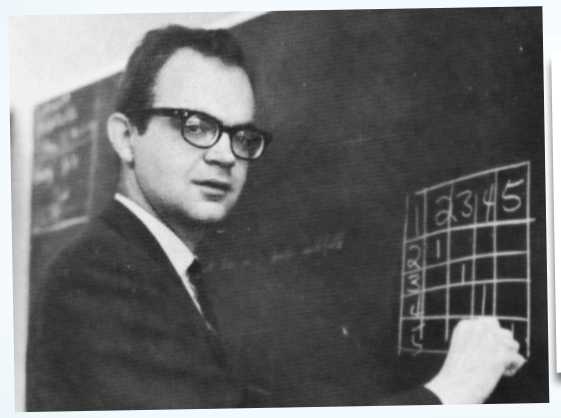
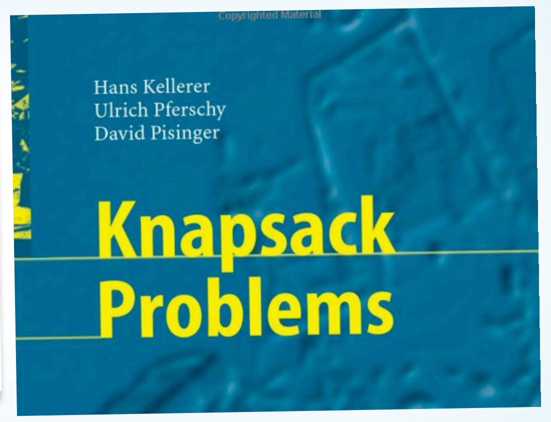


$$
\begin{aligned}
\text{maximize} \quad & \sum_{j=1}^{n} p_j x_j \\
\text{subject to} \quad & \sum_{j=1}^{n} w_j x_j \leq c, \\
& x_j = 0 \text{ or } 1, \qquad j = 1, \ldots, n,
\end{aligned}
$$

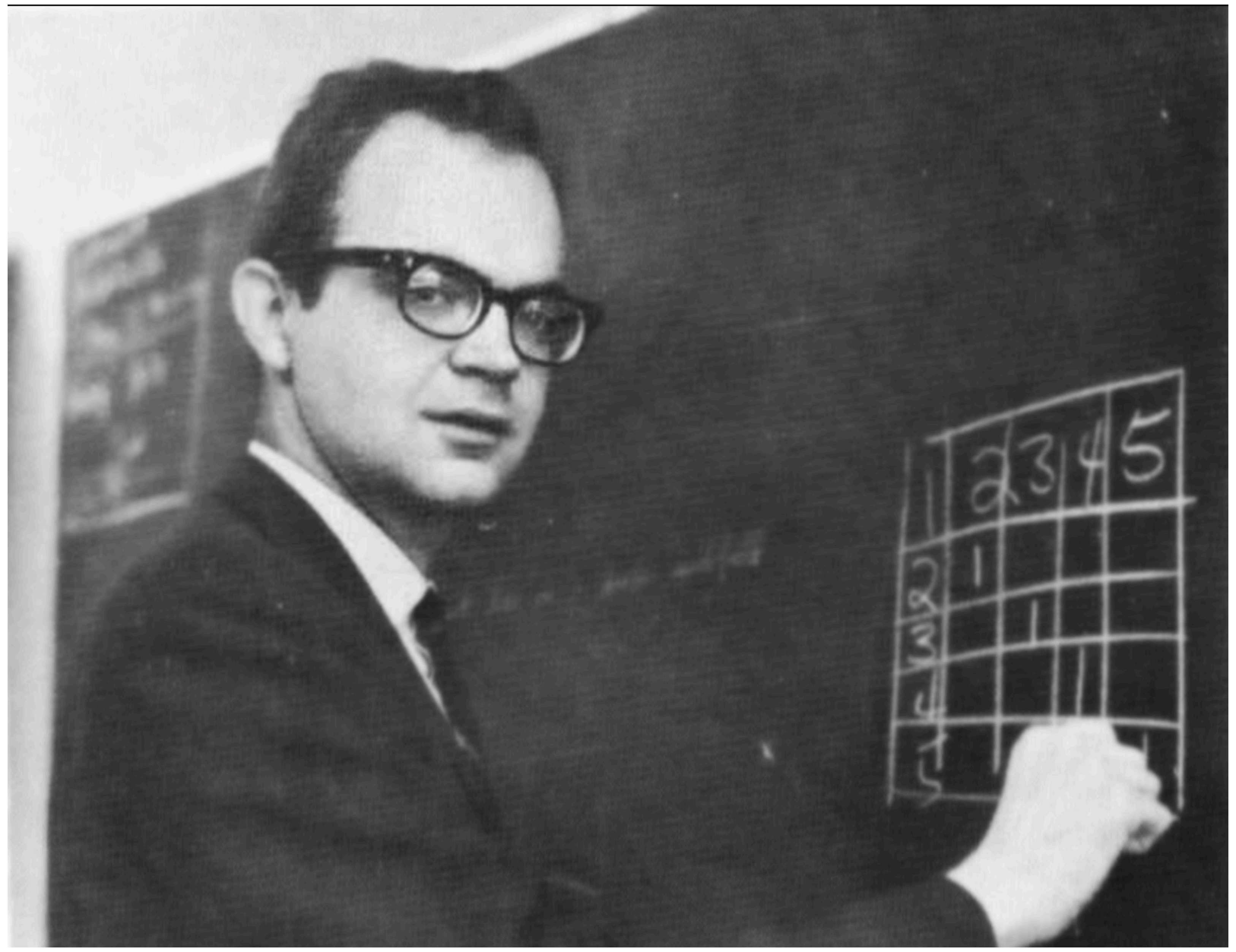


1 Knapsack-Probleme

Algorithmen und Datenstrukturen 2
Sommer 2024

Prof. Dr. Sándor Fekete

Die Zeit läuft!



Knut Donald, erster Studierender der Informatik

- 150 Minuten
- 20 Aufgaben
- 100 Punkte
- 50 Punkte zum Bestehen
- Die Zeit läuft!

30 Minuten später:

- 6 sichere Punkte
- 10 hoffnungslose Punkte
- Noch 44 Punkte zum Bestehen
- Restliche Aufgaben…

Restliche Aufgaben

Nummer	Minuten	Punkte
1	20	3
2	32	3
3	40	10
4	8	5
5	16	2
6	4	4
7	32	2
8	40	9
9	8	2
10	32	5
11	28	3
12	20	9
13	16	10
14	20	3
15	40	10
16	24	4

Kann Knut die Klausur bestehen?

Sortieren?

$\boxed{\text{nach Wert } \left(\frac{z_i}{p_i}\right) \text{ sortieren:}}$

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i$$

$$\sum_{i=1}^{n} p_i x_i$$

Sortieren?

$\boxed{\text{nach Wert } \left(\frac{z_i}{p_i}\right) \text{ sortieren:}}$

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 4$$

$$\sum_{i=1}^{n} p_i x_i = 4$$

Sortieren?

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 12$$

$$\sum_{i=1}^{n} p_i x_i = 9$$

Sortieren?

$\boxed{\text{nach Wert } \left(\frac{z_i}{p_i}\right) \text{ sortieren:}}$

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 28$$

$$\sum_{i=1}^{n} p_i x_i = 19$$

Sortieren?

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 48$$

$$\sum_{i=1}^{n} p_i x_i = 28$$

Sortieren?

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 56$$

$$\sum_{i=1}^{n} p_i x_i = 30$$

Sortieren?

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 96$$

$$\sum_{i=1}^{n} p_i x_i = 40$$

Sortieren?

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 120$$

$$\sum_{i=1}^{n} p_i x_i = 44$$

Knut kann die Klausur bestehen!

Teilaufgaben?!

Problem 1.3 (FRACTIONAL KNAPSACK).

Gegeben:

- *n Objekte* $1, \ldots, n$ *mit jeweils Größe* $z_i > 0$ *Gewinn* $p_i > 0$
- *Größenschranke Z*

Gesucht:

Für jedes Objekt ein Wert

$$\boxed{x_i \in [0, 1]}$$

sodass

$$\sum_{i=1}^{n} z_i x_i \leq Z$$

und

$$\sum_{i=1}^{n} p_i x_i = Maximal$$

Teilaufgaben?!

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 4$$

$$\sum_{i=1}^{n} p_i x_i = 4$$

Teilaufgaben?!

$\boxed{\text{nach Wert } \left(\frac{z_i}{p_i}\right) \text{ sortieren:}}$

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 12$$

$$\sum_{i=1}^{n} p_i x_i = 9$$

Teilaufgaben?!

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 28$$

$$\sum_{i=1}^{n} p_i x_i = 19$$

Teilaufgaben?!

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 48$$

$$\sum_{i=1}^{n} p_i x_i = 28$$

Teilaufgaben?!

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 56$$

$$\sum_{i=1}^{n} p_i x_i = 30$$

Teilaufgaben?!

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 96$$

$$\sum_{i=1}^{n} p_i x_i = 40$$

Teilaufgaben?!

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	40	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	10	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 96$$

$$\sum_{i=1}^{n} p_i x_i = 40$$

x_{15}

Teilaufgaben?!

nach Wert $\left(\frac{z_i}{p_i}\right)$ sortieren:

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	**24**	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	**6**	9	4	5	3	3	2	3	3	2

$$\sum_{i=1}^{n} z_i x_i = 120$$

$$\sum_{i=1}^{n} p_i x_i = 46$$

$x_{15} = 0{,}6$

Algorithmus

Algorithmus 1.4. *(Greedy-Algorithmus)*

Eingabe: $z_i, \ldots, z_n, Z, p_i, \ldots, p_n$

Ausgabe: $x_i, \ldots, x_n \in [0, 1]$

mit

$$\sum_{i=1}^{n} z_i x_i \leq Z$$

und

$$\sum_{i=1}^{n} p_i x_i = Maximal$$

1: *Sortiere {1,...,n} nach* $\frac{z_i}{p_i}$ *aufsteigend;*
   Dies ergibt die Permutation $\pi(1), \ldots, \pi(n)$.
   Setze $j = 1$.
2: ***while*** $(\sum_{i=1}^{j} z_{\pi(i)} \leq Z)$ ***do***
3: $\quad x_{\pi(j)} := 1$
4: $\quad j := j + 1$
5: *Setze* $x_{\pi(j)} := \dfrac{Z - \sum_{i=1}^{j-1} z_{\pi(i)}}{z_{\pi(j)}}$
6: ***return***

Das klappt immer!

Satz 1.5. *Algorithmus* $\boxed{1.4}$ *liefert eine optimale Lösung für Problem* $\boxed{1.3}$.

Beweis:
O.B.d.A. sei $\sum_{i=1}^{n} z_{\pi(i)} > Z$ - sonst nimmt man einfach alles.

(1) Wir bekommen eine zulässige Lösung.
(2) Wir bekommen eine optimale Lösung.

