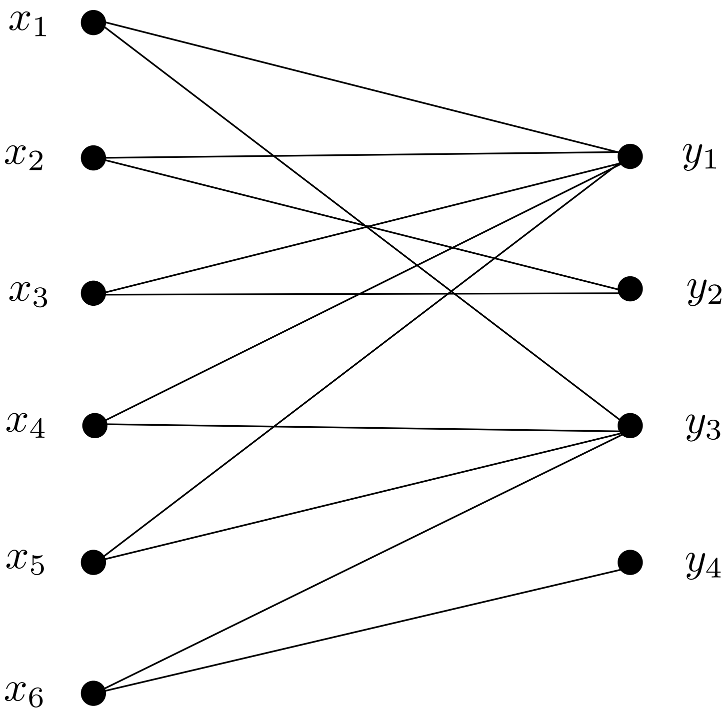


Figure 2: The graph G for exercise 2.

b) Consider the graph of Figure 2: $\{y_1, y_2, y_3, y_4\}$ corresponds to the set of workers, $\{x_1, \dots, x_6\}$ to the set of jobs and two jobs are joined when the corresponding worker can perform the corresponding job. Assume that the priority order on the jobs is $p(x_1) < p(x_2) < \dots < p(x_6)$. Give the optimal job assignment.

(3+2 P.)

Exercise 3: Consider the following problem: A salesclerk needs to give a customer change, a total of k Euros. He has unlimited amounts of coins, valued €2.00, €1.00, €.50, and €.10. Assume it is possible to obtain k from these coins (e.g., k could be €7.80, but not €7.79.) He uses the following greedy strategy:

1. While $k \geq 2$: give the customer a 2.00 coin, reduce k by 2.
2. While $k \geq 1$: give the customer a 1.00 coin, reduce k by 1.
3. While $k \geq .5$: give the customer a .50 coin, reduce k by .5.
4. Give the customer $k/.1$ coins with value .10.

These are the assigments:

a) Prove that the salesclerk always hands out the minimum number of coins that sum up to k.

b) Formulate the problem as an independence system. Prove whether it is a matroid.

(1+1 P.)