Collaborative transmission in wireless sensor networks

Alternative algorithmic approaches

Stephan Sigg

Institute of Distributed and Ubiquitous Systems
Technische Universität Braunschweig

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Overview and Structure

- Introduction to context aware computing
- Wireless sensor networks
- Wireless communications
- Basics of probability theory
- Randomised search approaches
- Cooperative transmission schemes
- Distributed adaptive beamforming
  - Feedback based approaches
  - Asymptotic bounds on the synchronisation time
  - Alternative algorithmic approaches
  - Alternative Optimisation environments
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  - Feedback based approaches
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  - Alternative algorithmic approaches
  - Alternative Optimisation environments
Outline

Alternative beamforming approaches

1. Hierarchical clustering
2. Local random search
3. An asymptotically optimal algorithm
4. Environmental changes
   - Velocity of nodes
   - Multiple receiver nodes
   - Increased population size
   - Receive beamforming
Alternative algorithmic approaches

Hierarchical clustering

- For feedback based distributed adaptive transmit beamforming:
  - $\text{RSS}_{\text{sum}}$ changes linear with the network size $n$.
  - Bound on the synchronisation time is more than linear in $n$. 

Alternative algorithmic approaches

Hierarchical clustering

\[ E[T_P] = \Theta(n \cdot k \cdot \log(n)) \]
Alternative algorithmic approaches
Hierarchical clustering

- Hierarchical clustering
  1. Determine clusters
  2. Synchronise clusters successively (with possibly increased transmit power for nodes)
  3. Build and synchronise overlay-cluster of representative nodes from all clusters.
  4. Nodes alter carrier phase by phase offset experienced by representative node:
     \[
     \zeta_i = \Re \left( m(t) \text{RSS}_i e^{j2\pi f_c t(\gamma_i + \phi_i + \psi_i)} \right) \quad \text{(before)}
     \]
     \[
     \zeta'_i = \Re \left( m(t) \text{RSS}_i e^{j2\pi f_c t(\gamma'_i + \phi_i + \psi_i)} \right) \quad \text{(after)}
     \]
     Node \( h \) from same cluster alters carrier signal
     \[
     \zeta_h = \Re \left( m(t) \text{RSS}_h e^{j2\pi f_c t(\gamma_h + \phi_h + \psi_h)} \right) \quad \text{to}
     \]
     \[
     \zeta'_h = \Re \left( m(t) \text{RSS}_h e^{j2\pi f_c t(\gamma_h + \phi_h + \psi_h + \gamma_i - \gamma'_i)} \right)
     \]
  5. Final synchronisation among all nodes

Ideal conditions: All nodes should now in phase
Alternative algorithmic approaches

Hierarchical clustering
Alternative algorithmic approaches

Hierarchical clustering

Potential problem: Phase noise
- Only one cluster synchronised at a time
- Due to practical properties of oscillators, phases of nodes in the inactive clusters experience phase noise and start drifting out of phase
- Sufficient synchronisation possible in the order of milliseconds

Positive:
- No inter-node communication required

Open Issue:
- More than one hierarchy stage might be optimal for optimisation time
- for energy consumption
- Optimum hierarchy depth and cluster size derived by integer programming in time $O(n^2)$
Alternative algorithmic approaches
Hierarchical clustering

Determine optimum cluster size and hierarchy depth:

- Expected optimisation time:
  \[ E[T_{\mathcal{P}n}] = c \cdot k \cdot n \cdot \log(n) \]

- Expected energy consumption:
  \[ E[\mathcal{E}_{\mathcal{P}n}] = c \cdot k \cdot n \cdot \log(n) \cdot \mathcal{E}_{\mathcal{P}n} \]

Hierarchy and cluster structure that minimises these formulae optimal
Alternative algorithmic approaches

Hierarchical clustering

Opt. cluster size and hierarchy depth (integer programming):

- For a cluster size of $m$:

\[
E[T_{Pn}] = E[T_{P \frac{n}{m}}] \cdot \frac{n}{m} \cdot E[T_{Pm}]
\]
\[
E[E_{Pn}] = E[E_{P \frac{n}{m}}] \cdot \frac{n}{m} \cdot E[E_{Pm}].
\]