(1) Zulässigkeit

> *Setze* $j = 1$.
> **while** $\left(\sum_{i=1}^{j} z_{\pi(i)} \le Z\right)$ **do**
> $\quad x_{\pi(j)} := 1$
> $\quad j := j+1$
> *Setze* $x_{\pi(j)} := \dfrac{Z - \sum_{i=1}^{j-1} z_{\pi(i)}}{z_{\pi(j)}}$
> **return**

i	6	4	13	12	9	3	15	8	16	10	1	14	5	11	2	7
z_i	4	8	16	20	8	40	24	40	24	32	20	20	16	28	32	32
p_i	4	5	10	9	2	10	6	9	4	5	3	3	2	3	3	2

$\sum_{i=1}^{n} z_i x_i$ = 120

$\sum_{i=1}^{n} p_i x_i$ = 46

x_{15} = 0,6

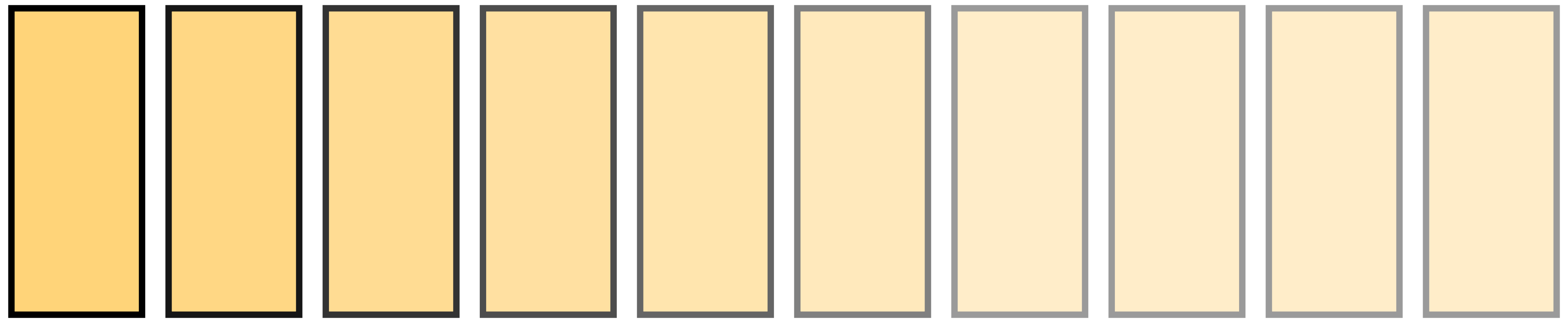
Sei j^* der letzte Index, für den die **while**-Bedingung getestet wird.

Dann ist $\sum_{i=1}^{j^*-1} z_{\pi(i)} \le Z$ und $\sum_{i=1}^{j^*} z_{\pi(i)} > Z$

Also ist $x_{\pi(1)}, \ldots, x_{\pi(j^*-1)} = 1$ und $x_{\pi(j^*)} \in [0,1[$ und $\sum_{i=1}^{j^*} x_{\pi(i)} z_{\pi(i)} = Z$

