Approximation Algorithms
Chapter 6: Review and Relay Placement

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Department of Computer Science
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1. Introduction
2. Review
3. Extra Packing: Dispersion
4. Extra Tours: Lawn Mowing
5. Relay Placement
6. Coordinated Motion Planning

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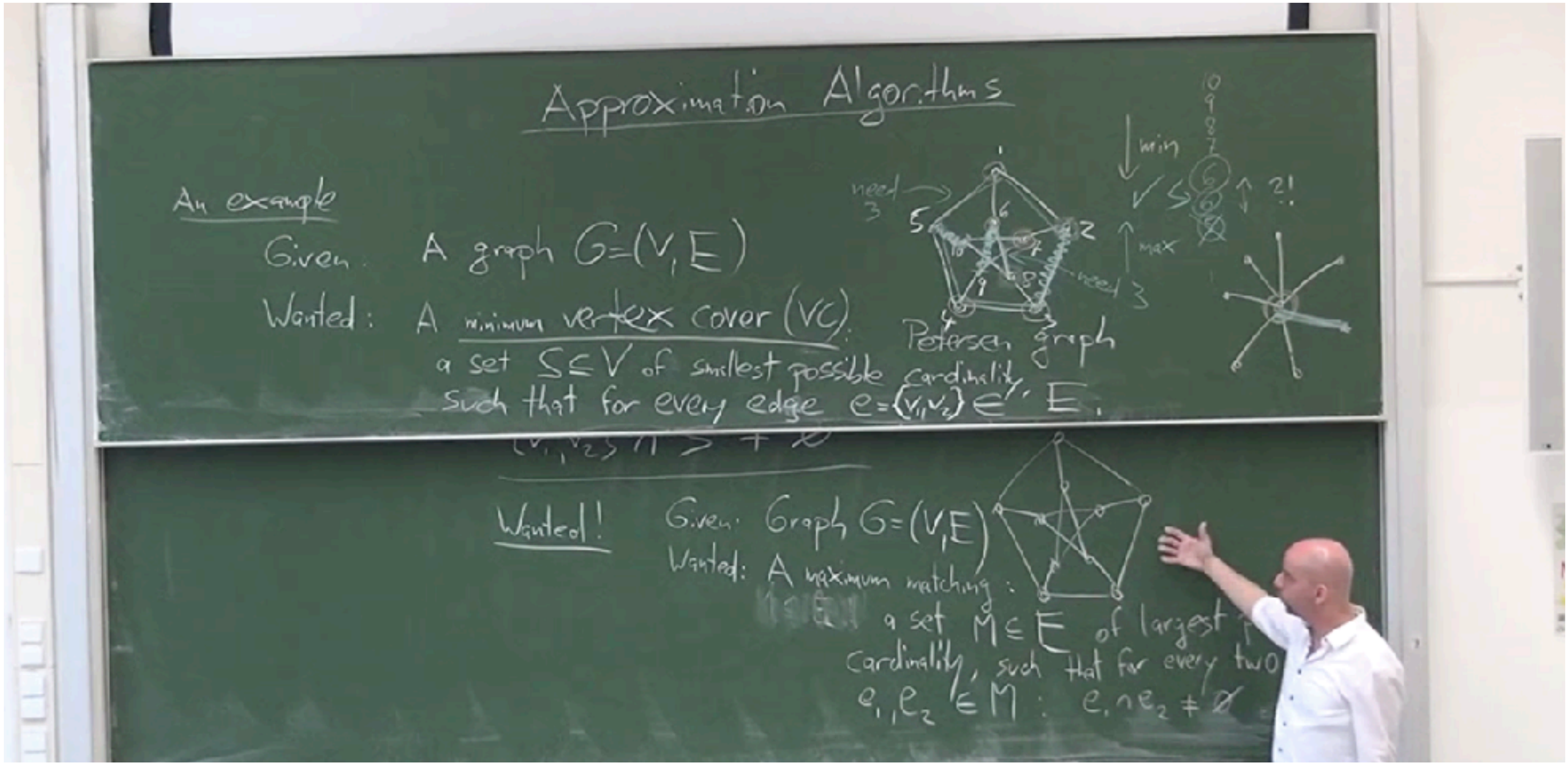
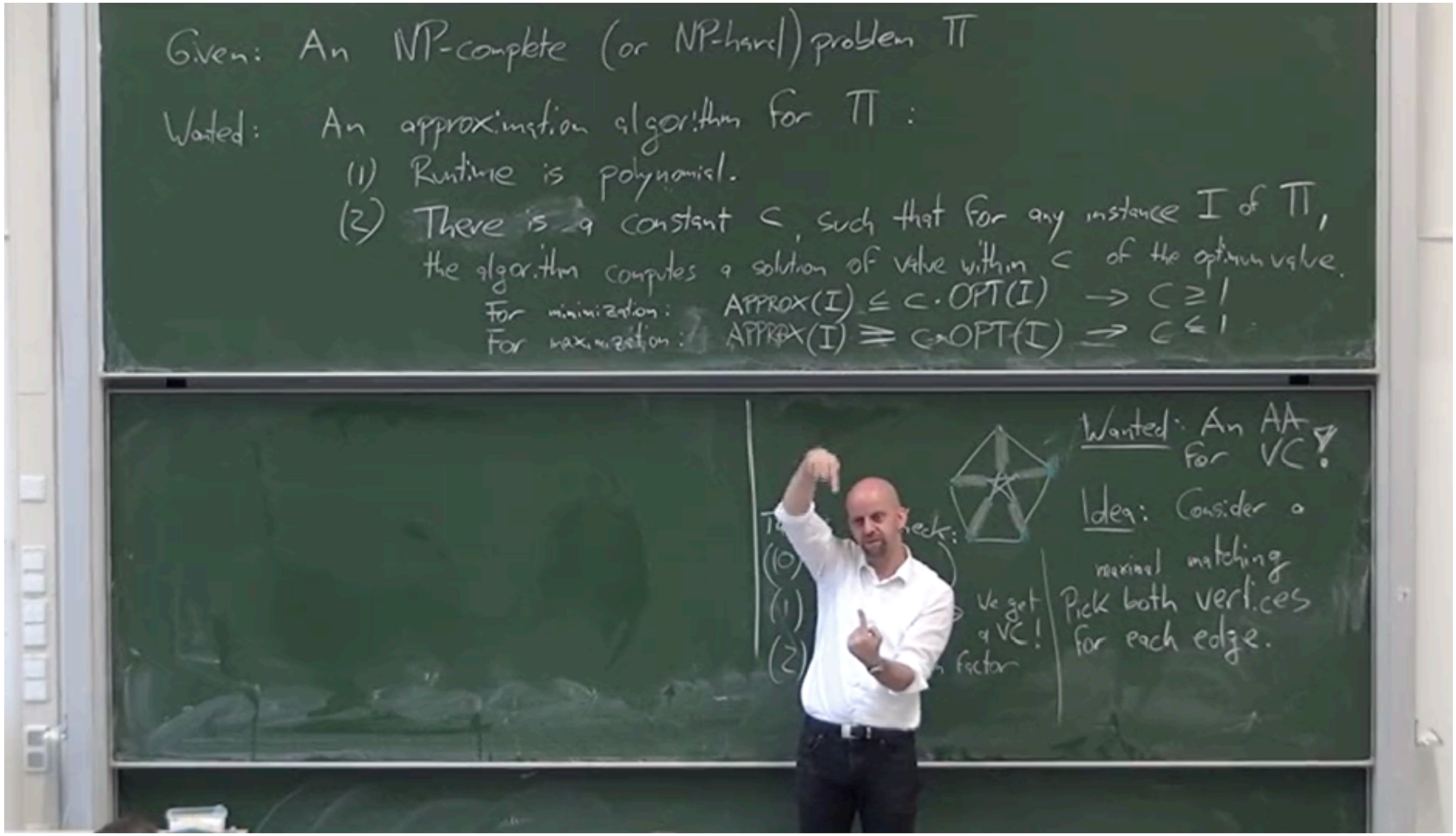
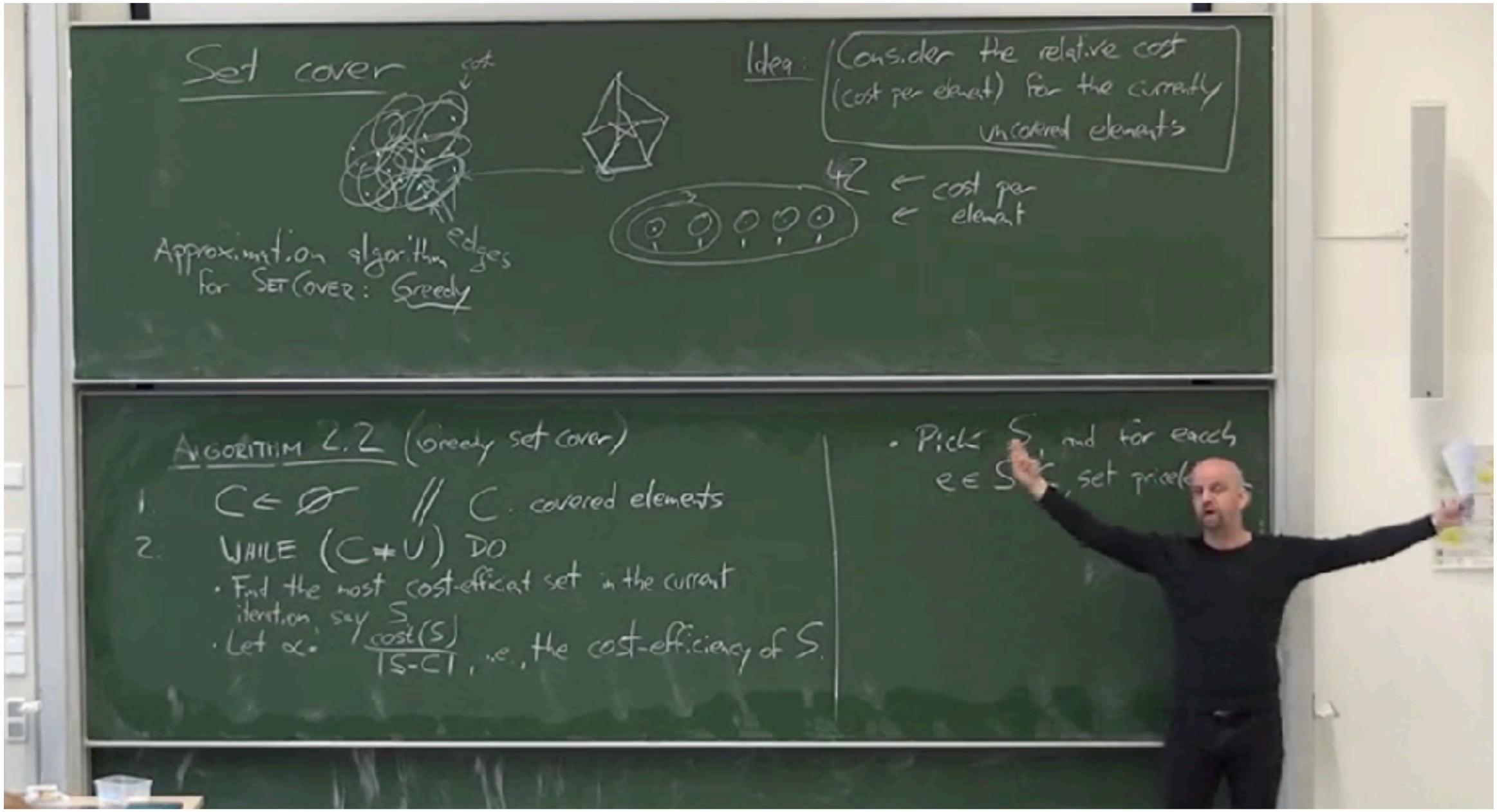
Review: Matching & Vertex Cover



