5.4.3 Rekursionen allgemeiner:

(1) Beispiel MERGESORT:

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \Theta(n)$$

Spezialfall von

$$T(n) = \sum_{i=1}^{m} T(\alpha_i \cdot n) + \Theta(n^k)$$

$$\left(\text{mit} \quad m = 2, \quad \alpha_1 = \alpha_2 = \frac{1}{2}, \quad k = 1\right)$$

→ frühere Terme kommen linear vor!

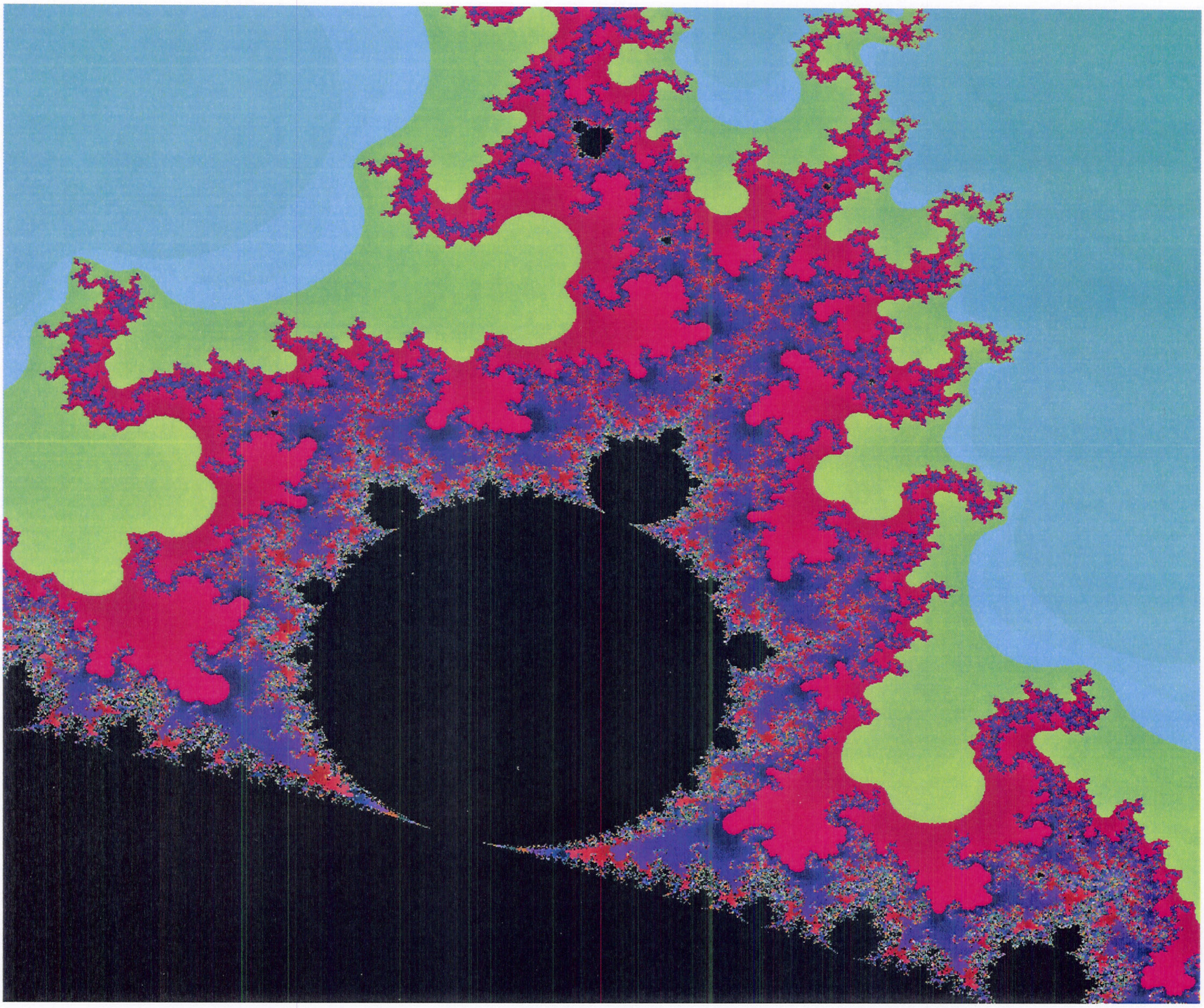
↳ gleich mehr dazu!