(2) Optimalität

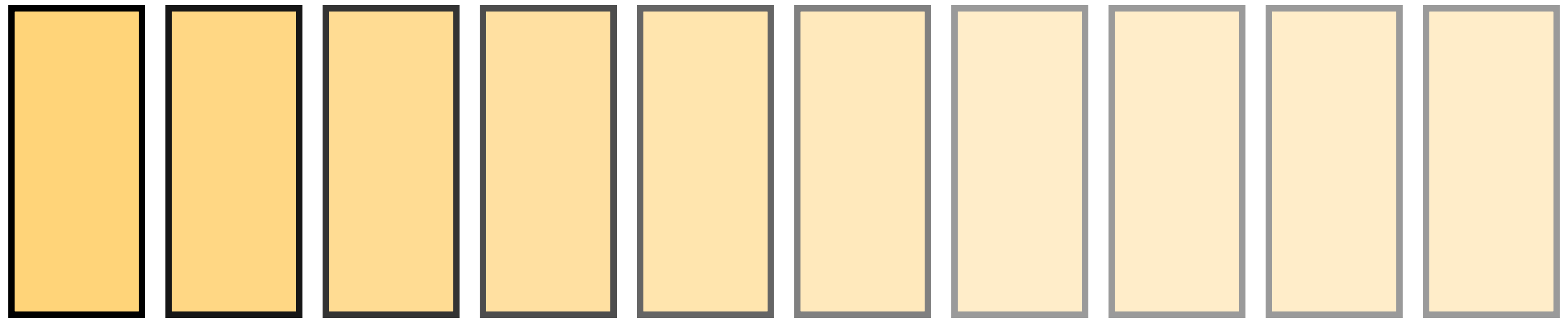
$\frac{p_i}{z_i}$



(2) Optimalität

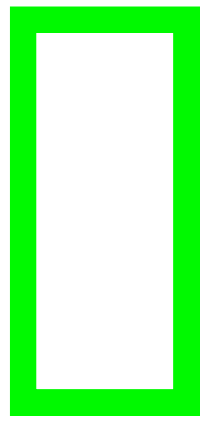
x

$\frac{p_i}{z_i}$

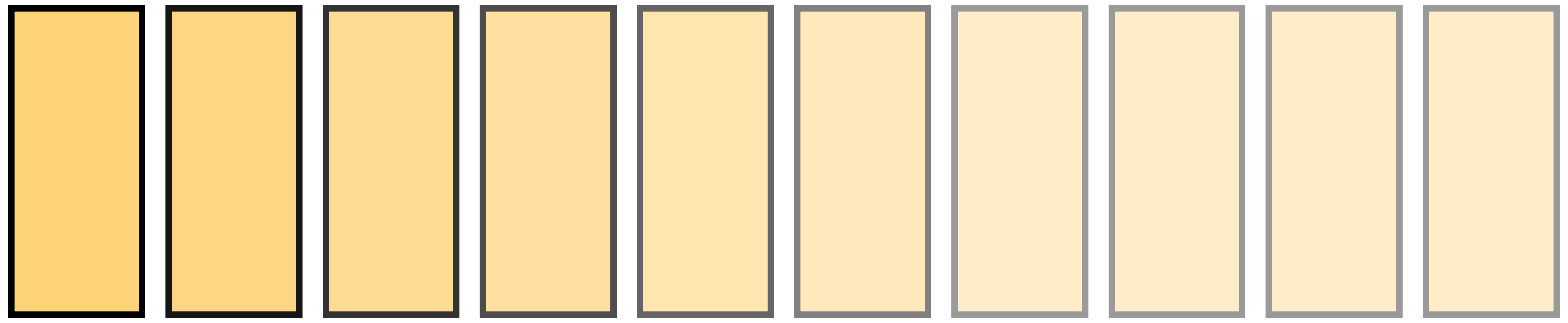


(2) Optimalität

x



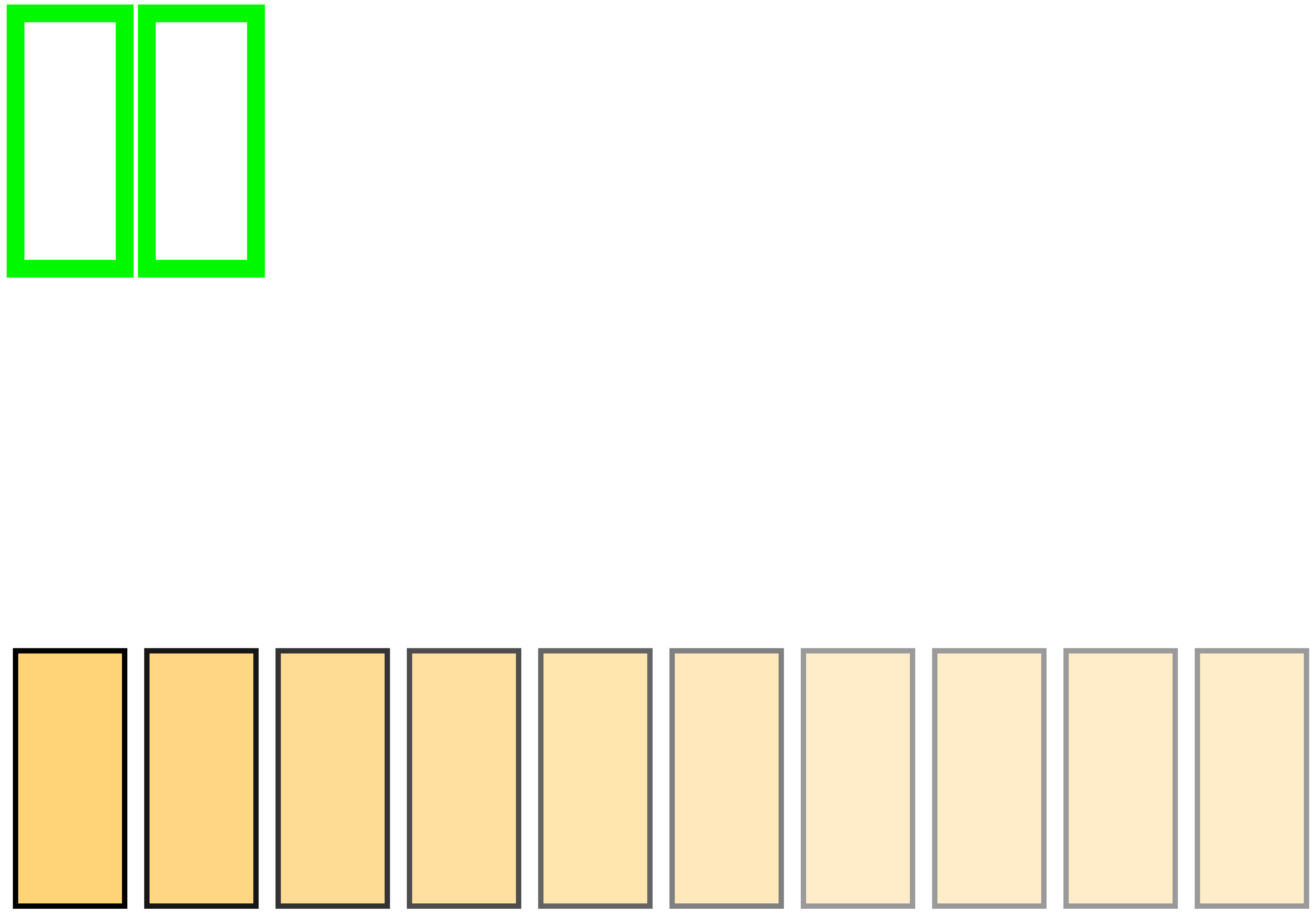
$\frac{p_i}{z_i}$

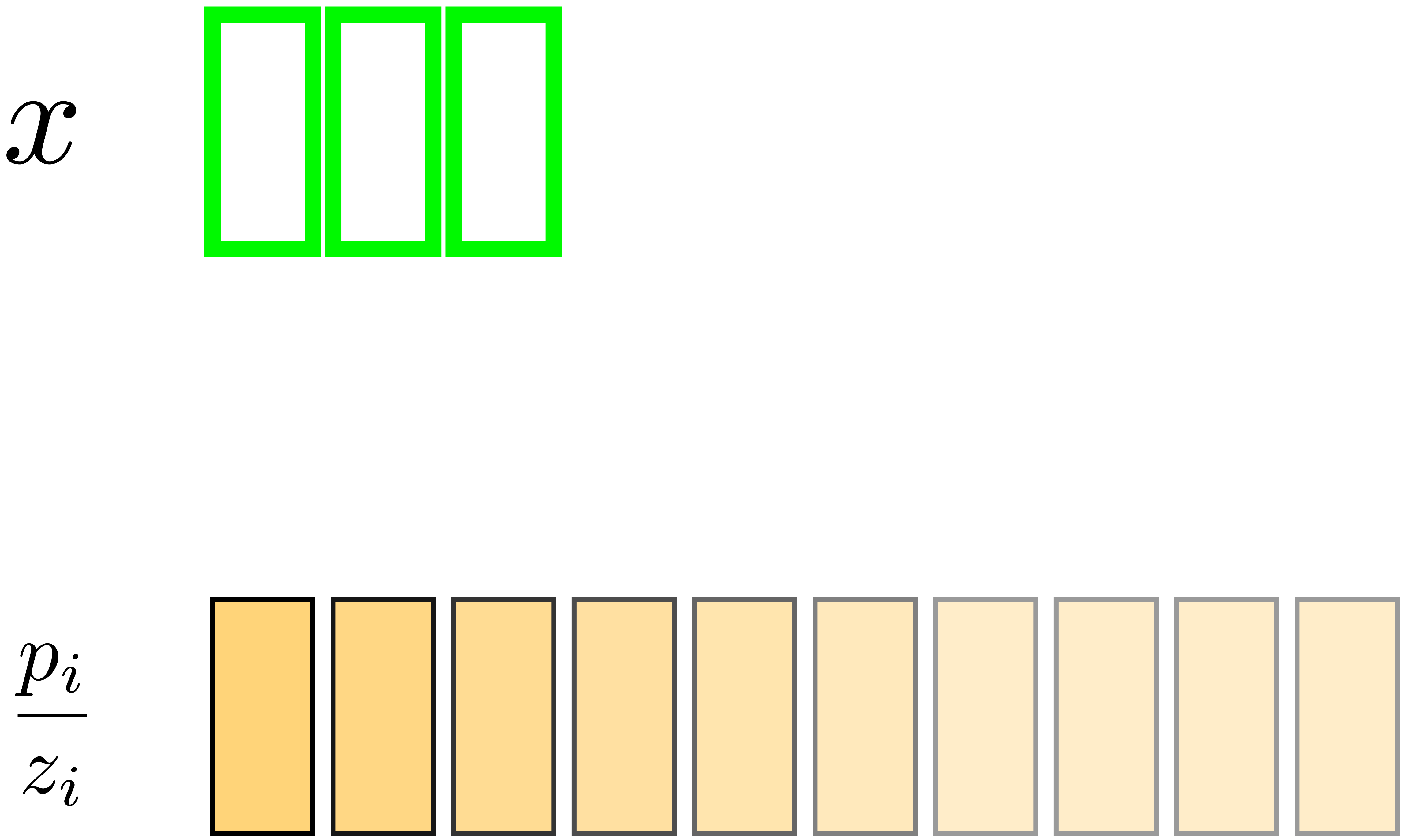


(2) Optimalität