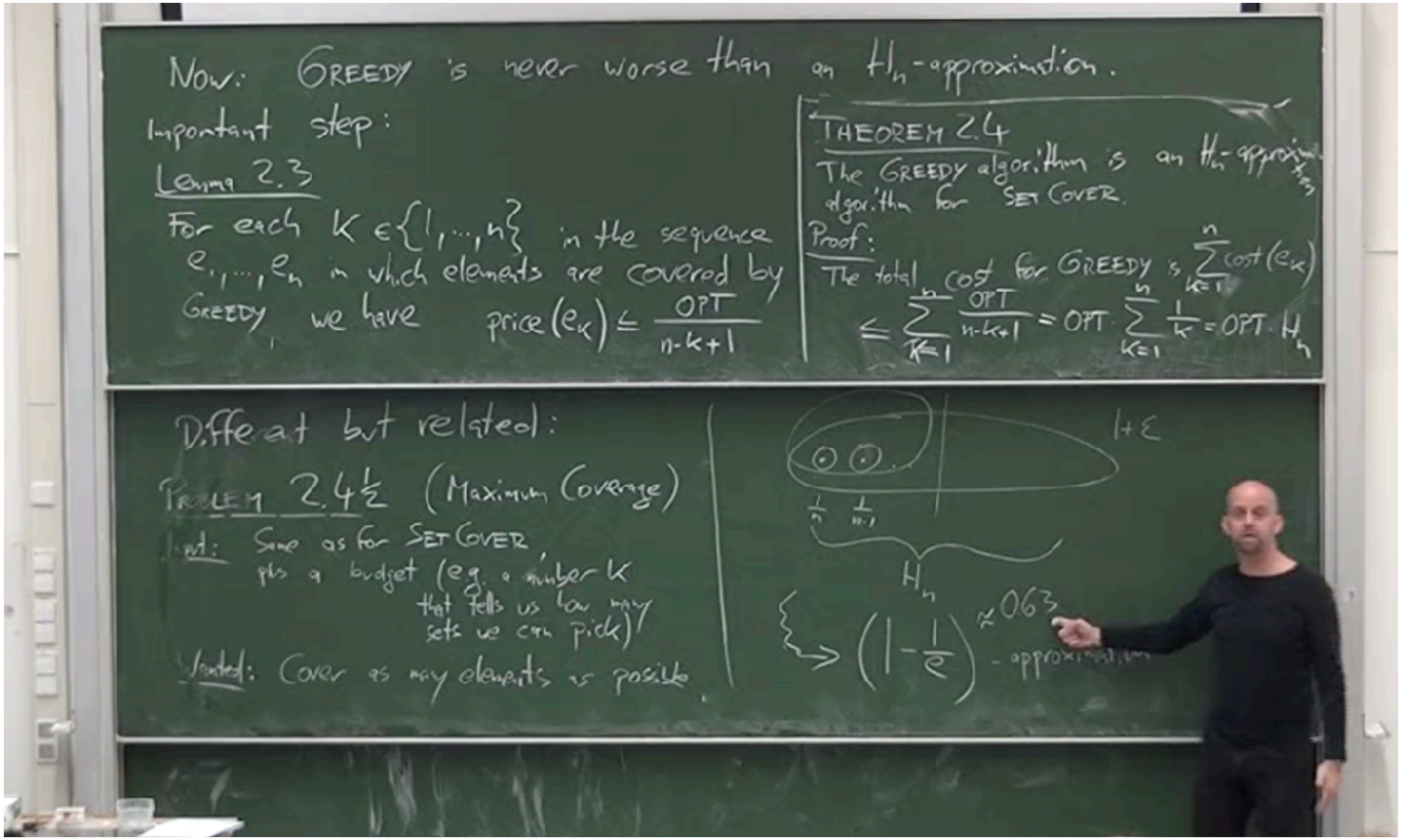
Review: Approximation Algorithms



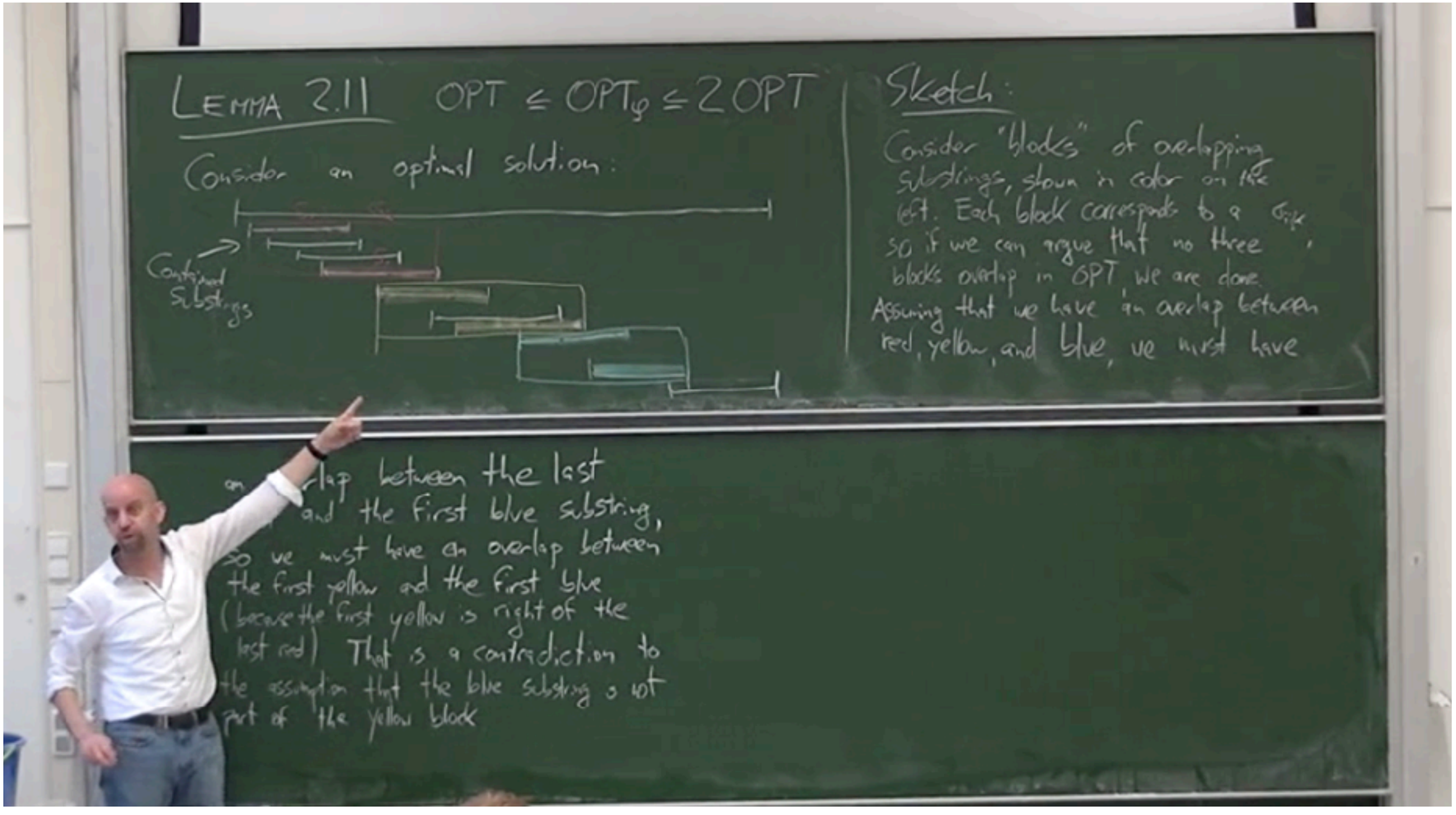
Review: Set Cover



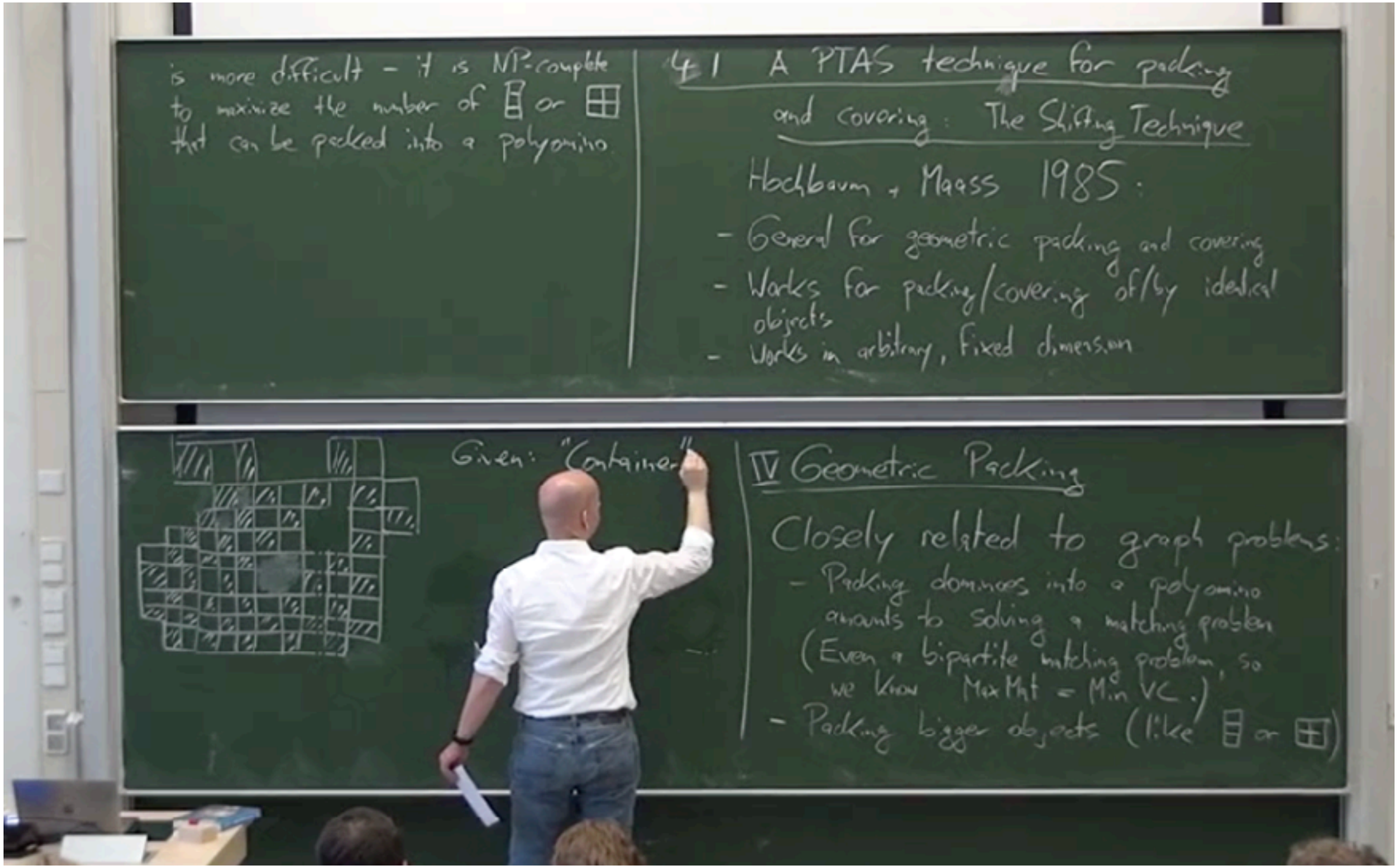
Review: Maximum Coverage



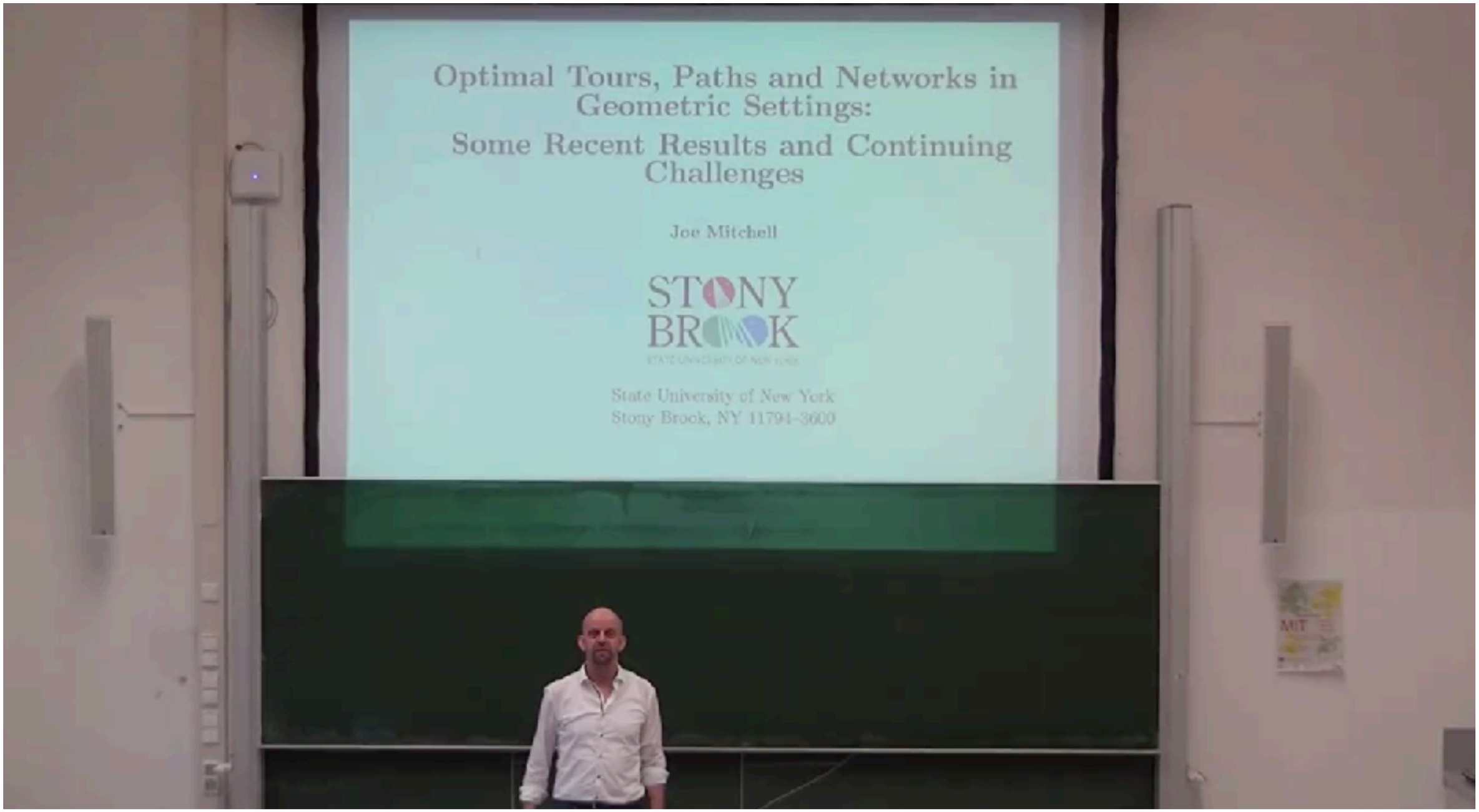
Review: Shortest Superstring



Review: Packing



Review: Tour Problems



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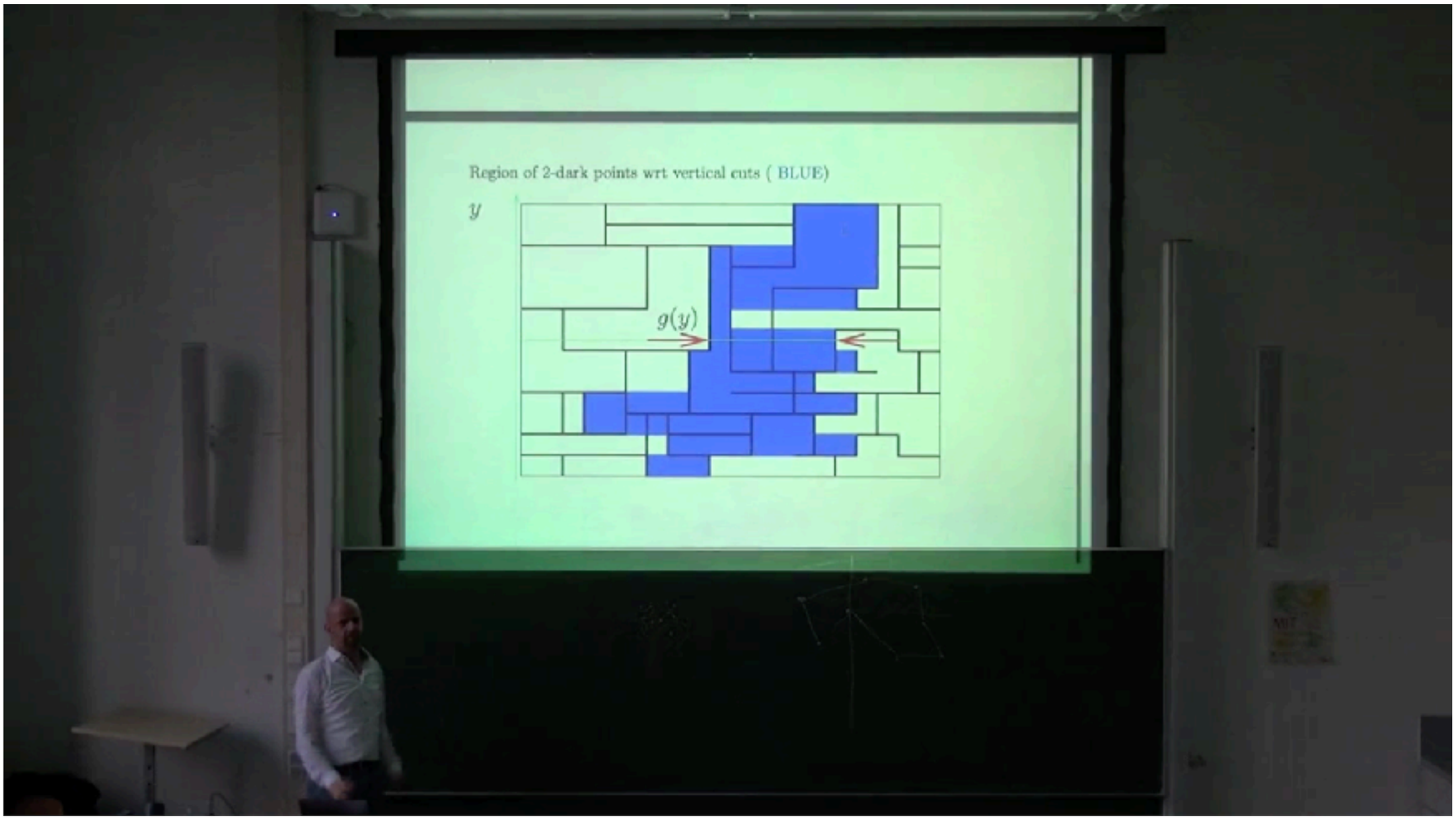
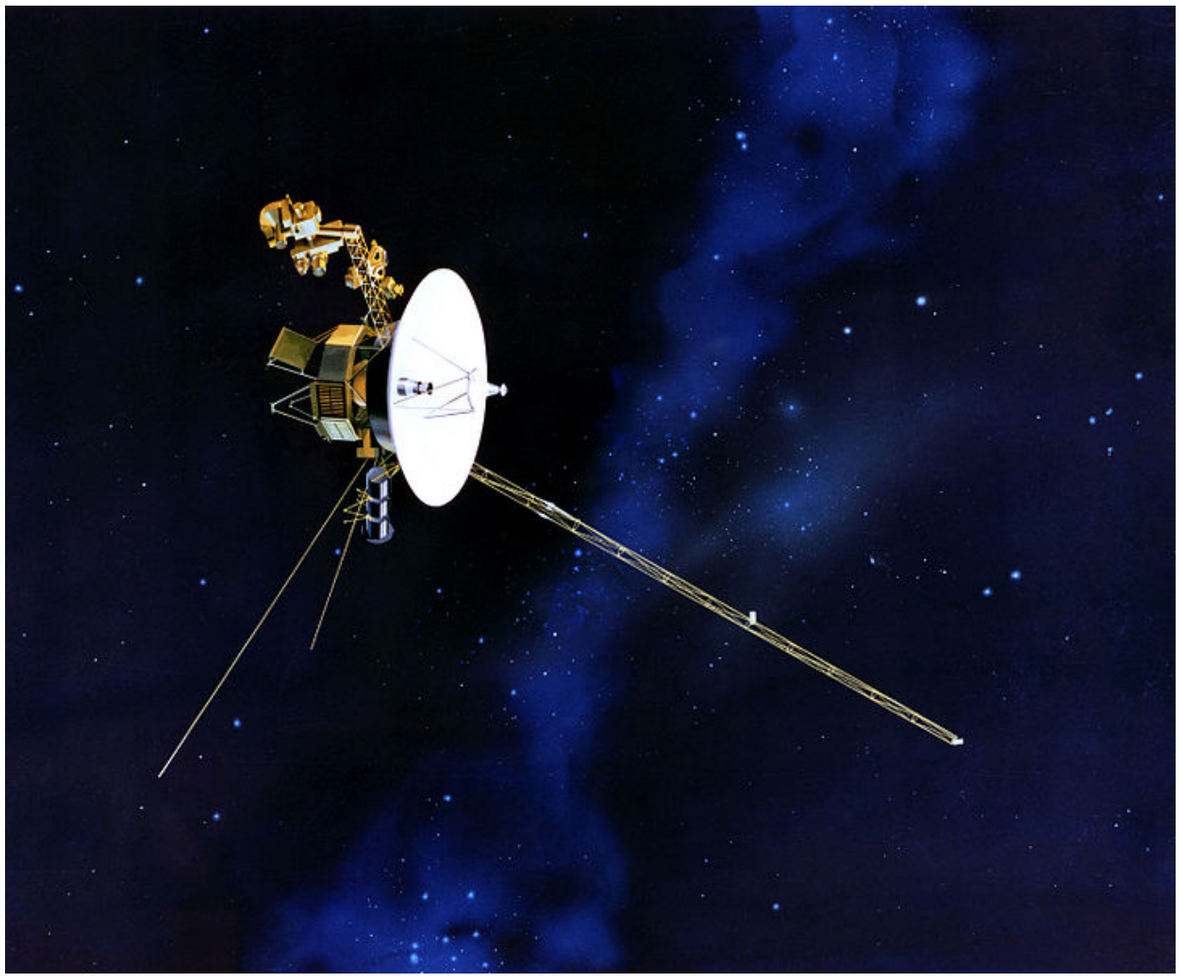


The Freeze-Tag Problem: How to Wake Up a Swarm of Robots

Estie Arkin[1]
Michael Bender[1]
Sándor Fekete[2]
Joe Mitchell[1]
Martin Skutella[3]

[1] University at Stony Brook
[2] TU Braunschweig
[3] TU Berlin

Motivation



- Highly focused antennas.
- Expensive rotations.

How can we quickly distribute information from one satellite to all others?

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Institute of Operating Systems and Computer Networks

Technische Universität Braunschweig

Student Presentation

Minimum Scan Cover with Angular Transition Costs

Sándor Fekete

Linda Kleist

Dominik Krupke

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Dispersion (1)

Algorithmica (2001) 30: 451–470
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Algorithmica

Approximation of Geometric Dispersion Problems[1]

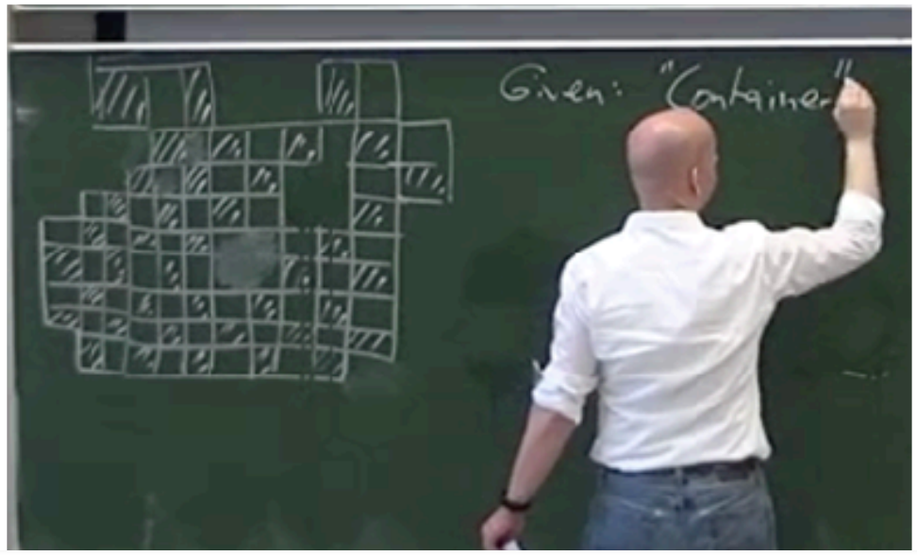
C. Baur[2] and S. P. Fekete[3]

Abstract. We consider problems of distributing a number of points within a polygonal region P, such that the points are "far away" from each other. Problems of this type have been considered before for the case where the possible locations form a discrete set. Dispersion problems are closely related to packing problems. While Hochbaum and Maass [20] have given a polynomial-time approximation scheme for packing, we show that geometric dispersion problems cannot be approximated arbitrarily well in polynomial time, unless P = NP. A special case of this observation solves an open problem by Rosenkrantz et al. [31]. We give a $\frac{2}{3}$ approximation algorithm for one version of the geometric dispersion problem. This algorithm is strongly polynomial in the size of the input, i.e., its running time does not depend on the area of P. We also discuss extensions and open problems.

Key Words. Packing, Dispersion, Location problems, Geometric optimization, Bounds on approximation factors.

Dispersion (2)

PACK(k, L)
Input: A polygonal region P with n vertices, a parameter k, a parameter L.
Question: Can k many L-squares be packed into P?



$\max_k$PACK(L)
Input: A polygonal region P with n vertices.
Task: Pack k many L-squares into P, such that k is as big as possible.

$\max_L$PACK(k)
Input: A polygonal region P with n vertices.
Task: Pack k many $L \times L$ squares into P, such that L is as big as possible.