- Define recursion by

\[
E_{\text{opt}}[T_{Pn}] = \min_{m} \left[ E_{\text{opt}}[T_{P \frac{n}{m}}] \cdot \frac{n}{m} \cdot E_{\text{opt}}[T_{Pm}] \right]
\]
\[
E_{\text{opt}}[E_{Pn}] = \min_{m} \left[ E_{\text{opt}}[E_{P \frac{n}{m}}] \cdot \frac{n}{m} \cdot E_{\text{opt}}[E_{Pm}] \right]
\]

- Start of recursion ($\eta$ min feasible cluster size):
  - $E_{\text{opt}}[T_{P\eta}]$
  - $E_{\text{opt}}[E_{P\eta}]$
Alternative algorithmic approaches
Hierarchical clustering

Opt. cluster size and hierarchy depth (integer programming) : 
- Time required for calculation is quadratic.
  - With a network of $n$ nodes, at most $n^2$ distinct terms
    - $E_{\text{opt}}[T_{\mathcal{P}i}]$
    - $E_{\text{opt}}[E_{\mathcal{P}i}]$
- Start calculation at
  - $E_{\text{opt}}[E_{\mathcal{P}\eta}]$
  - $E_{\text{opt}}[T_{\mathcal{P}\eta}]$
- All other values by table loop-up in time $O(n^2)$ according to
  - $E_{\text{opt}}[T_{\mathcal{P}n}]$
  - $E_{\text{opt}}[E_{\mathcal{P}n}]$ in time $O(n^2)$
Alternative algorithmic approaches
Hierarchical clustering

- Reduction of synchronisation time and transmission power
- Calculation of optimum cluster size and depth in $\mathcal{O}(n^2)$
Outline
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1. Hierarchical clustering
2. Local random search
3. An asymptotically optimal algorithm
4. Environmental changes
   - Velocity of nodes
   - Multiple receiver nodes
   - Increased population size
   - Receive beamforming
Global random search:
- Synchronisation performance might deteriorate when the optimum is near
- With small local search space:
  - Majority of worse points excluded
Local random search
An upper bound on the synchronisation performance

Assumptions:
Mutation probability: $n^{-1}$
Uniform phase alteration

Initial distance to the optimum:
$\geq \frac{n \cdot \log(k)}{2}$ (Chernoff)

Technical assumption:
Fitness space divided in $k$ slices of identical width

Superimposed sum signal at the receiver

Received Signal Strength $P_x$

- Expected sum signal
- Initial received sum signal
- Received sum signal after 6000 iterations

Time [s]
Local random search
An upper bound on the synchronisation performance

Analysis in two phases for the synchronisation process

Phase 1: Optimum outside search neighbourhood for at least one node

Phase 2: Optimum within search neighbourhood for all nodes
Local random search
An upper bound on the synchronisation performance

**Phase 1:** Optimum is outside the neighbourhood
- Reach search point with improved fitness: $\geq \frac{1}{2}$
Local random search
An upper bound on the synchronisation performance

When \( i \) signals synchronised:
- Improve \( n - i \) non-optimal signals
- \( i \) already optimal ones unchanged:

\[
(n - i) \cdot \frac{1}{n} \cdot \frac{1}{2} \cdot (1 - \frac{1}{n})^i = \frac{n-i}{2n} \cdot (1 - \frac{1}{n})^i
\]

- since \((1 - \frac{1}{n})^n < e < (1 - \frac{1}{n})^{n-1}\)

\[
s_i \geq \frac{n-i}{2en}
\]

- Expected number of mutations to increase fitness bounded by \( s_i^{-1} \).