x

$\frac{p_i}{z_i}$

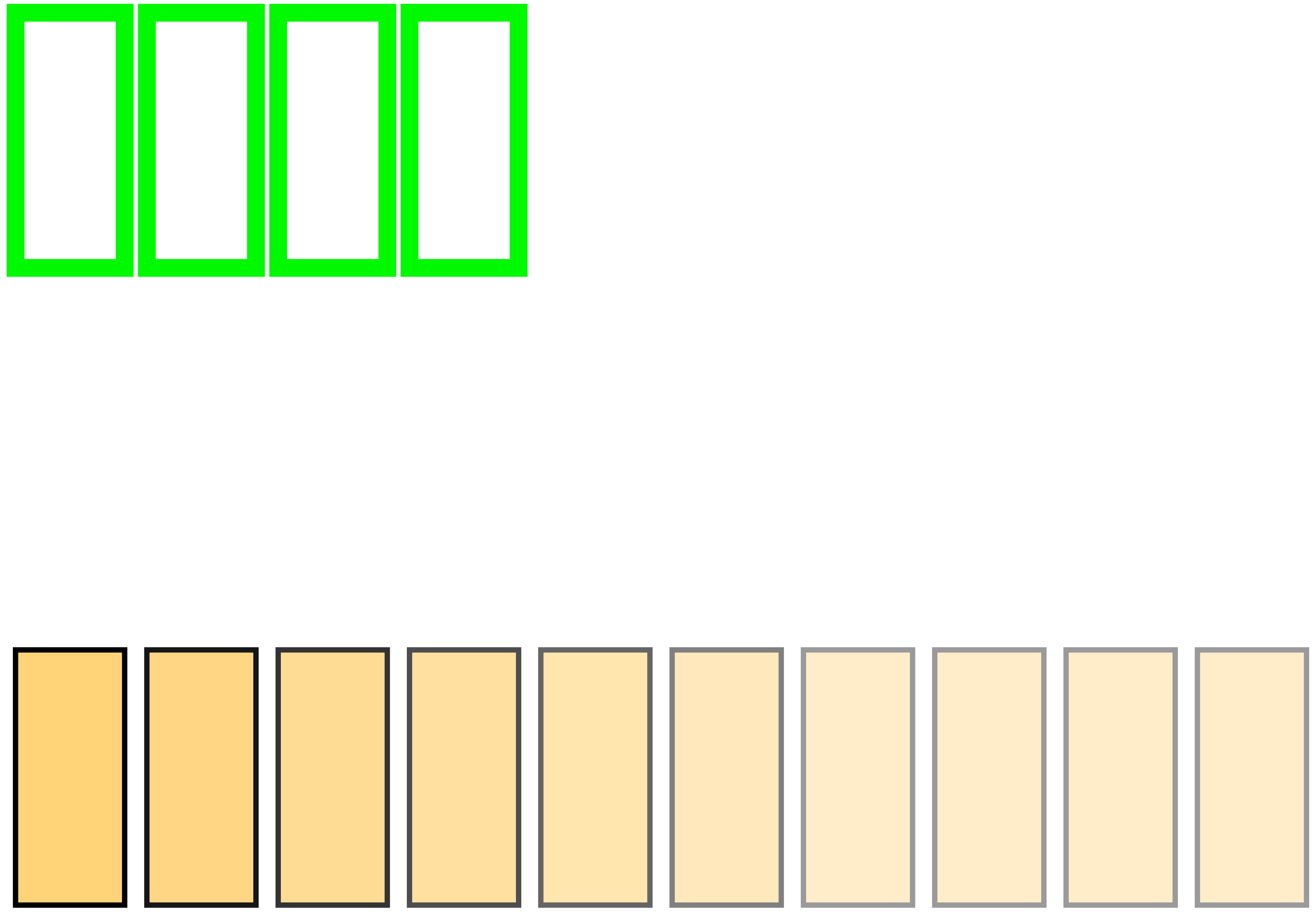



x
$\frac{p_i}{z_i}$

(2) Optimalität

x

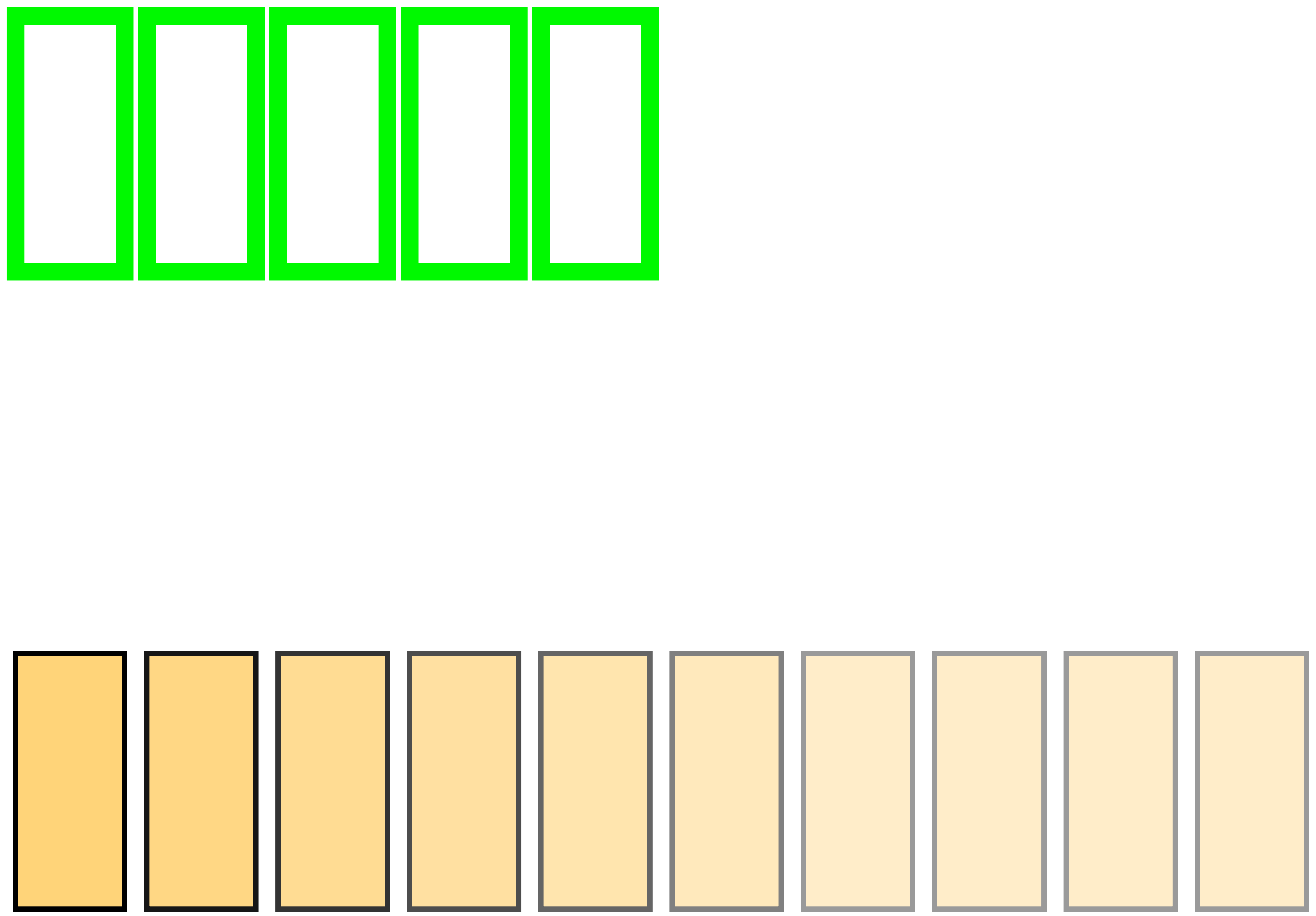
$\frac{p_i}{z_i}$



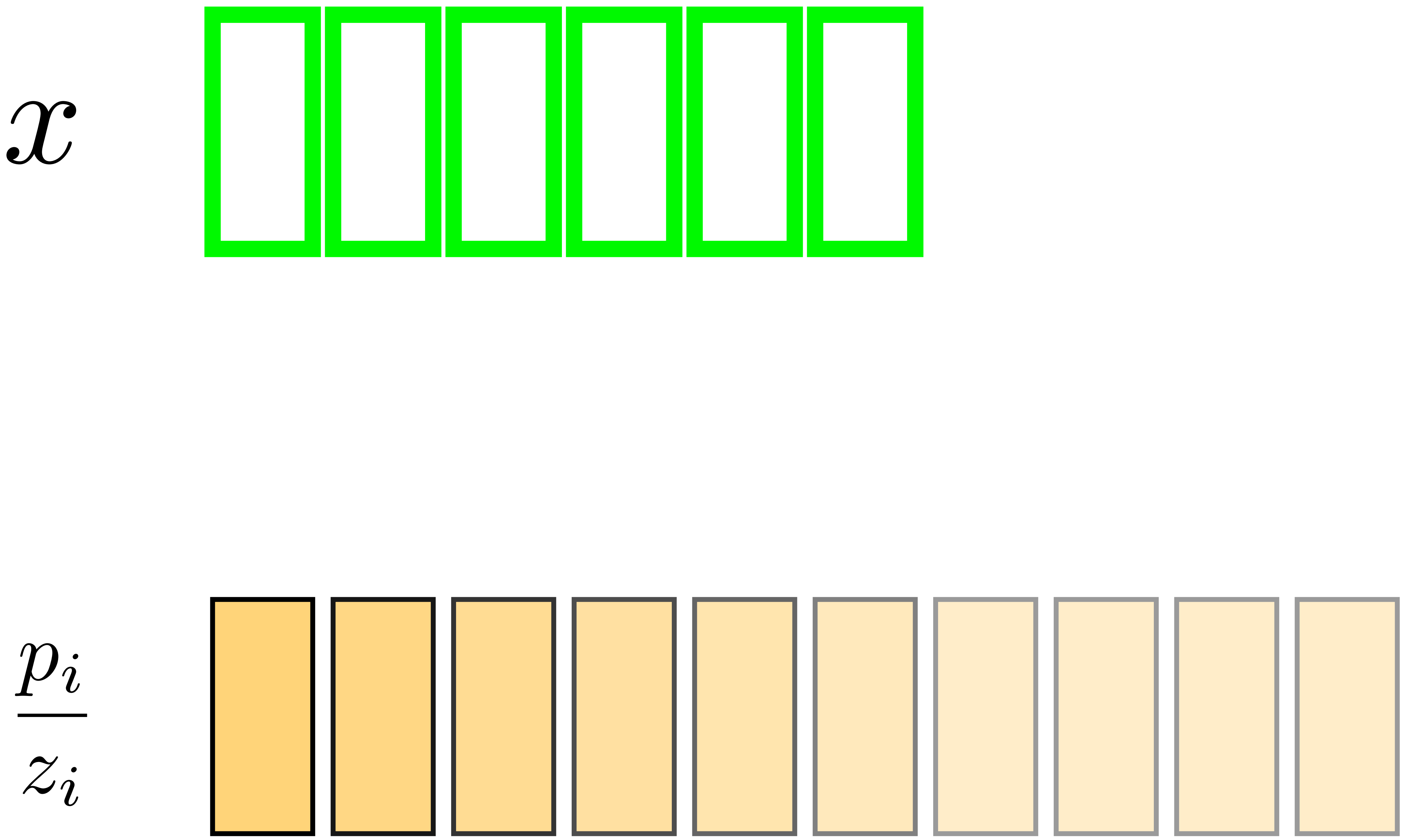
(2) Optimalität

x

$\frac{p_i}{z_i}$

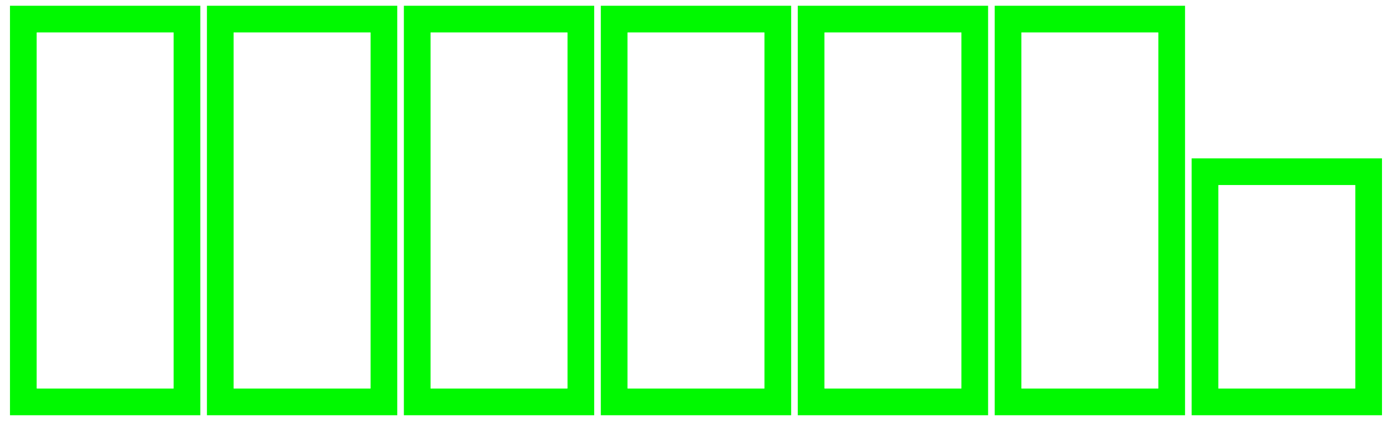


(2) Optimalität

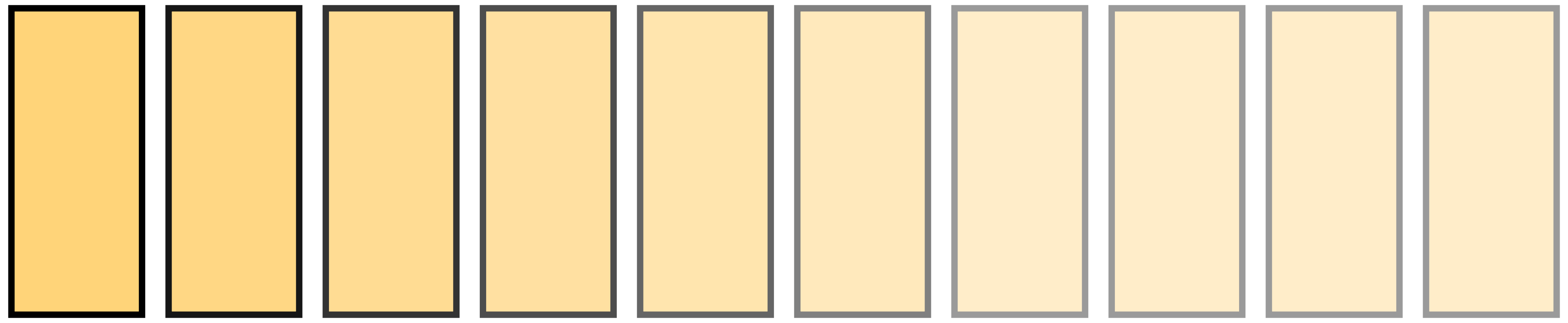