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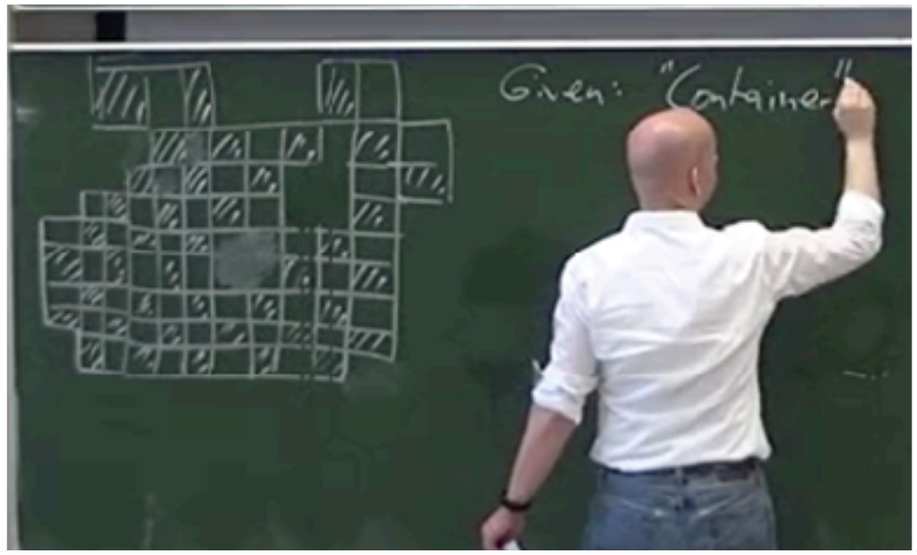

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Dispersion (3)

THEOREM 1. *Unless $P = NP$, there is no polynomial algorithm that finds a solution within more than $\frac{13}{14}$ of the optimum for rectilinear geometric dispersion with boundaries.*

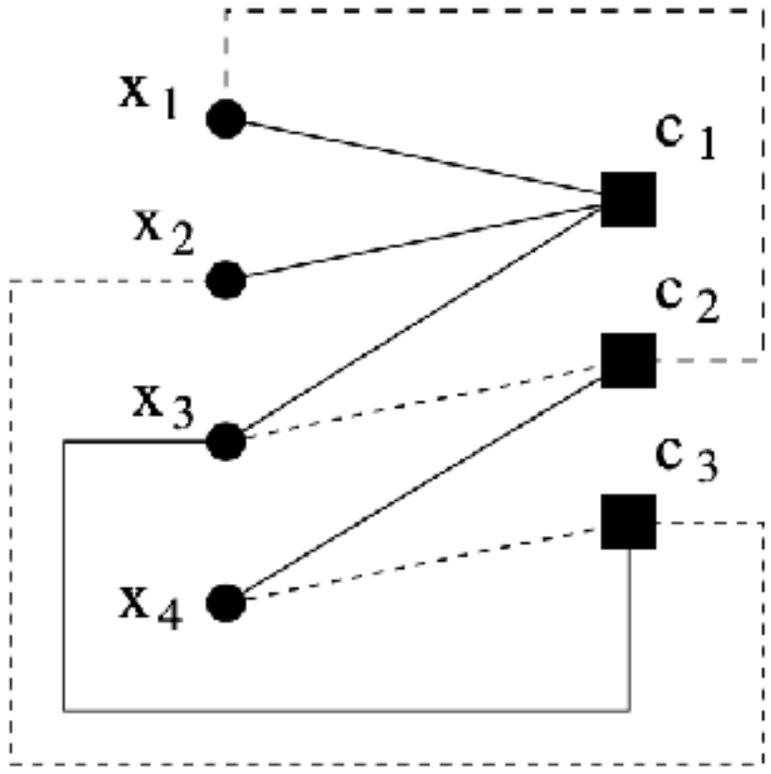

Fig. 1. The graph G_I representing the PLANAR 3SAT instance $(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_3 \vee x_4) \wedge (\bar{x}_2 \vee x_3 \vee \bar{x}_4)$. Edges are distinguished according to the logical parity of the corresponding literals.

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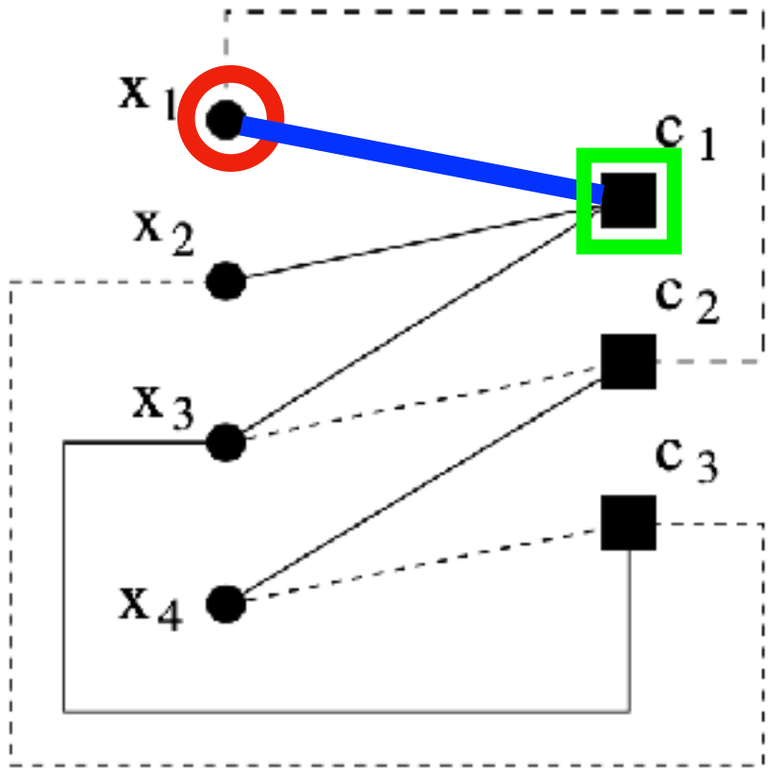