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Local random search

An upper bound on the synchronisation performance

- Time until optimum is within the neighbourhood?
- Constant time to leave slice
- $k$ distinct slices

$$E[T_P] \leq c \cdot \sum_{i=0}^{k} \frac{2en}{n-i} = 2cen \cdot \sum_{i=1}^{k+1} i^{-1}$$

$$< 2cen \cdot \ln(k + 1) = \mathcal{O}(n \cdot \log(k))$$
Local random search
An upper bound on the synchronisation performance

Phase 2: Optimum within search neighbourhood
- Worst case: Increase fitness with probability $\frac{1}{N}$
- Similar to consideration above:

$$O(N \cdot n \cdot \log(k))$$

Overall synchronisation time:

$$O(N \cdot n \cdot \log(k)).$$
Scenario analysis and algorithmic improvement

Local random search

A lower bound on the synchronisation time:

- Method of the expected progress
- Similar to estimation for global random search
- Basically: Substitute network size $n$ by neighbourhood size $N$
Scenario analysis and algorithmic improvement
Local random search

A lower bound on the synchronisation time:

- Method of the expected progress
- Similar to estimation for global random search
- Basically: Substitute network size $n$ by neighbourhood size $N$
  - Probability to alter individual bit

\[
\frac{1}{N \cdot \log(k)}
\]

- Instead of

\[
\frac{1}{n \cdot \log(k)}
\]
Scenario analysis and algorithmic improvement

Local random search

A lower bound on the synchronisation time:

- With similar arguments as for global random search, lower bound

\[ \Omega(N \cdot \log(k) \cdot \Delta) \]
Mathematical simulation environment
Impact of the node choice

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node distribution area</td>
<td>$30m \times 30m$</td>
</tr>
<tr>
<td>Location of the receiver</td>
<td>$(15m, 15m, 30m)$</td>
</tr>
<tr>
<td>Mobility</td>
<td>stationary nodes</td>
</tr>
<tr>
<td>Base band frequency</td>
<td>$f_{\text{base}} = 2.4$ GHz</td>
</tr>
<tr>
<td>Transmission power of nodes</td>
<td>$P_{\text{tx}} = 1$ mW</td>
</tr>
<tr>
<td>Gain of the transmit antenna</td>
<td>$G_{\text{tx}} = 0$ dB</td>
</tr>
<tr>
<td>Gain of the receive antenna</td>
<td>$G_{\text{rx}} = 0$ dB</td>
</tr>
<tr>
<td>Iterations per simulations</td>
<td>6000</td>
</tr>
<tr>
<td>Identical simulation runs</td>
<td>10</td>
</tr>
<tr>
<td>Random noise power [46]</td>
<td>$-103$ dBm</td>
</tr>
<tr>
<td>Pathloss calculation ($P_{\text{rx}}$)</td>
<td>$P_{\text{tx}} \left( \frac{\lambda}{2\pi d} \right)^2 G_{\text{tx}} G_{\text{rx}}$</td>
</tr>
</tbody>
</table>

- Fitness measure:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{t=0}^{T} \left( \sum_{i=1}^{n} s_i + s_{\text{noise}}(i) - s^* \right)^2}.
\]
Scenario analysis and algorithmic improvement

Local random search

 Median fitness values (Network size: 100 nodes)

- Hillclimber, mutation probability: 0.01, Neighbourhood size: $0.6\pi$
- Uniform distribution, Mutation probability: 0.01
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Multivariable equations

**Received sum signal**

- Reduce the amount of randomness in the optimisation
- Improve the synchronisation performance
- Improve the synchronisation quality
Scenario analysis and algorithmic improvements

Search space

- **Search space:**
  - Spanned by all Configurations of carrier phase offsets $\gamma_i$
- **Search point / Configuration:**
  - One possible configuration of carrier phase offsets
Multivariable equations

Received sum signal

- Fitness function observed by single node
- Constant carrier phase offset for \( n - 1 \) nodes
- Fitness function:

\[
\mathcal{F}(\Phi_i) = A \sin(\Phi_i + \phi) + c
\]
Multivariable equations

Received sum signal

Approach:
- Measure feedback at 3 points
- Solve multivariable equations
- Apply optimum phase offset calculated

\[ F(\Phi_i) = A \sin(\Phi_i + \phi) + c \]
Problem:

- Calculation not accurate when two or more nodes alter the phase of their transmit signals
An active node will:

1. Transmit with three distinct phase offsets $\gamma_1 \neq \gamma_2 \neq \gamma_3$ and measure feedback.
2. From these three feedback values and phase offsets, estimate feedback function and optimum phase offset $\gamma_i^*$.
3. Transmit a fourth time with $\gamma_4 = \gamma_i^*$.
4. If the deviation is less than 1% save $\gamma_i^*$ as optimal phase offset, otherwise discard it.