(2) Optimalität

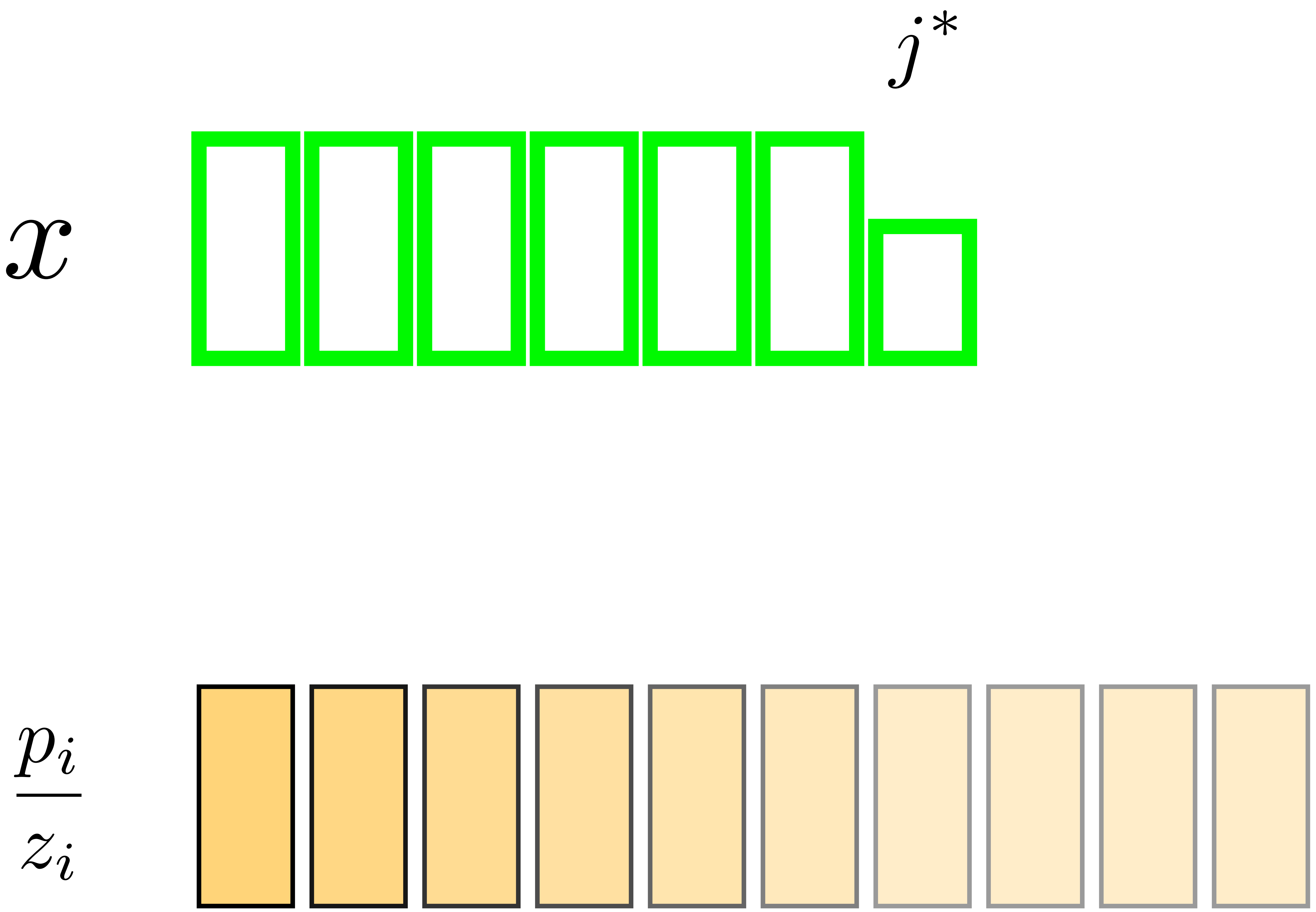
x



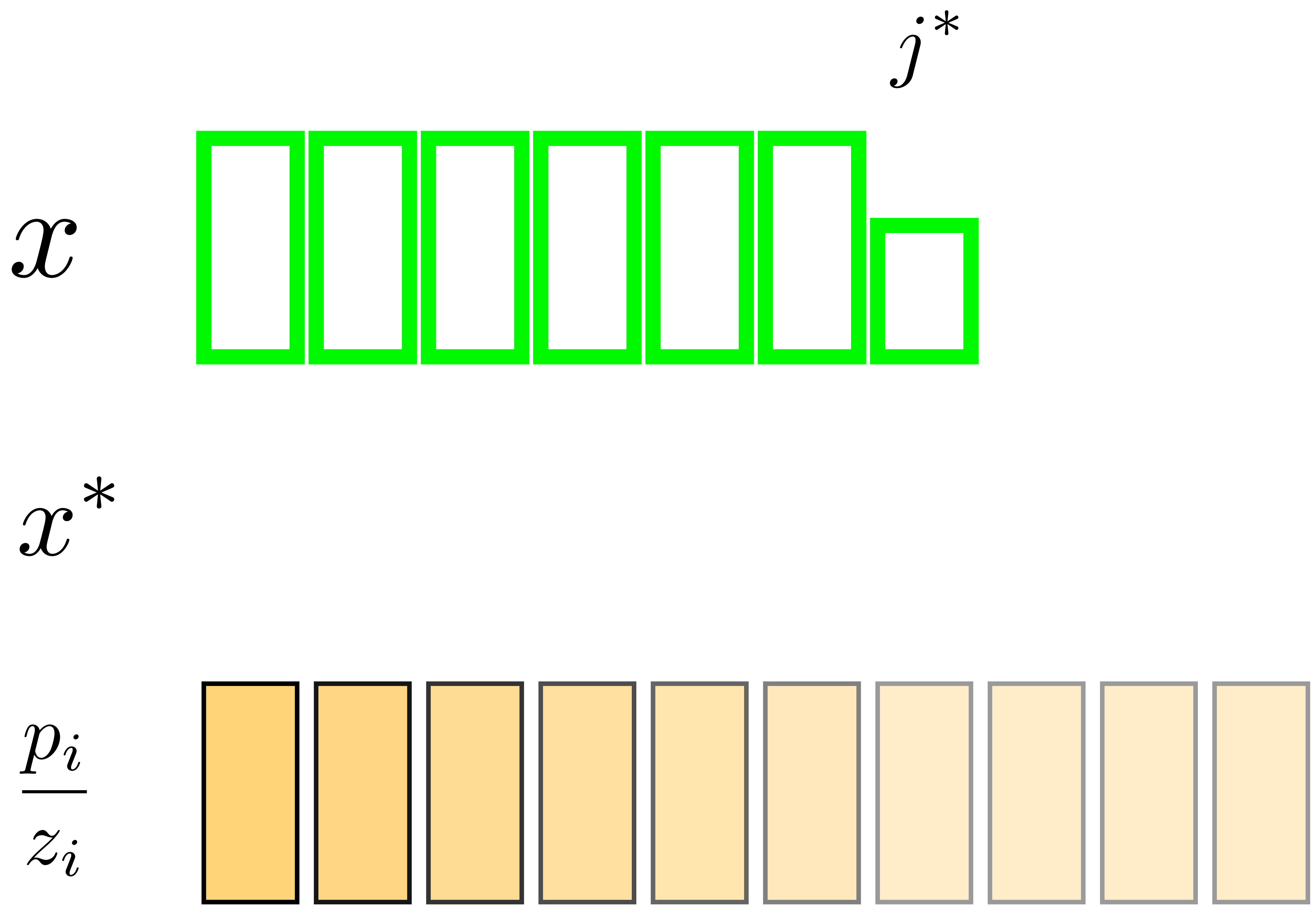
$\frac{p_i}{z_i}$



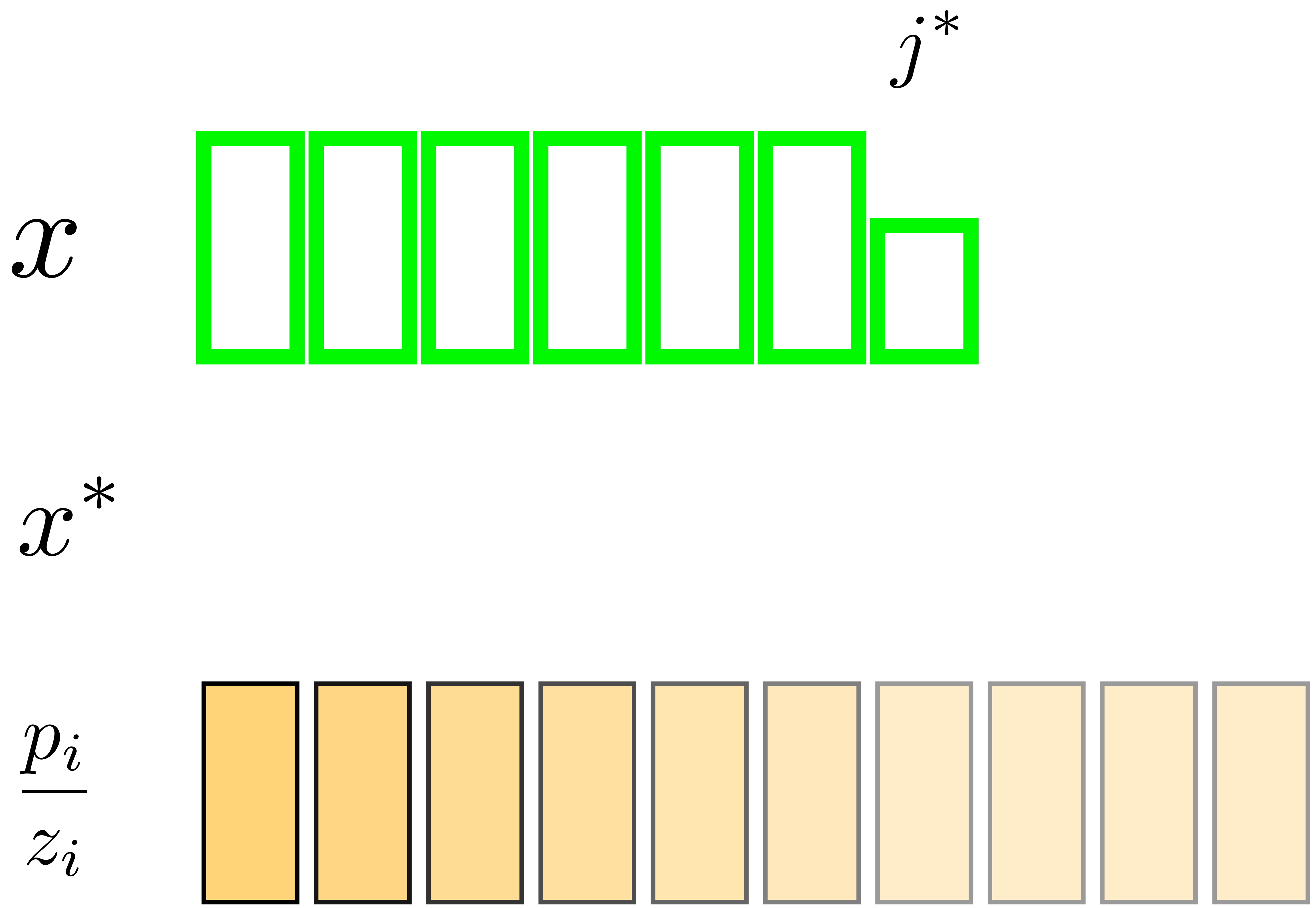
(2) Optimalität



(2) Optimalität

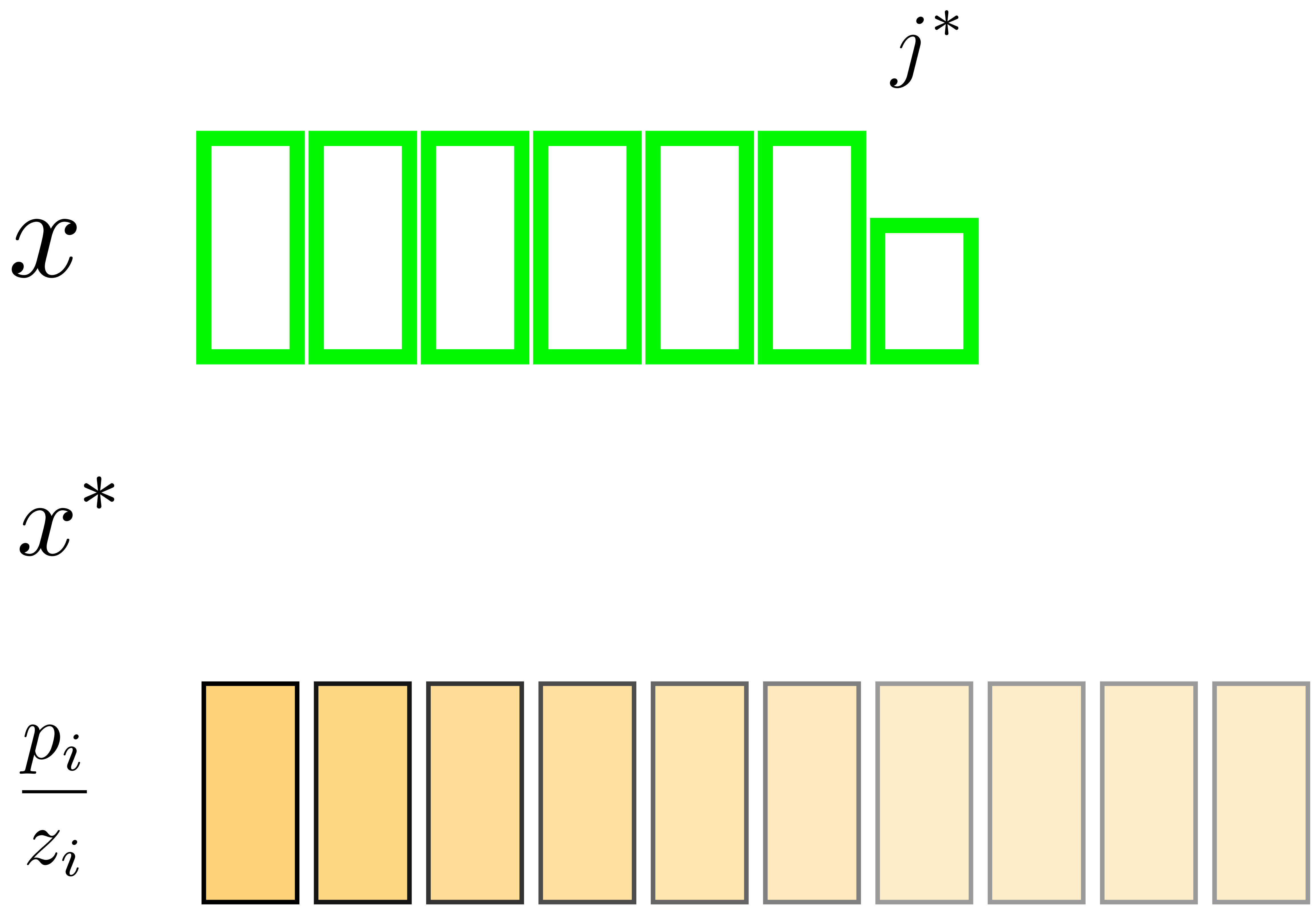


(2) Optimalität



$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