(2) Anderes Beispiel: Quadratisch!

$$f(n+1) = f(n)^2 + c \qquad , \quad f(0) = 0$$

Wie kompliziert wird das ?!

↳ siehe nächste Folie!



Ausschnitt aus der Mandelbrot-Menge

Erklärung:

- c ist komplex! ~~ebenso wie~~
- Darstellung in der komplexen Zahlenebene:
  - Punkt ist schwarz, wenn Zahlenfolge nicht beliebig groß wird
  - Sonst Farbe nach Wachstumsverhalten

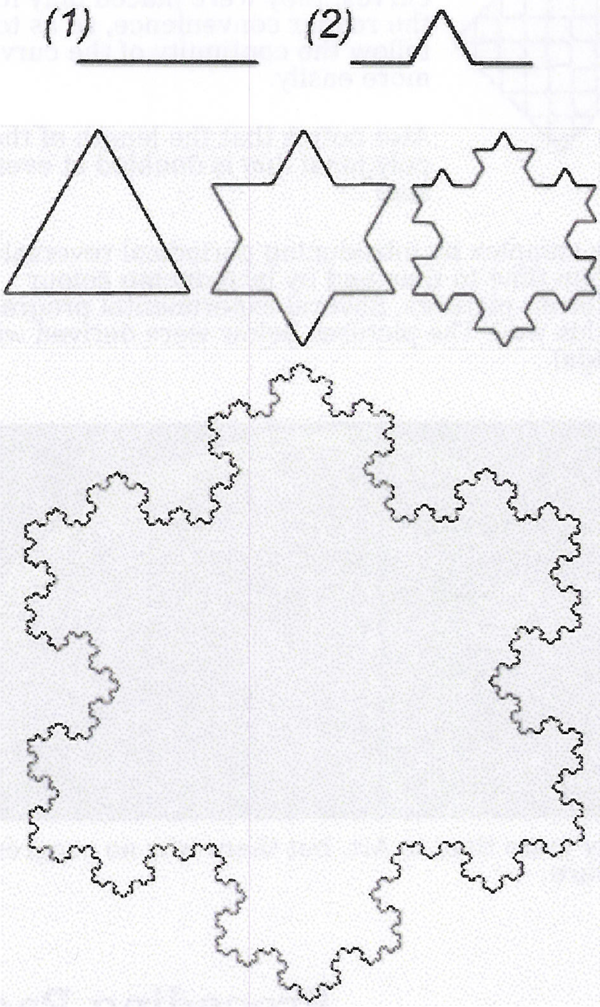
Ergebnisse:

- Schwarze Menge ist sehr komplex aufgebaut!
  ↳ „Selbstähnlich", „Fraktal"
- Musterbeispiel für „chaotisches" Verhalten dynamischer Systeme
  (→ Kleine Unterschiede bewirken riesige Unterschiede!)
  ↑ zunächst ↑ später

- Fraktale entstehen durch rekursive Anwendung einfacher Iterationsregeln, ~~und führen~~ die zu komplizierten, selbstähnlichen Strukturen führen können!

Von Koch Curves

Preceding Page



The genuine Von Koch curve, also called snowflake curve, is derived as the limit of a polygonal contour. At every step, as shown on the left, the middle third of every side of the polygone is replaced with two linear segments at angles 60° and 120°.

Starting from an equilateral triangle, the two first steps lead to the star-like curves plotted on the left. If one goes on long enough one finally gets the curve right below. Ideally the process should go on indefinitely, but, in practice, the curve displayed on the screen no longer changes when the elementary side becomes less than the pitch, and then the iterations can be stopped.

