Abteilung Algorithmik
Institut für Betriebssysteme und Rechnerverbund
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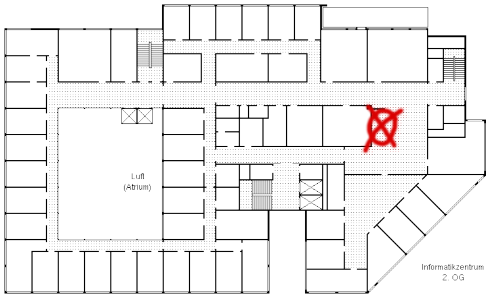
winter 10/11

Combinatorial Algorithms
homework set #2, 18. 11. 2010

Solutions are due thursday, December 2nd, 2010, either

- at the beginning of the tutorial in room IZ161 or
- until 16:40 in the cupboard for handing in practice sheets.

Please put your name on all pages!



Exercise 1: Construct a simple example to show that two nonisomorphic graphs can have the same matroid.
(To be precise here: We say two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic ($G_1 \simeq G_2$), if there is a one-to-one node mapping $\varphi : V_1 \to V_2$ that respects edges, i.e., $uv \in E_1 \iff \varphi(u)\varphi(v) \in E_2$.
Similar for independence systems $M_1 = (E_1, \mathcal{I}_1)$, $M_2 = (E_2, \mathcal{I}_2)$: They are isomorphic ($M_1 \simeq M_2$), if there is a one-to-one mapping $\varphi : E_1 \to E_2$ between the ground sets that respects independence: $F \in \mathcal{I}_1 \iff \{\varphi(e) : e \in F\} \in \mathcal{I}_2$.
So in this assignment you need to come up with two graphs G_1, G_2 where $G_1 \not\simeq G_2$ but $M(G_1) \simeq M(G_2)$.) **(1 P.)**

Exercise 2: Let A be the following matrix over the field $\mathbb{R}$ of real numbers.

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Thus, $E = \{1, 2, 3, 4, 5\}$.

a) Give the bases of $M[A]$.

b) Let G be the graph in Figure 1. And let $M = M(G) = (E_G, \mathcal{I}_G)$. Name E_G and the bases of M.

c) Is $M[A]$ graphic? (Prove your claim.)

(1+1+1 P.)

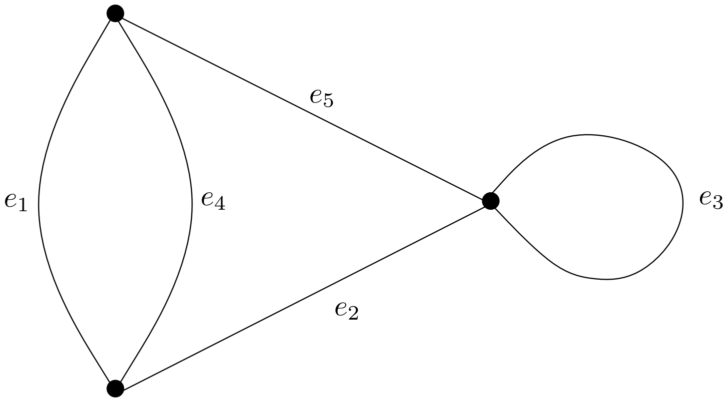


Figure 1: The graph G for exercise 2.

Exercise 3: The finite Galois field $\mathrm{GF}(p)$ for a prime p consists of the elements $\{0, \ldots, p-1\}$ with the usual operations $+$, $\times$, but computed modulo p. I.e., $p = 0 \mod p$, and $-1 = p - 1 \mod p$ etc.
Let

$$A := \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \in \mathrm{GF}(3)^{2\times 4} \tag{1}$$

The columns of A are denoted by $v_1, \ldots, v_4$.

a) Name the circuits and the bases of $M_{GF(3)}[A]$.

b) Is $U_{2,4} \simeq M[A]$? (Prove your claim.)

c) Is $U_{2,4}$ representable over $\mathrm{GF}(2)$, the field of two elements? That is, does there exist a matrix $A' \in \mathrm{GF}(2)^{n\times m}$ with $U_{2,4} \simeq M[A']$? (Prove your claim.)

(1+1+1 P.)

Exercise 4: A circuit in a matroid $M = (E, \mathcal{I})$ is a minimal dependent subset $C \subseteq E$. The set of all circuits in M is denoted by $\mathcal{C}$ or $\mathcal{C}(M)$.
Prove that the following conditions hold for the family $\mathcal{C}$ of a matroid $M = (E, \mathcal{I})$:

(C1) $\emptyset \notin \mathcal{C}$

(C2) $C_1, C_2 \in \mathcal{C}$ and $C_1 \subseteq C_2 \Longrightarrow C_1 = C_2$

(C3) $\forall C_1 \neq C_2 \in \mathcal{C} \quad \forall e \in C_1 \cap C_2 \quad \exists C_3 \in \mathcal{C}: \quad C_3 \subseteq (C_1 \cup C_2)\setminus\{e\}$.

(1+1+1 P.)