A passive node will:

1. Transmit 4 times with identical phase offset $\gamma_i$. 
Multivariable equations

Solution

- Node estimates the quality of the function estimation itself
- Transmit with optimum phase offset and measure channel again
- When Expected fitness deviates significantly from measured fitness, discard altered phase offset
- Deviation:
  1 node: $\approx 0.6\%$
  2 nodes: $\approx 1.5\%$
  3 nodes: $> 3\%$
Multivariable equations
Synchronisation process

1. Transmit with phase offsets $\gamma_1 \neq \gamma_2 \neq \gamma_3$; measure feedback
2. Estimate feedback function and calculate $\gamma_i^*$
3. Transmit with $\gamma_4 = \gamma_i^*$
4. If deviation smaller 1% finished, otherwise discard $\gamma_i^*$
Asymptotic synchronisation time:

\[ O(n) \]

Classic approach:\(^1\)

\[ \Theta(n \cdot k \cdot \log(n)) \]

\(^1\)Sigg, El Masri and Beigl, A sharp asymptotic bound for feedback based closed-loop distributed adaptive beamforming in wireless sensor networks (submitted to Transactions on Mobile Computing)
Multivariable equations
Performance estimation

Median fitness values (Network size: 100 nodes)

- Multivariable equations, uniform probability to change the phase of a carrier signal: 0.01
- Normal distributed probability to change the phase of a carrier signal: 0.01, Variance: 0.5 π
Multivariable equations
Performance estimation

Relative phase shift (Network size: 100)

Phase shift [π]

Iterations

0 1000 2000 3000 4000 5000 6000
Phase offset of distinct nodes is within $+/- 0.05\pi$ for up to 99% of all nodes.
Multivariable equations
Performance estimation

- Asymptotically optimal synchronisation time
- Simulations: $\approx 12n$
- Further improvement:
  - 3 iterations per turn
  - Utilise last transmission from previous iteration
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Environmental changes

Introduction

- Velocity of nodes
- Multiple receiver nodes
- Increased population size
- Receive beamforming
Environmental changes
Velocity of nodes

Moving receiver:
- Straight line
- Random walk

Moving transmitter:
- Straight line
- Random walk
Environmental changes
Velocity of nodes

Moving receiver:
- Straight line
- Random walk

Aspects:
- Only one moving node
- Simple case
- Also applicable when all transmitters move identically
Environmental changes
Velocity of nodes

Moving transmit nodes:
- Straight line
- Random walk

Aspects:
- Multiple nodes moving
- Hard case
Environmental changes

Velocity of nodes

Phase offset deviation from the optimal phase offset (Number of nodes: 100)

Random walk – receiver :

- Maximum velocity for classic algorithm: 5m/sec
Environmental changes

Velocity of nodes

Random walk – receiver:

- Max. velocity for Multivariable equations: 5m/sec easily supported
Environmental changes

Velocity of nodes

Random walk – transmitter:

- Maximum velocity for classic algorithm: 2m/sec
Environmental changes

Velocity of nodes

Random walk – transmitter :

- Max. velocity for Multivariable equations: 5m/sec supported
Environmental changes
Velocity of nodes

- Maximum velocity for classic algorithm: 30m/sec
- Regardless if transmitter or receiver move
Environmental changes
Velocity of nodes

- Maximum velocity for Multivariable equations algorithm: 60m/sec
- Regardless if transmitter or receiver move
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Environmental changes
Multiple receiver nodes

Runde 1:
Erstes Netzwerk optimiert sein Trägersignal, zweites Netzwerk gibt Feedback.

(Summensignal)

Phasenanpassung wie beim Szenario mit digitalem Feedback.
Feedback wird durch Vorhandensein von einem Signal gegeben.

(Einzelsignal)

Runde 2:
Zweites Netzwerk optimiert sein Trägersignal, erstes Netzwerk gibt Feedback.