What is thus obtained was long considered a mathematical monster, a curve plotted in a bounded domain, but with an infinite length (one easily sees that the length is multiplied by 4/3 at every step), continuous but nowhere differentiable (i.e. nowhere a tangent can be defined). It is now regarded as an elementary example of fractal –"elementary" because of the simplicity of the construction.

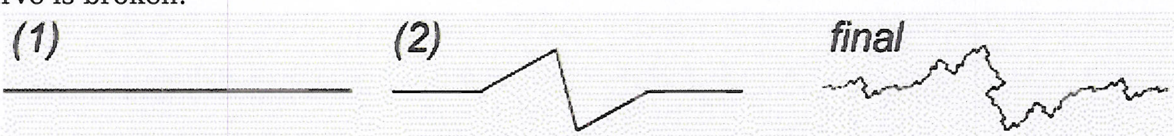
The pattern  can be seen everywhere along the curve, at every scale, from visible to infinitesimal. This feature is called self-similarity.

For a while, it was thought that such a curve was a possible starting point for the design of tools for drawing complex natural curves (such as the rocky coast of the celebrated example of Benoît Mandelbrot), with a little number of control parameters. Here, the parameters are just the 6 relative coordinates of the indentation points and the result looks really complex when one goes out from the field of classical geometry and its perfectly smooth curves.

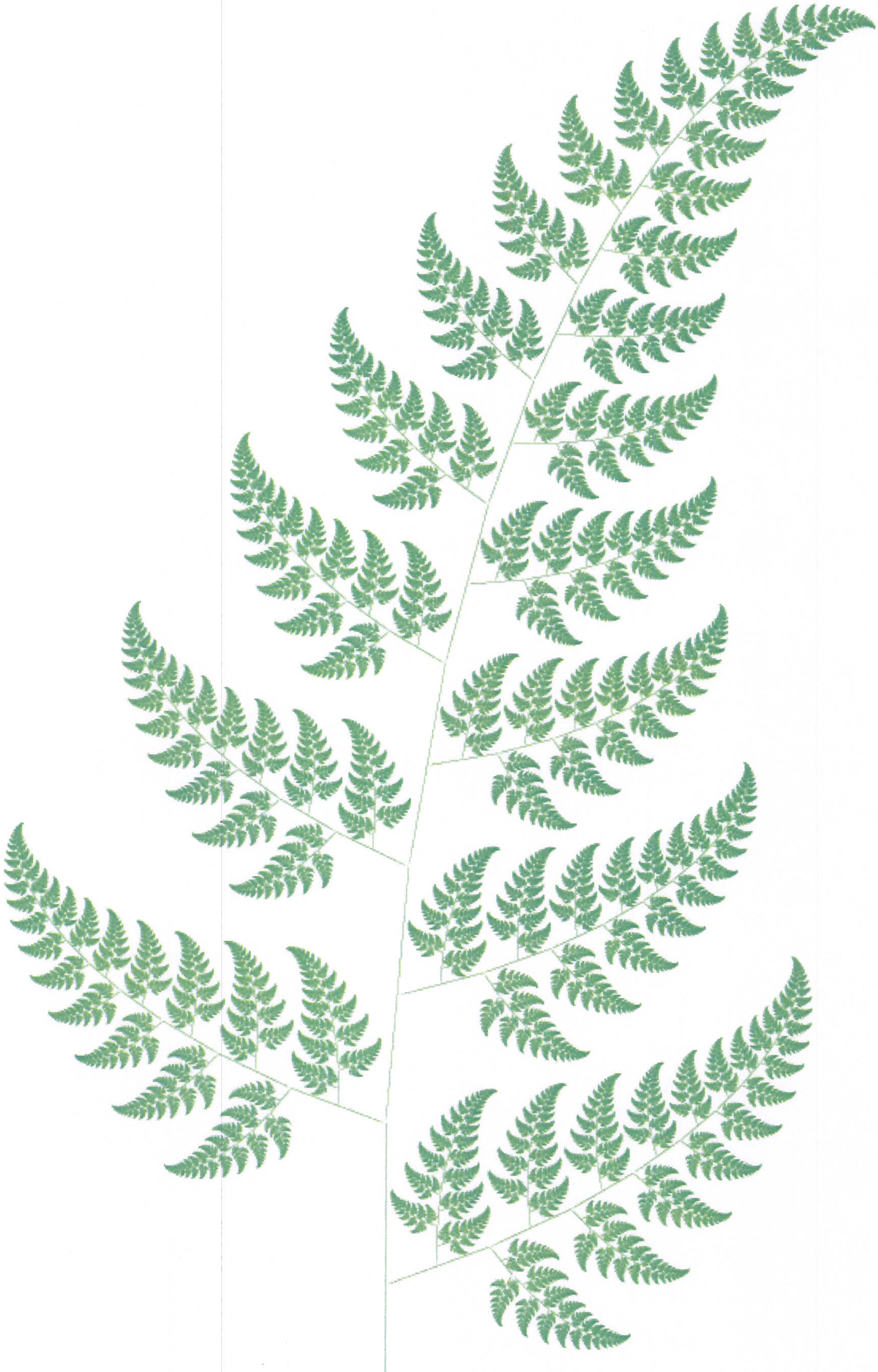
Of course, the Von Koch curve does not look like a "natural" curve. It is too regular in its irregularity to simulate the randomness of a rocky coast. This regularity comes from the strictness of its construction process. It can be loosened by introducing random fluctuations (for instance random moves of the indentation points) but, according to Mandelbrot, one gets better simulations tools with other recipes. .