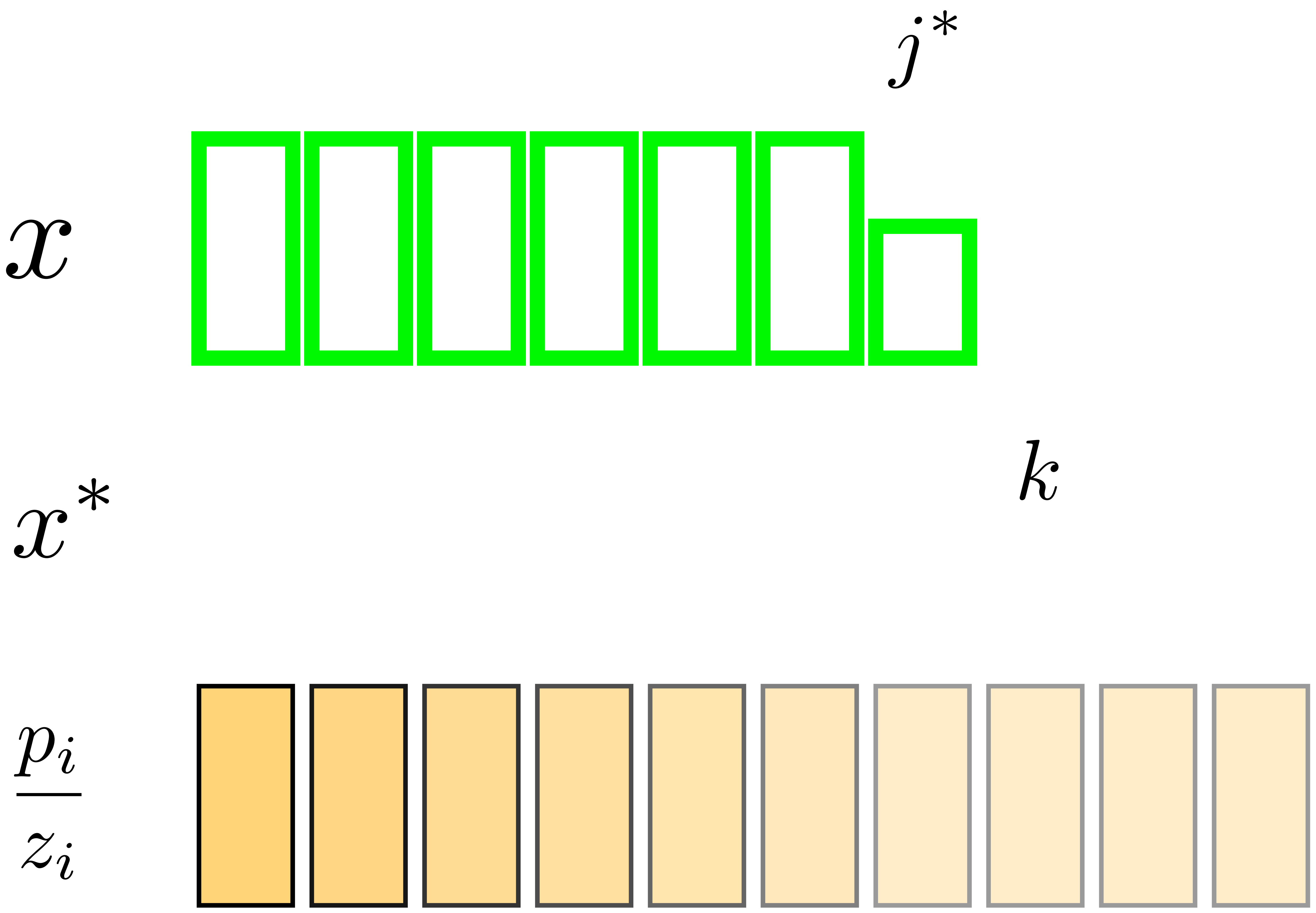
(2) Optimalität



$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

$$p_i > 0$$

(2) Optimalität



$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

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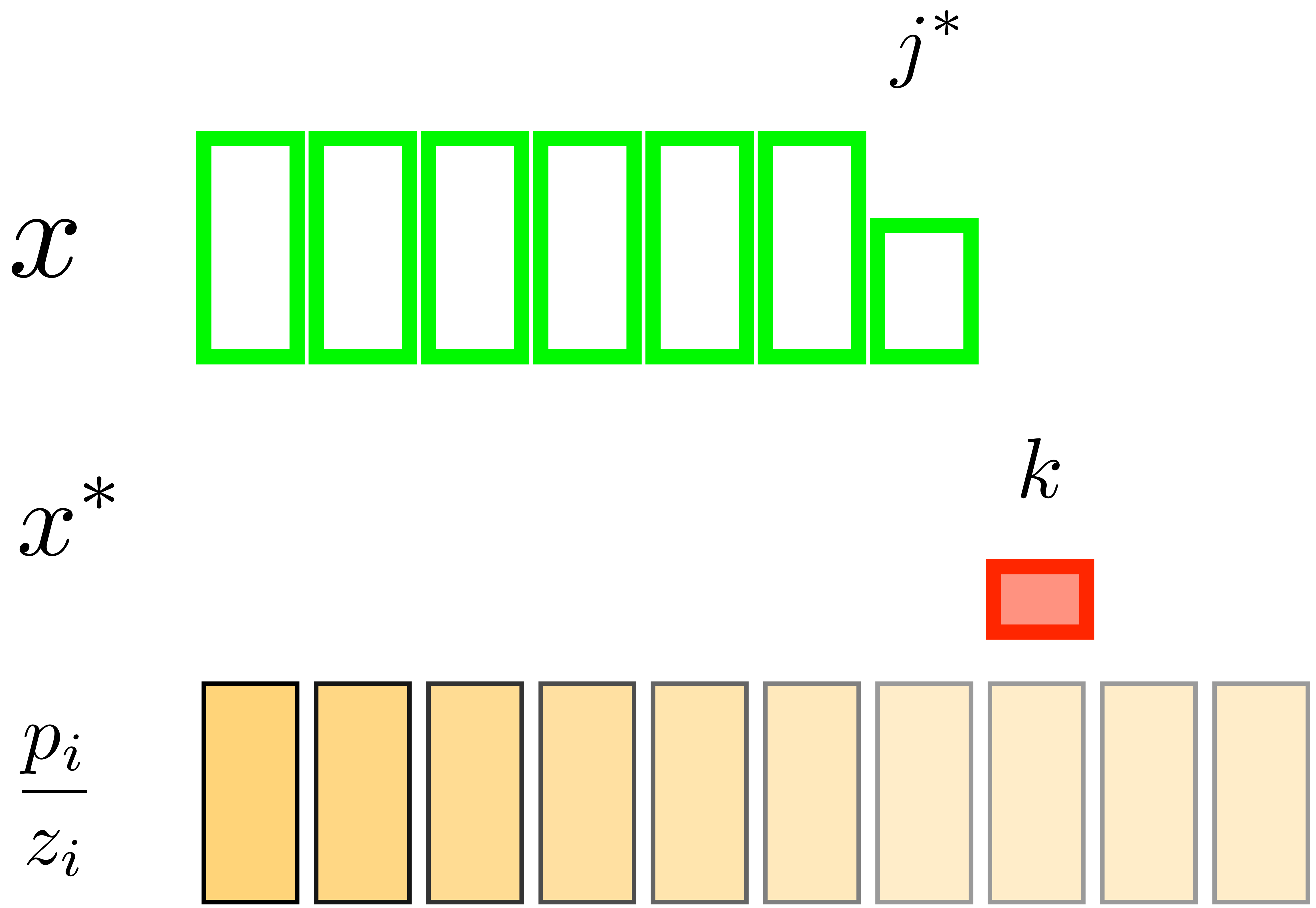
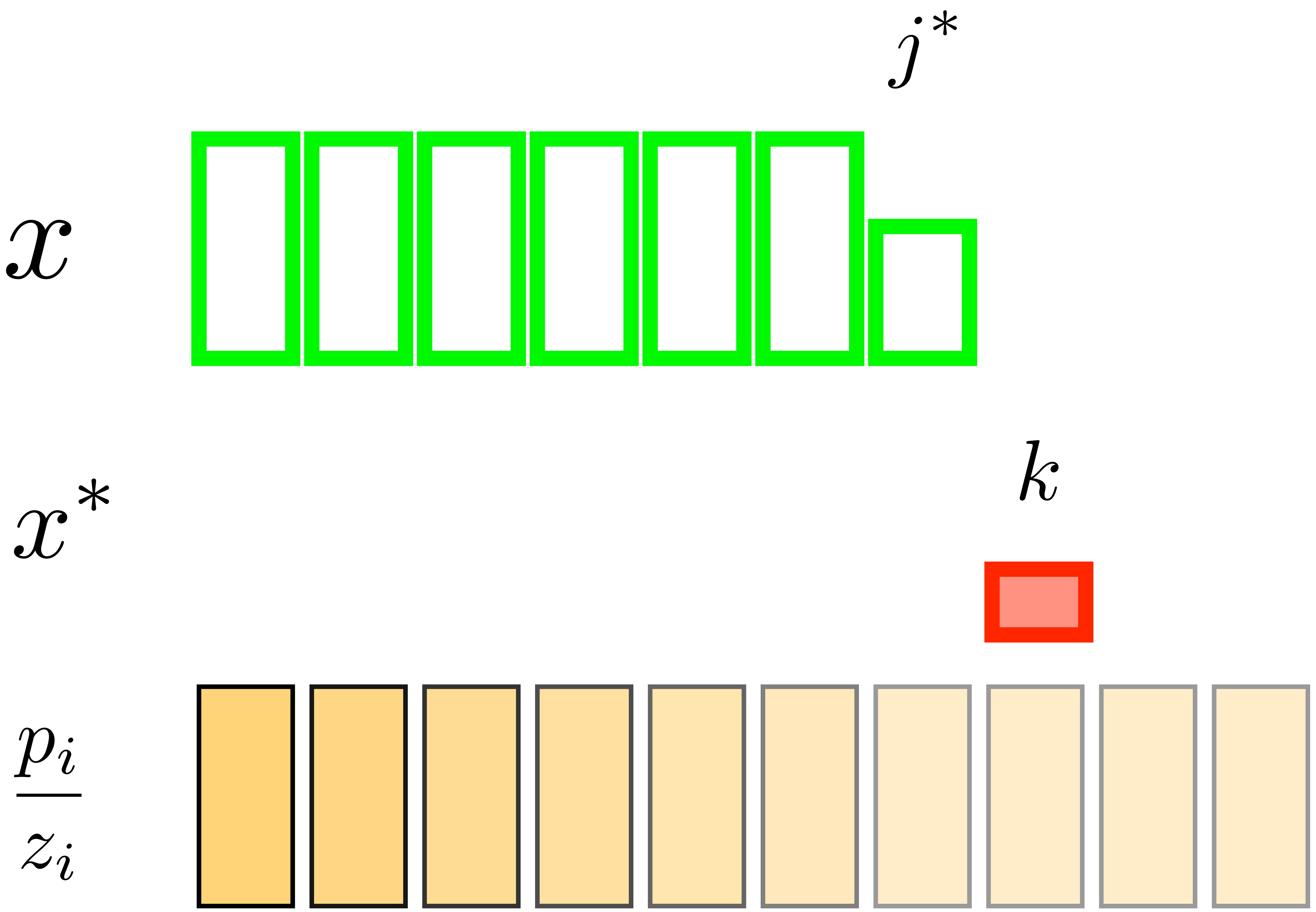
(2) Optimalität