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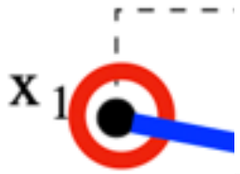
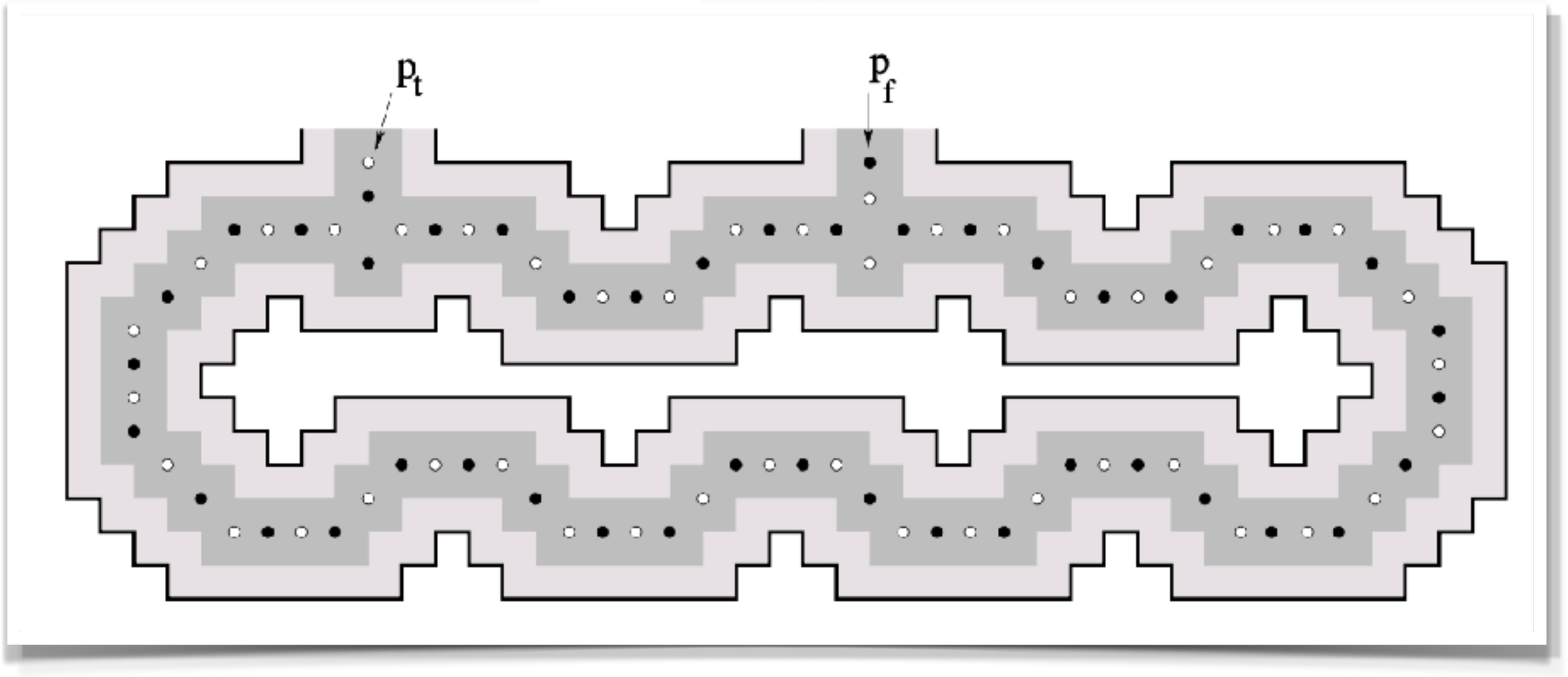
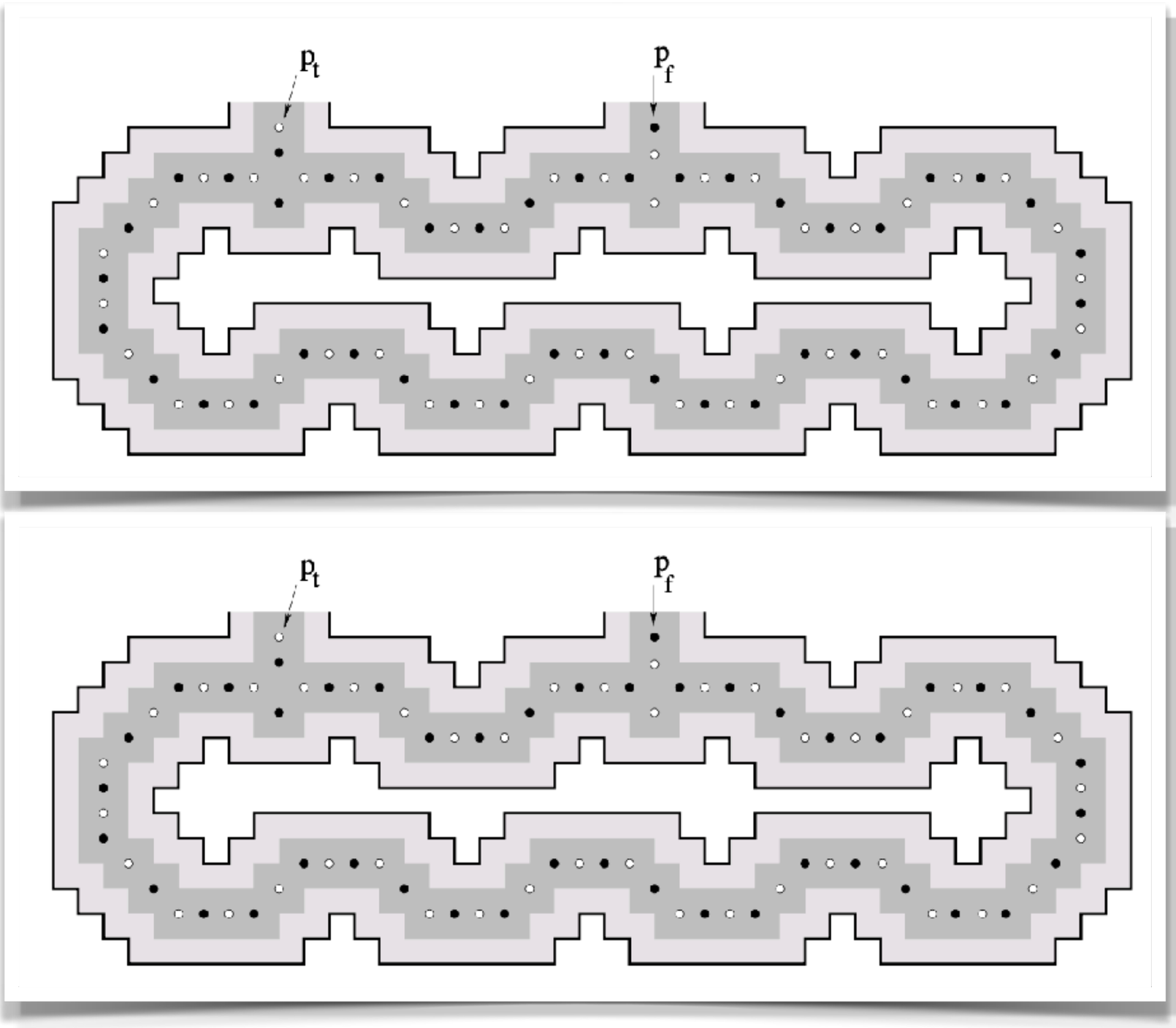
Variables (1)