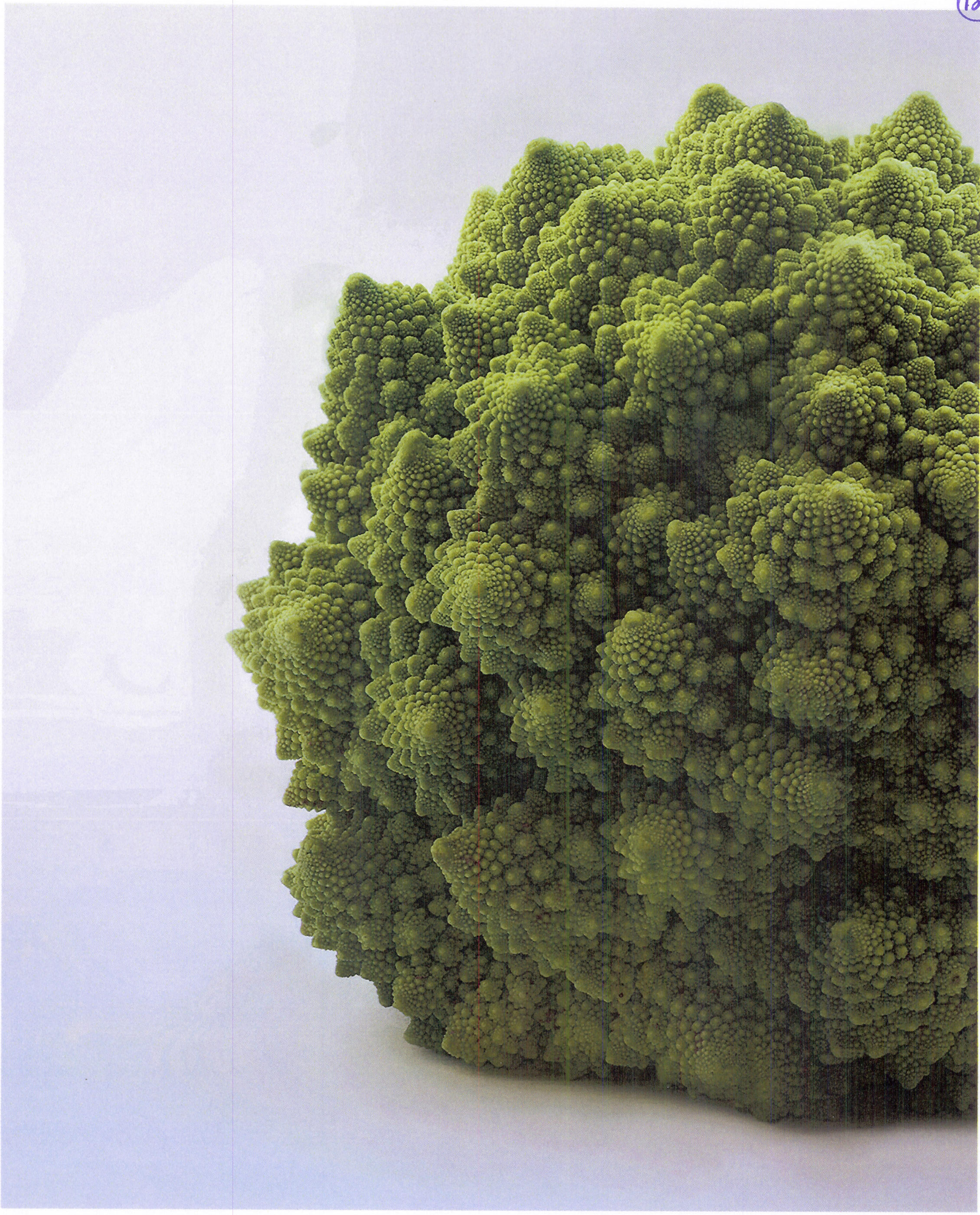
Generalized Von Koch curves

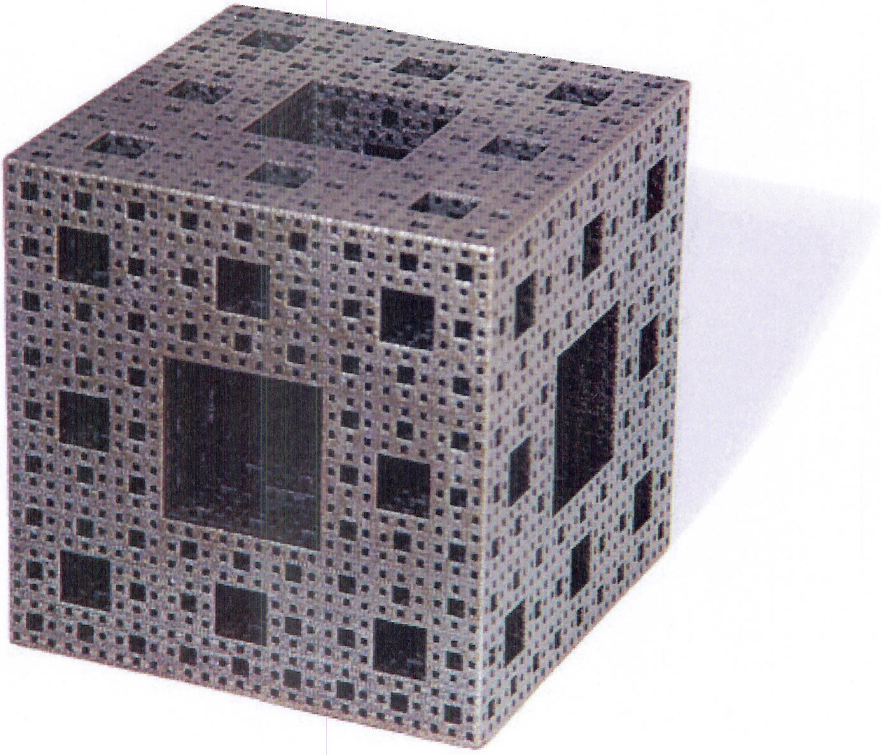
The final curve can be strongly modified when the iteration process is changed. Mandelbrot gives the following example, where the symmetry of the original Von Koch curve is broken:

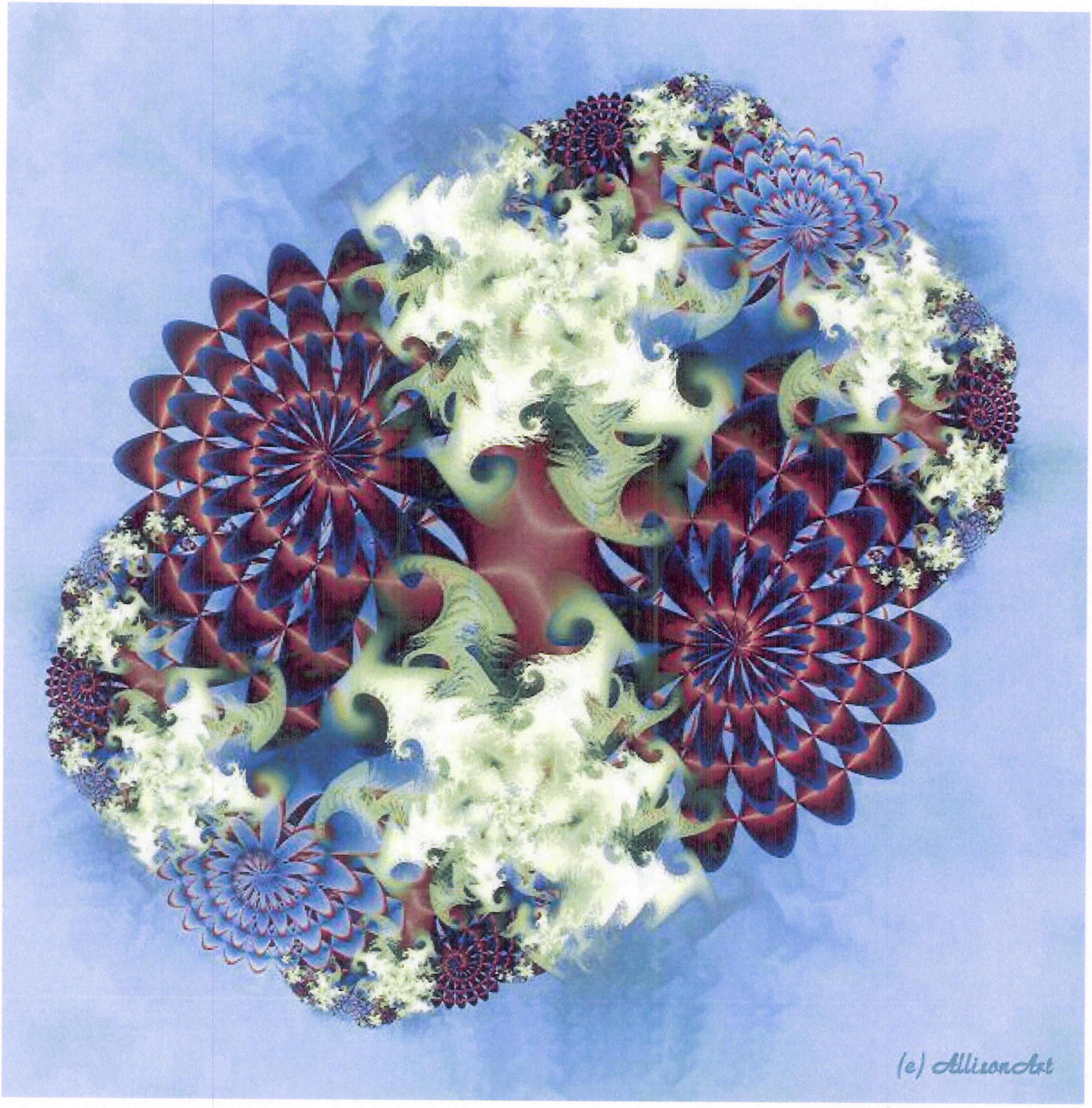


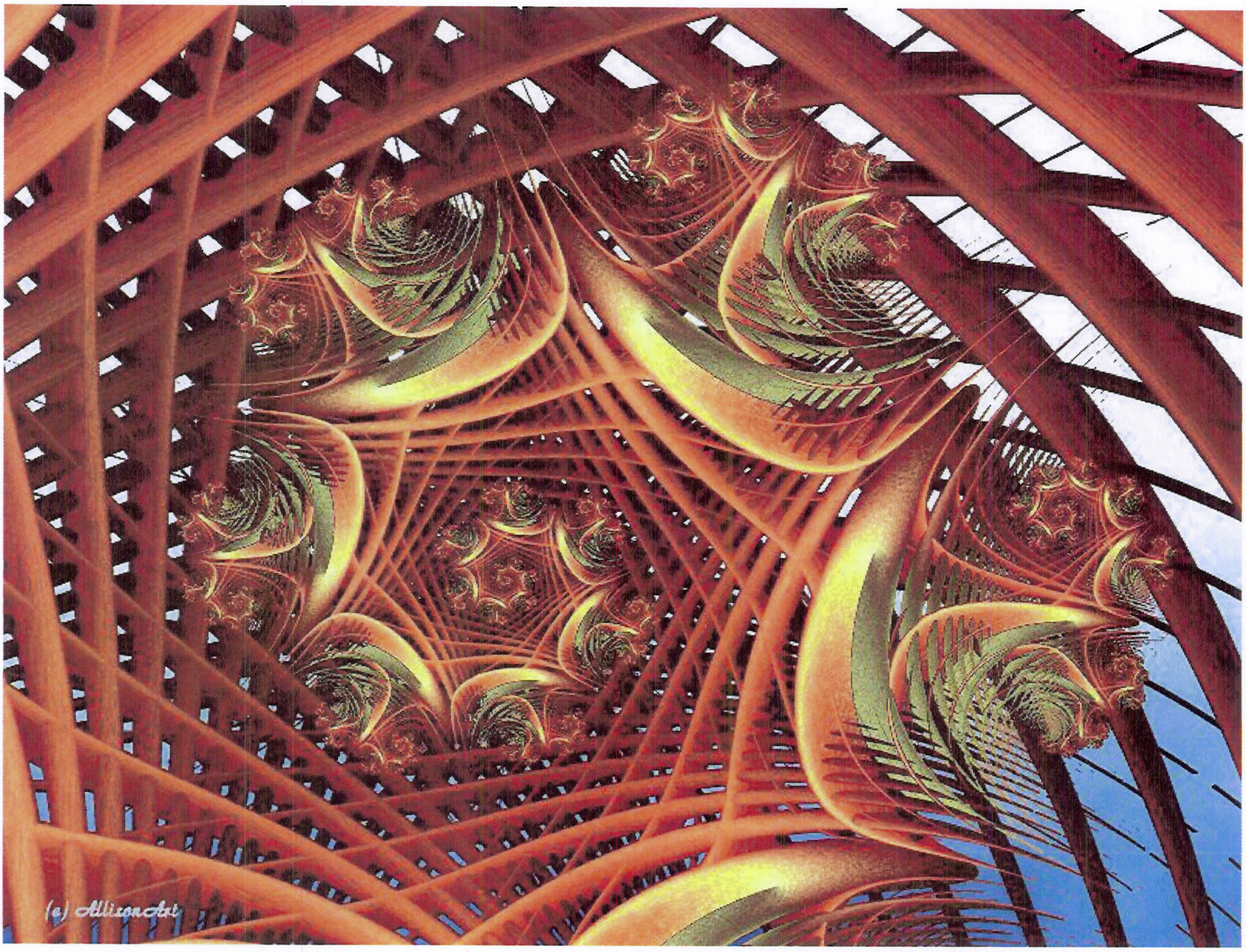
But one can think of quite more dramatic changes. The following pattern leads to the famous Peano curve, the continuous curve which completely fills a square. Only the three








(c) AllisonArt


(c) AllisonArt