$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

$$p_i > 0$$

(2) Optimalität

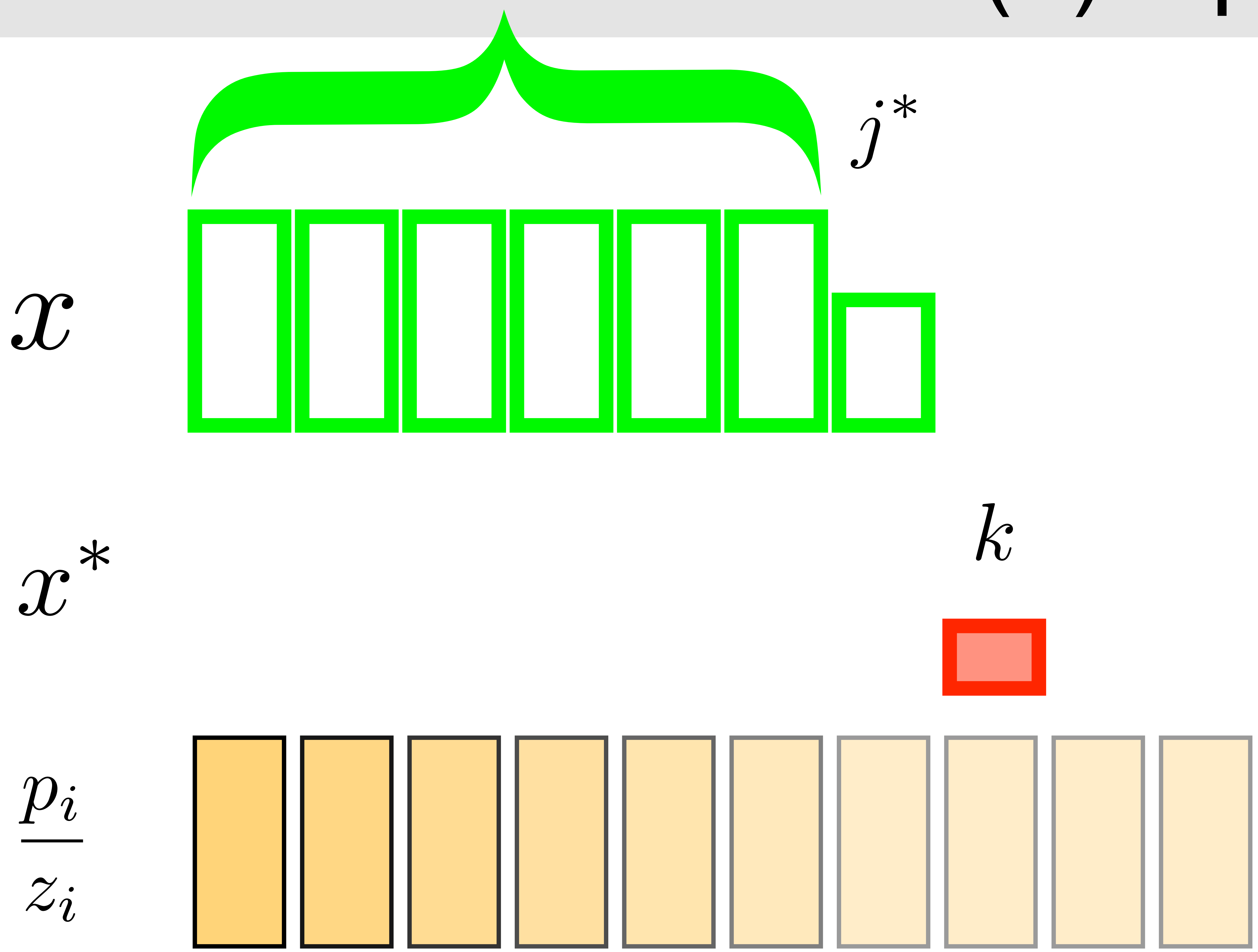


$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

$$p_i > 0$$

$$\Longrightarrow \exists\, k \in \{1, \ldots, n\} : \; x_{\pi(k)} < x^*_{\pi(k)}$$

(2) Optimalität



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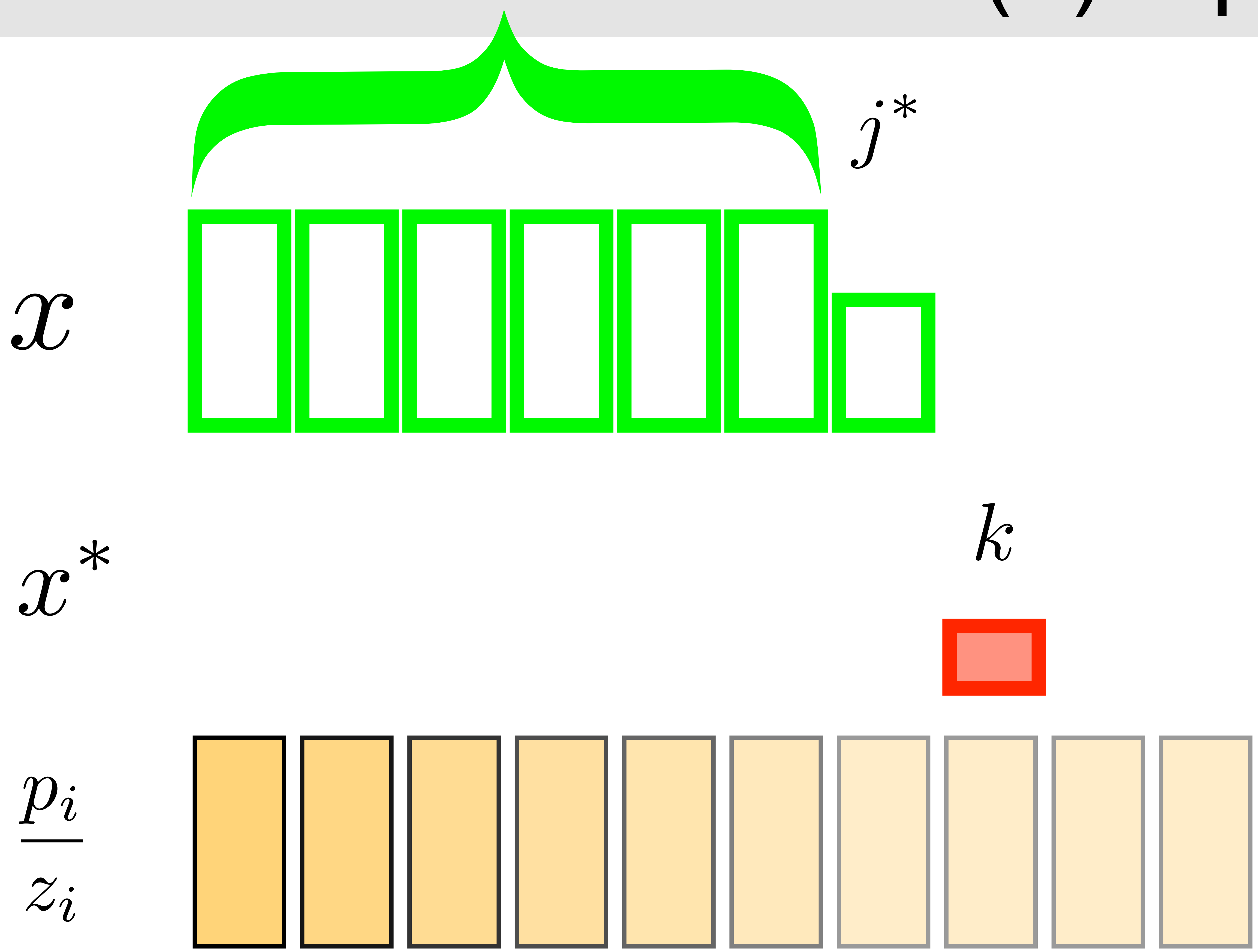
(2) Optimalität



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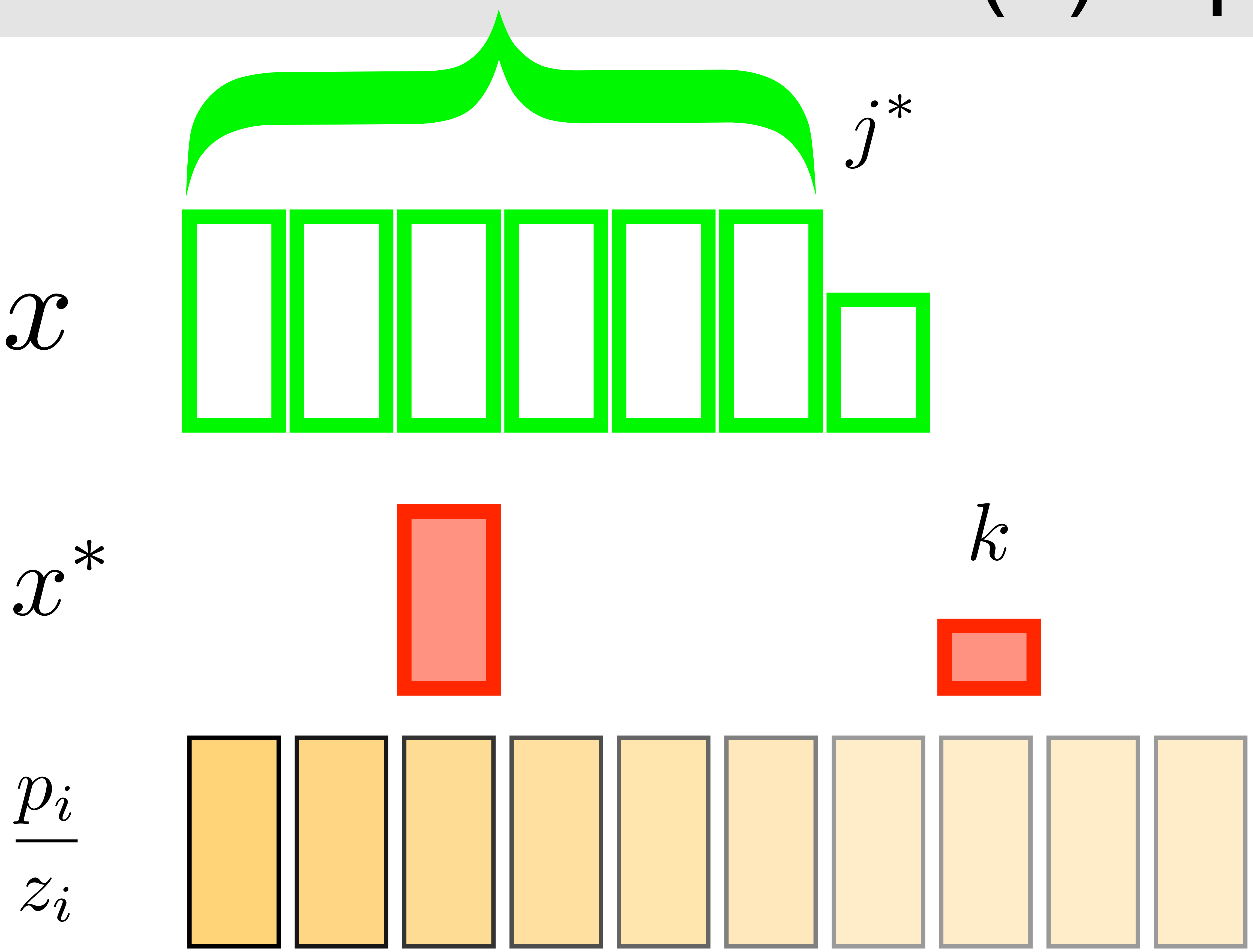
$$\Longrightarrow \exists\, k \in \{1, \ldots, n\} : \; x_{\pi(k)} < x^*_{\pi(k)}$$

$$\Longrightarrow \exists\, k \in \{j^*, \ldots, n\} : \; x_{\pi(k)} < x^*_{\pi(k)}$$