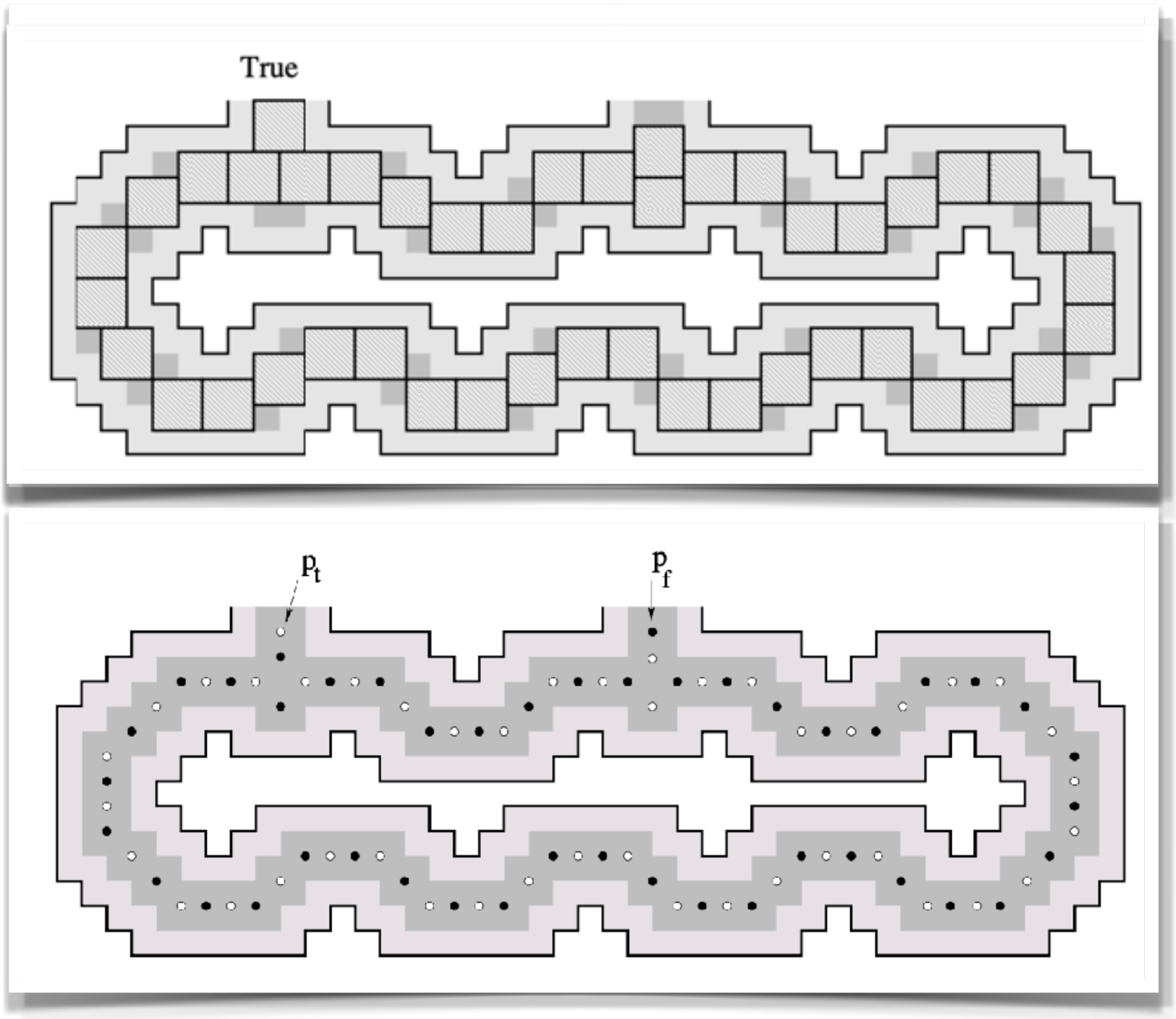




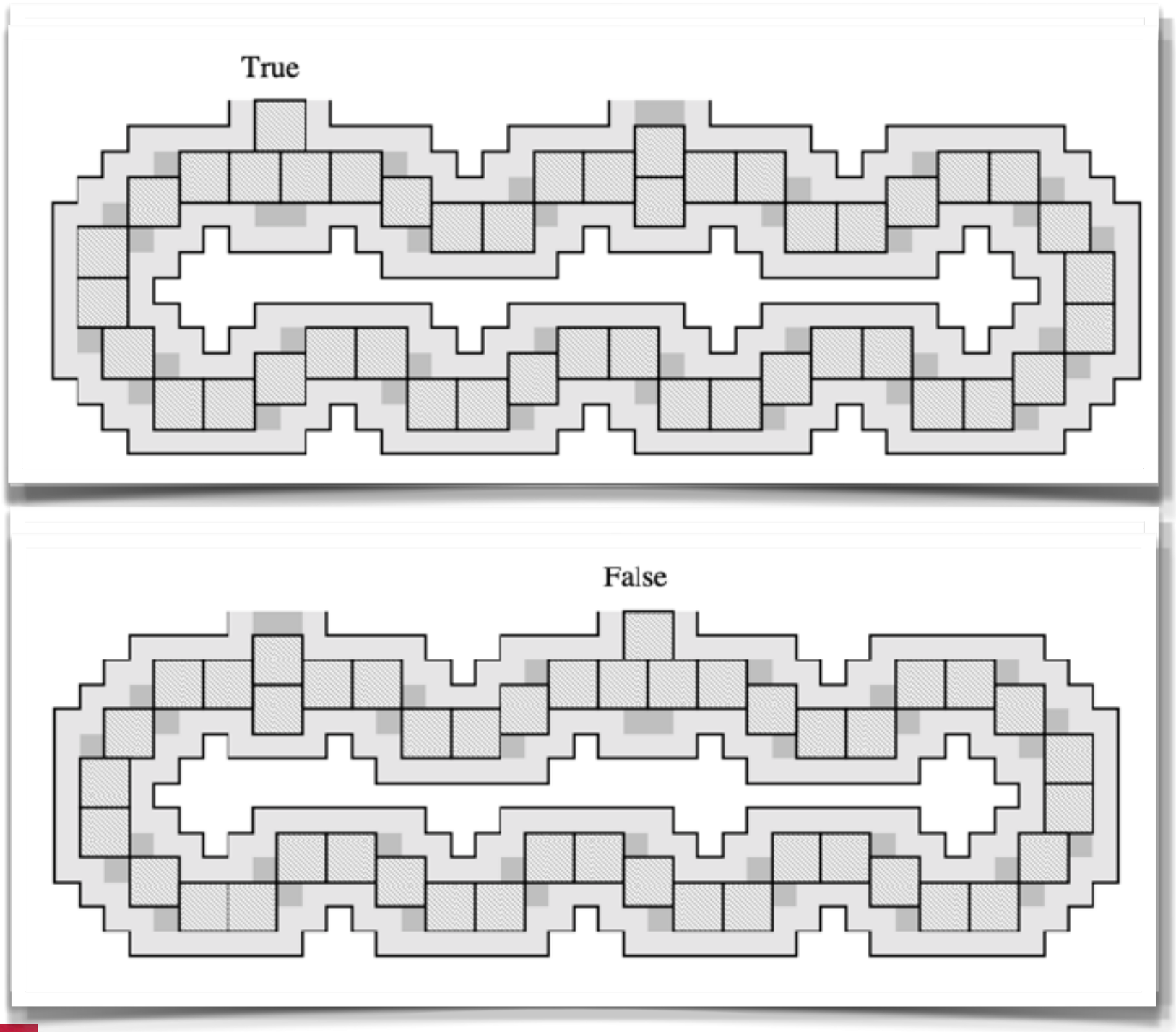
Variables (2)



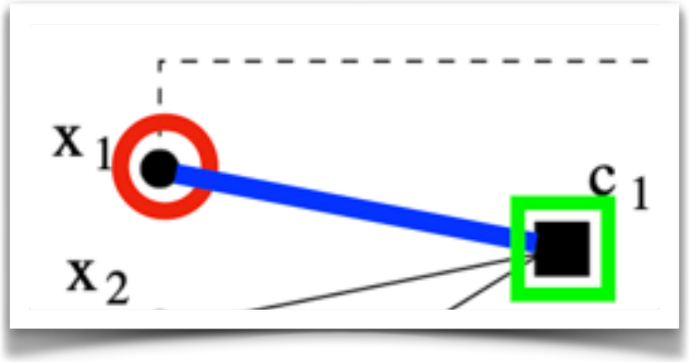
Variables (2)



Variables (2)



Connectors



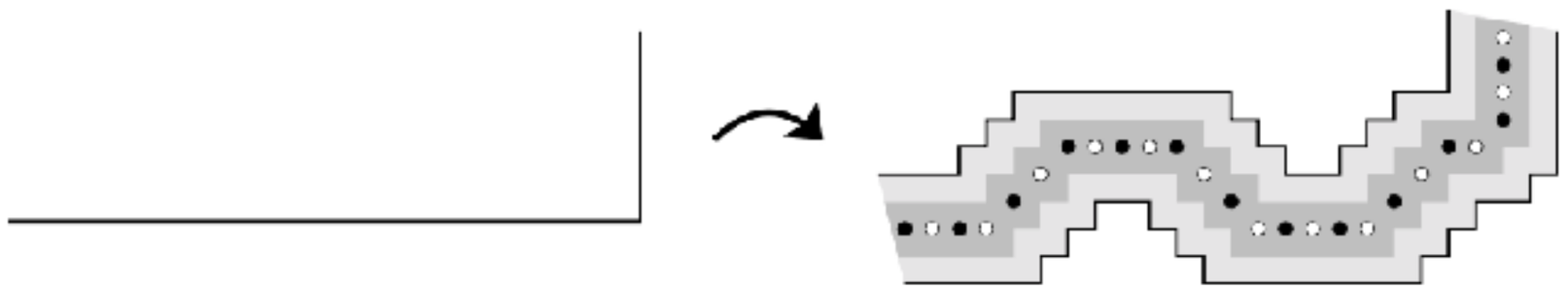
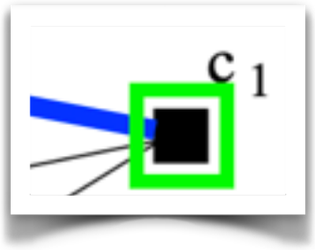