(Summensignal)
## Environmental changes

### Multiple receiver nodes

#### 3m

<table>
<thead>
<tr>
<th>Knoten</th>
<th>( n0 )</th>
<th>( n1 )</th>
<th>( n2 )</th>
<th>( m0 )</th>
<th>( m1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain zur Anfangsamplitude (Median) [dB]</td>
<td>0,96</td>
<td>2,39</td>
<td>1,40</td>
<td>1,46</td>
<td>1,10</td>
</tr>
<tr>
<td>Gain zu einem Knoten (Median) [dB]</td>
<td>2,33</td>
<td>2,32</td>
<td>2,37</td>
<td>3,50</td>
<td>4,05</td>
</tr>
<tr>
<td>Anzahl letztes Feedback</td>
<td>5/11</td>
<td>3/11</td>
<td>3/11</td>
<td>8/11</td>
<td>7/11</td>
</tr>
<tr>
<td>Amplitude nach Synchronisation [%]</td>
<td>92,4</td>
<td>51,4</td>
<td>65,3</td>
<td>91,0</td>
<td>90,7</td>
</tr>
</tbody>
</table>

#### 12m

<table>
<thead>
<tr>
<th>Knoten</th>
<th>( n0 )</th>
<th>( n1 )</th>
<th>( n2 )</th>
<th>( m0 )</th>
<th>( m1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain zur Anfangsamplitude (Median) [dB]</td>
<td>1,24</td>
<td>0,63</td>
<td>1,39</td>
<td>2,06</td>
<td>1,47</td>
</tr>
<tr>
<td>Gain zu einem Knoten (Median) [dB]</td>
<td>2,53</td>
<td>1,09</td>
<td>2,00</td>
<td>2,74</td>
<td>4,18</td>
</tr>
<tr>
<td>Anzahl letztes Feedback</td>
<td>2/10</td>
<td>4/10</td>
<td>4/10</td>
<td>5/10</td>
<td>5/10</td>
</tr>
<tr>
<td>Amplitude nach Synchronisation [%]</td>
<td>57,1</td>
<td>92,0</td>
<td>86,5</td>
<td>86,4</td>
<td>86,6</td>
</tr>
</tbody>
</table>

#### 24m

<table>
<thead>
<tr>
<th>Knoten</th>
<th>( n0 )</th>
<th>( n1 )</th>
<th>( n2 )</th>
<th>( m0 )</th>
<th>( m1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain zur Anfangsamplitude (Median) [dB]</td>
<td>1,12</td>
<td>2,33</td>
<td>2,76</td>
<td>3,61</td>
<td>1,67</td>
</tr>
<tr>
<td>Gain zu einem Knoten (Median) [dB]</td>
<td>1,2</td>
<td>2,54</td>
<td>2,03</td>
<td>5,15</td>
<td>3,76</td>
</tr>
<tr>
<td>Anzahl letztes Feedback</td>
<td>4/5</td>
<td>0/5</td>
<td>1/5</td>
<td>4/5</td>
<td>3/5</td>
</tr>
<tr>
<td>Amplitude nach Synchronisation [%]</td>
<td>94,2</td>
<td>80,0</td>
<td>61,4</td>
<td>95,8</td>
<td>97,9</td>
</tr>
</tbody>
</table>
Environmental changes

Multiple receiver nodes

3m
Environmental changes
Multiple receiver nodes

12m
Environmental changes
Multiple receiver nodes

Multiple receiver nodes – issues:
- Only binary feedback value
  - Therefore only classic optimisation approach
- Distance between transmit and receive nodes relative to spatial diversity of nodes in one network
  - Better synchronisation when nodes in one network in spatial proximity
  - When nodes in one network communicate: No issue
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Environmental changes
Increased population size

Increased population size – Discussion:

How to achieve population size greater than one?
- Separate transmit times
- WCDMA
- Distinct frequencies simultaneously

Only separate transmit times feasible for WSN
More time for each iteration

- Initial solution: Random search
- Not clear if performance improvement possible by crossover
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Receive beamforming – Discussion:

- Transmit node transmits only once
- Receiver nodes combine received signal fragments in the network
- Tradeoff:
  - Transmission power for in-network communication
  - Transmission over several iterations with receiver node
- More complex computation of transmit nodes
Questions?

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