Fig. 3. Part of a connector component (right) for representing an edge between variable nodes and clause nodes (left).

Clauses



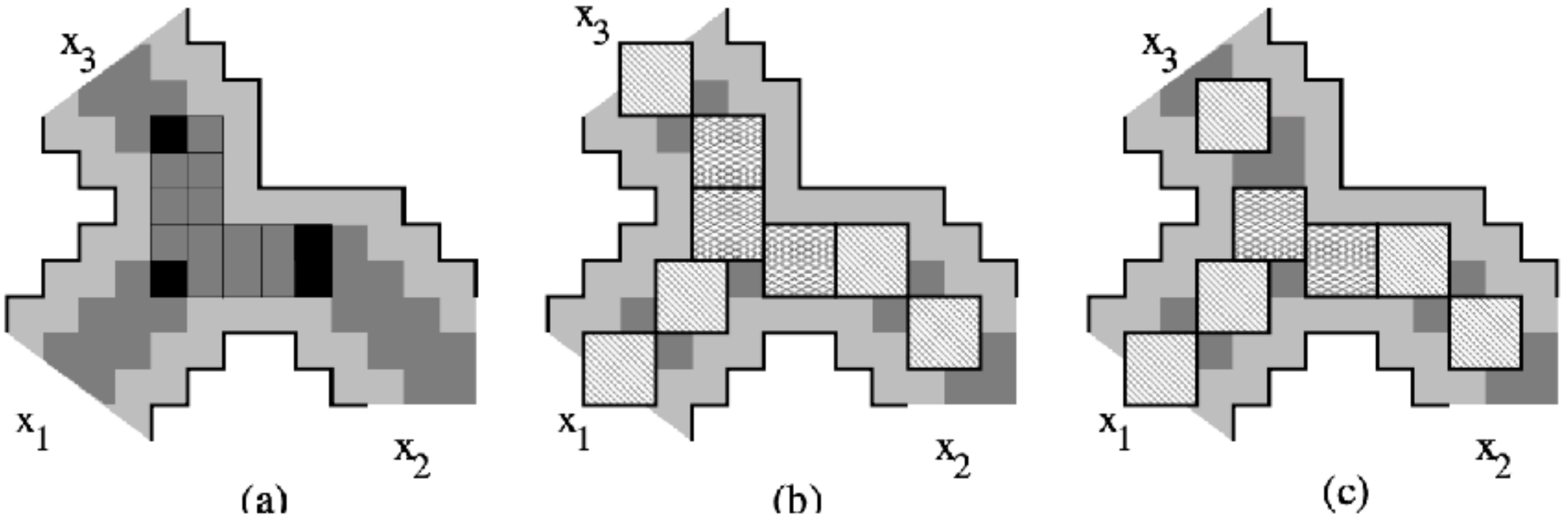
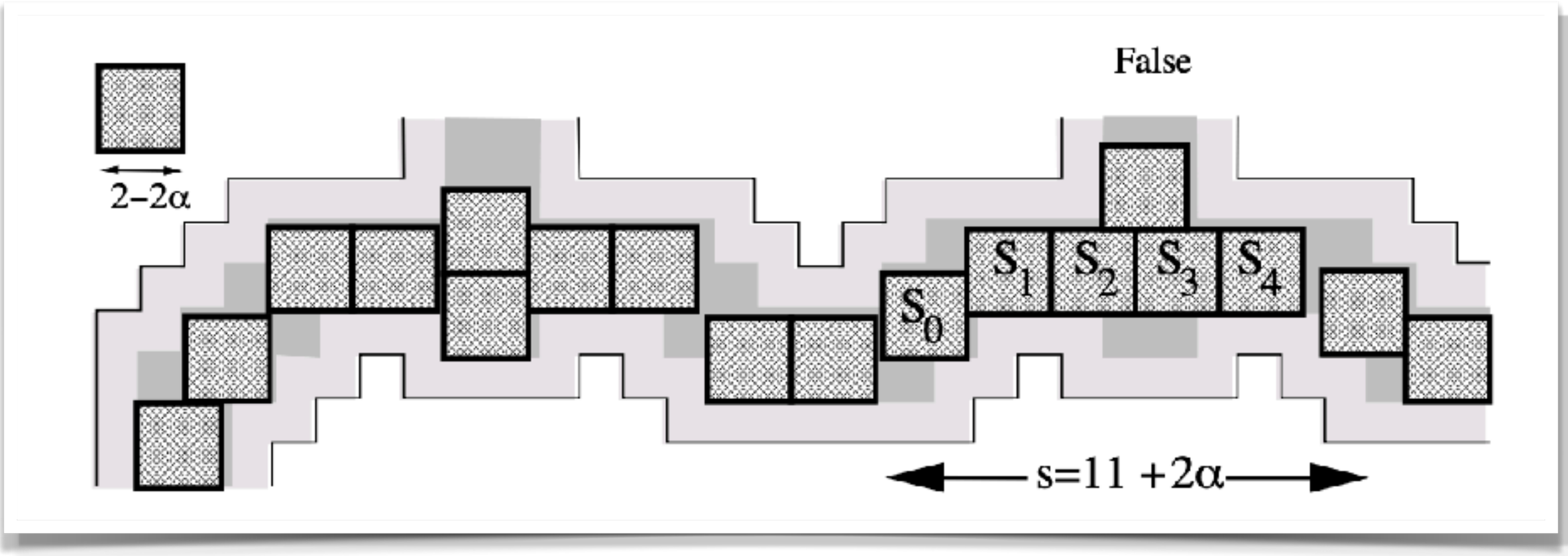
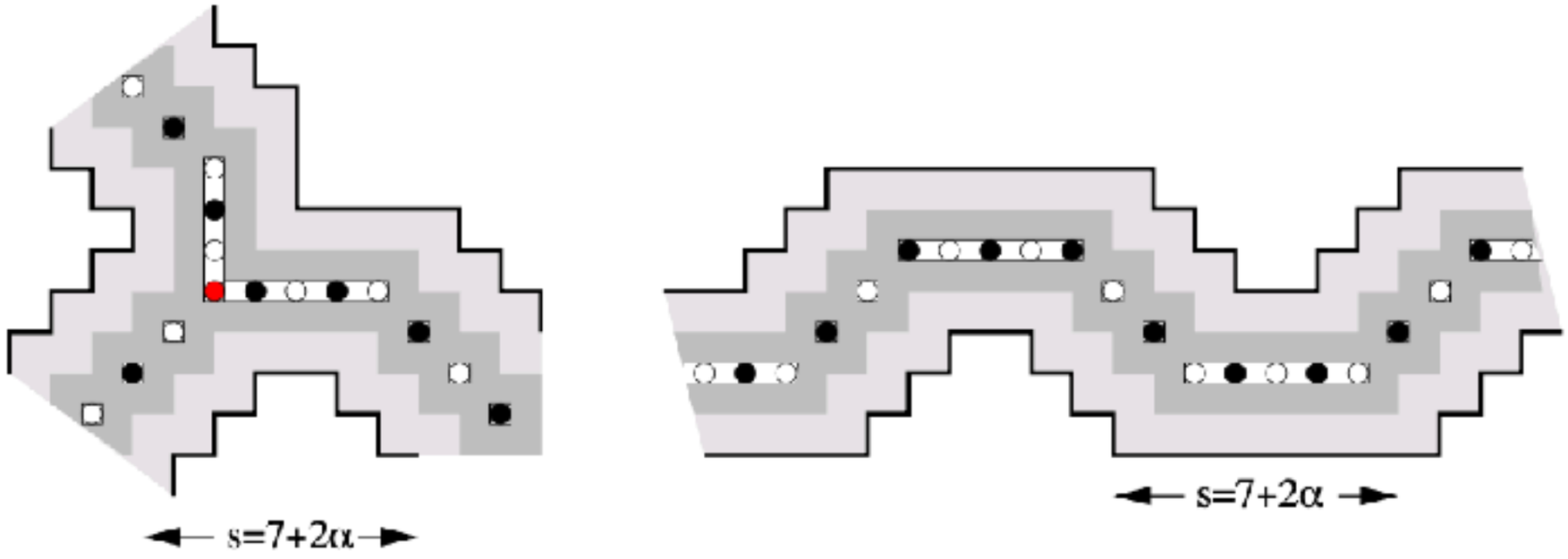


Fig. 4. A clause component for dispersion with boundaries and its receptor region (a); a satisfying placement (b); and an unsatisfying placement (c).

Smaller Squares (1)



Smaller Squares (2)



(a) (b)

Fig. 7. An upper bound on the approximation factor: clause components (a) and connector components (b).

Other Metrics

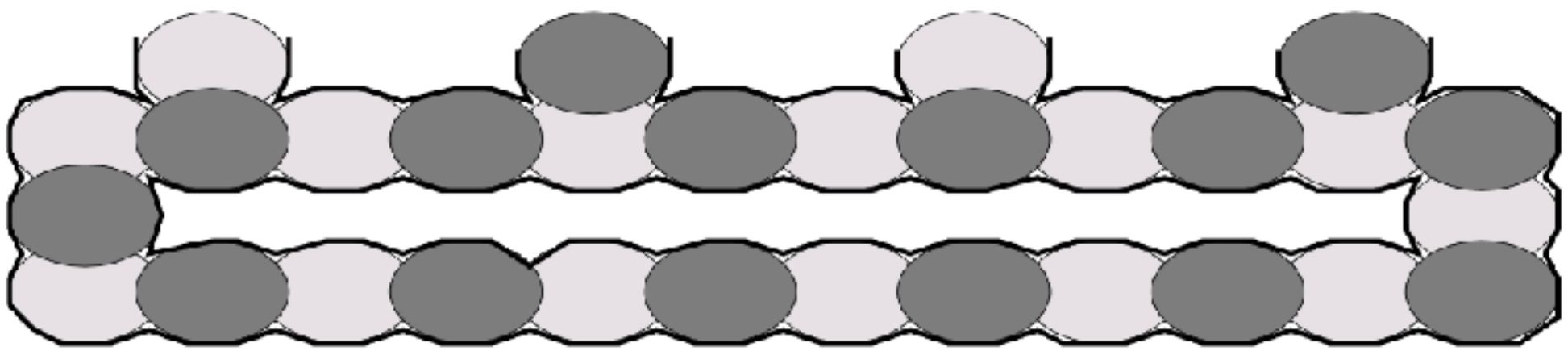


Fig. 9. A variable gadget for the case where the unit ball is an ellipse.

Approximation

THEOREM 9. *For rectilinear geometric dispersion with boundaries of k locations in a rectilinear polygon P with n vertices, Algorithm 8 computes a solution* $A_{Dis}(P, k)$, *such that*

$$A_{Dis}(P, k) \geq \tfrac{2}{3} OPT(P, k).$$

The running time is strongly polynomial.

ALGORITHM 8.

Input: rectilinear polygon P, positive integer k.

Output: a set of k locations, such that $A_{Dis}(P, k) := d$ is the minimum L_∞ distance between a location and the boundary, or between two locations.

1. **For all** $(e_i, e_j) \in Par(P)$ **do**
   (a) Perform binary search for the smallest integer m, $2 \leq m \leq k + 1$, with the following property:
   - For $d_{ijm} := Dist(e_i, e_j)/m$, $AS(P - d_{ijm}/2, d_{ijm}, 6)$ returns a feasible solution for at least k locations at distance d_{ijm}.

   (b) Let d_{ij} be the distance d_{ijm} for the critical value m.
2. Let d be the maximum d_{ij} for any (e_i, e_j).

Approximation

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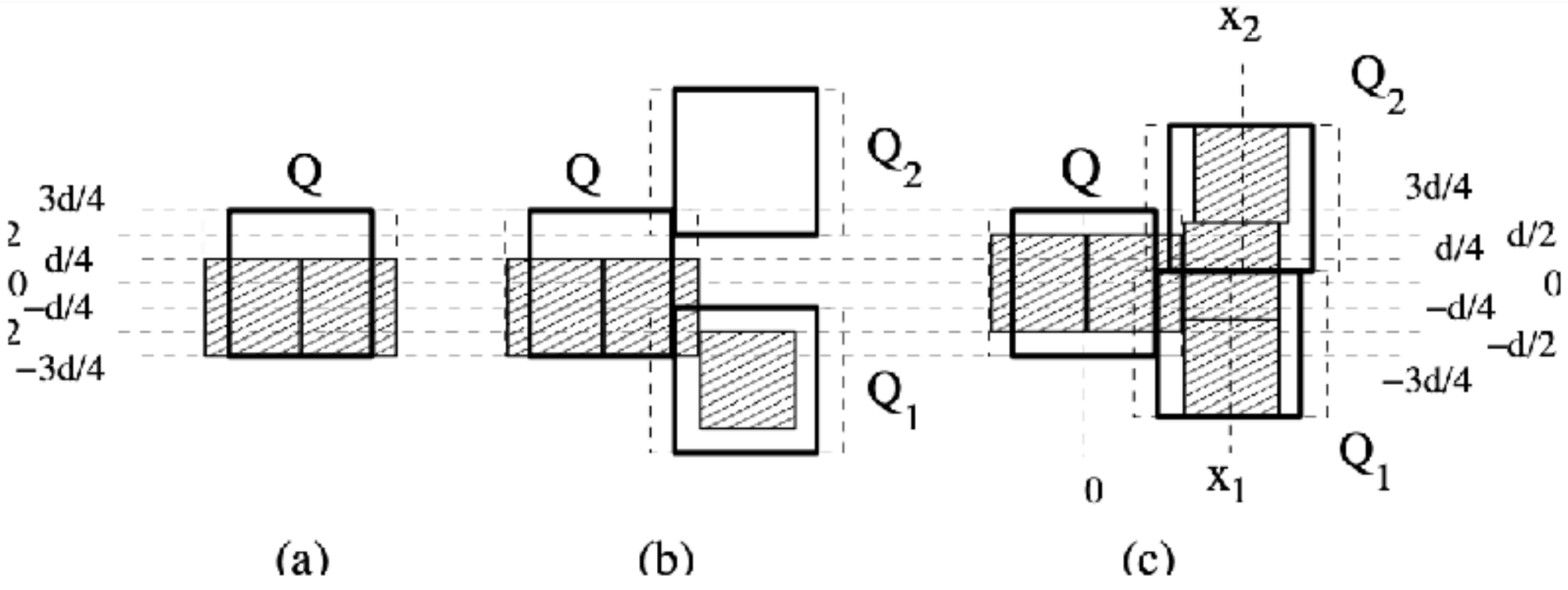

Fig. 12. Constructing a packing of d-squares.

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Thank you!