(2) Optimalität



$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

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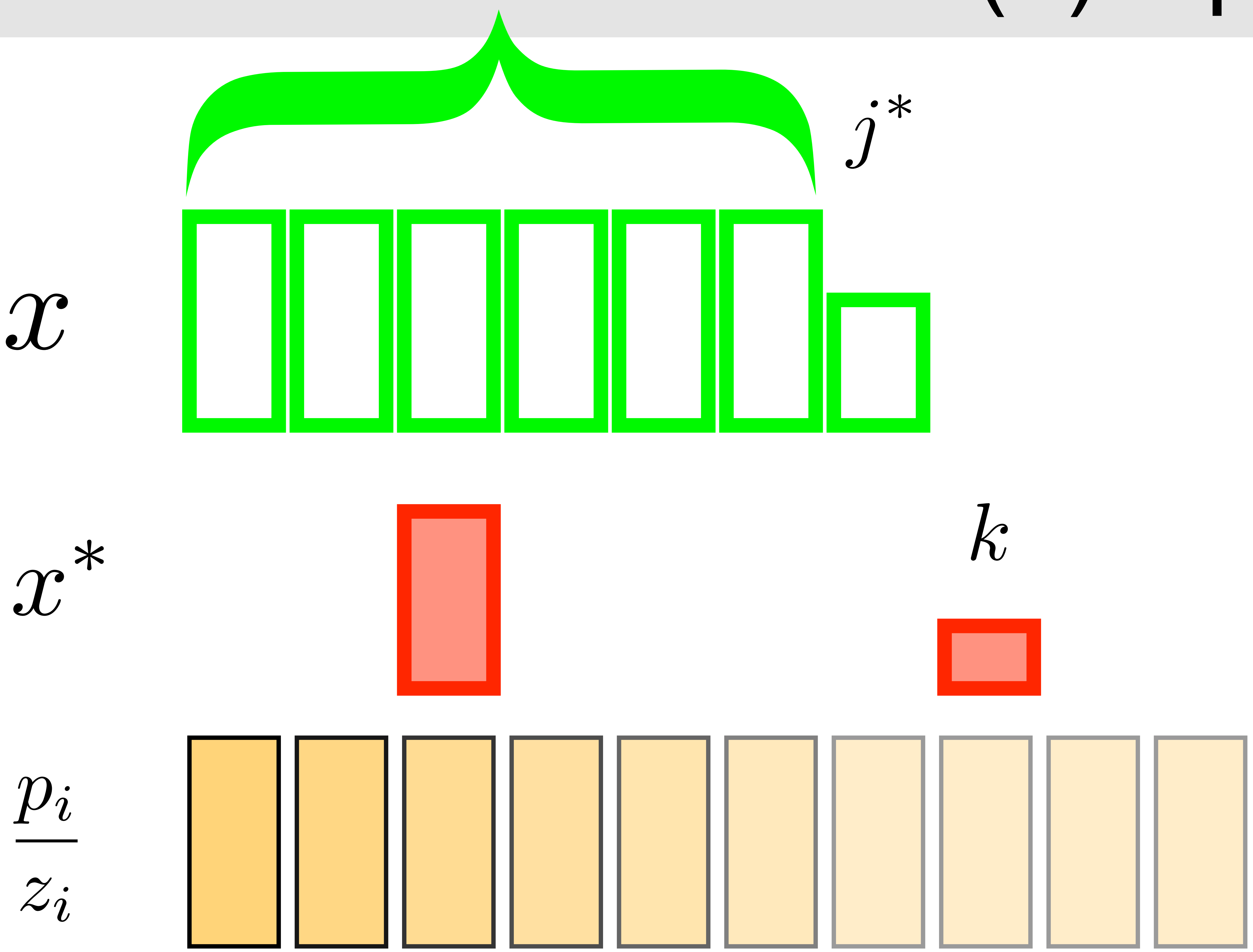
(2) Optimalität



$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

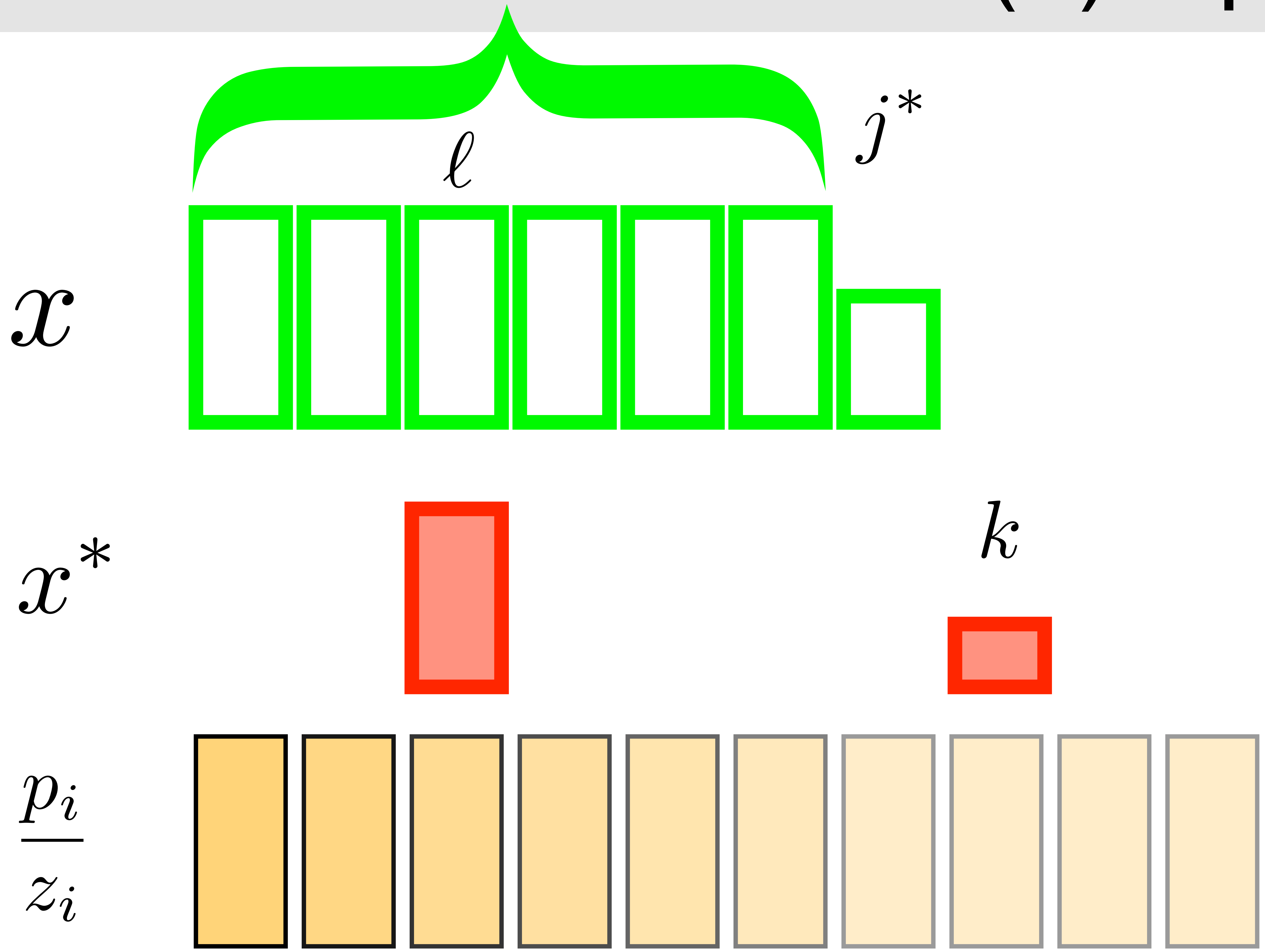
$$p_i > 0$$

$$\Longrightarrow \exists\, k \in \{1, \ldots, n\} : \; x_{\pi(k)} < x^*_{\pi(k)}$$

$$\Longrightarrow \exists\, k \in \{j^*, \ldots, n\} : \; x_{\pi(k)} < x^*_{\pi(k)}$$

$$\Longrightarrow \exists\, \ell \in \{1, \ldots, j^*\} : \; x_{\pi(\ell)} > x^*_{\pi(\ell)}$$

(2) Optimalität



$$\sum_{i=1}^{n} x_i p_i < \sum_{i=1}^{n} x_i^* p_i$$

$$p_i > 0$$

$$\Longrightarrow \exists\, k \in \{1, \ldots, n\}: \; x_{\pi(k)} < x^*_{\pi(k)}$$

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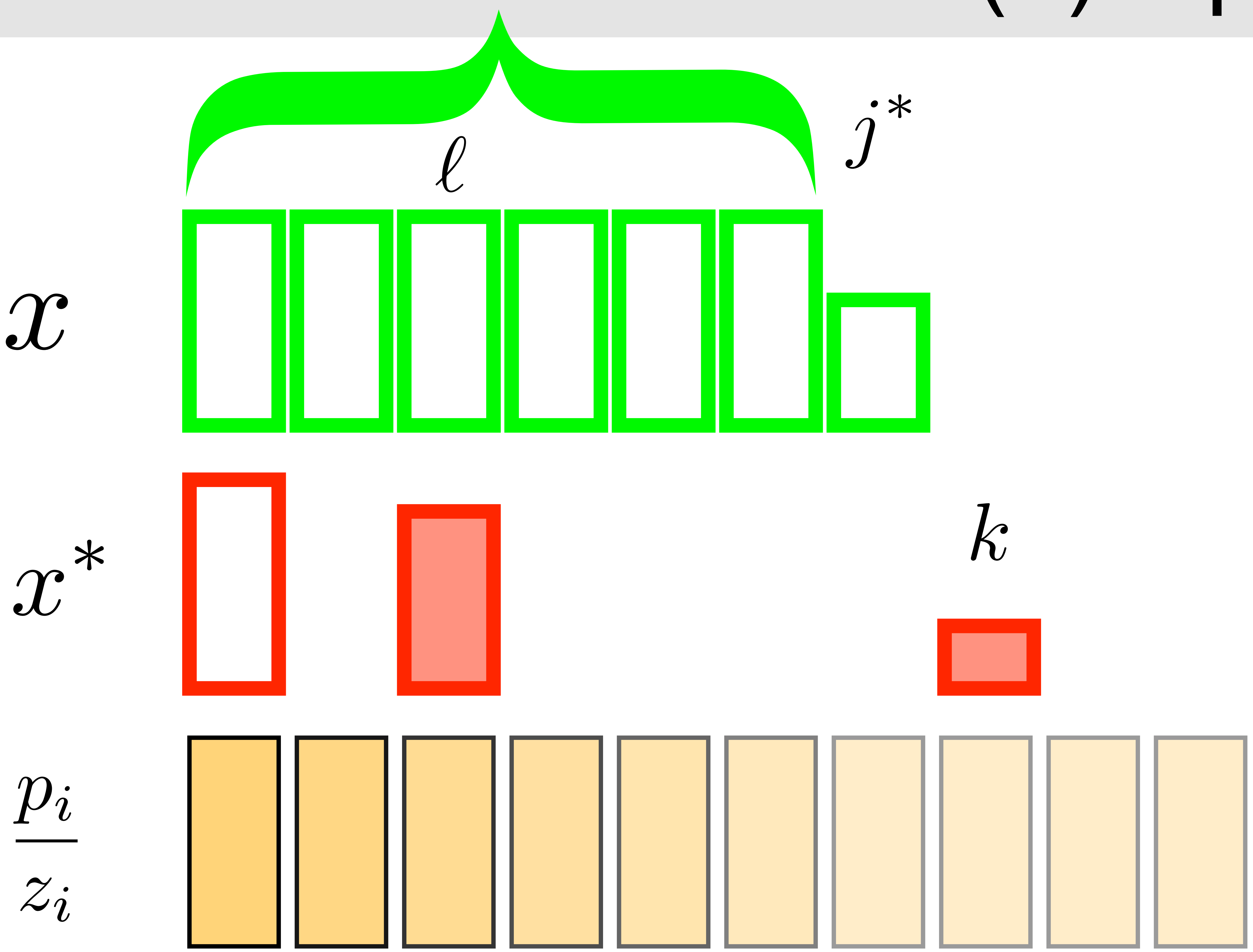
(2) Optimalität



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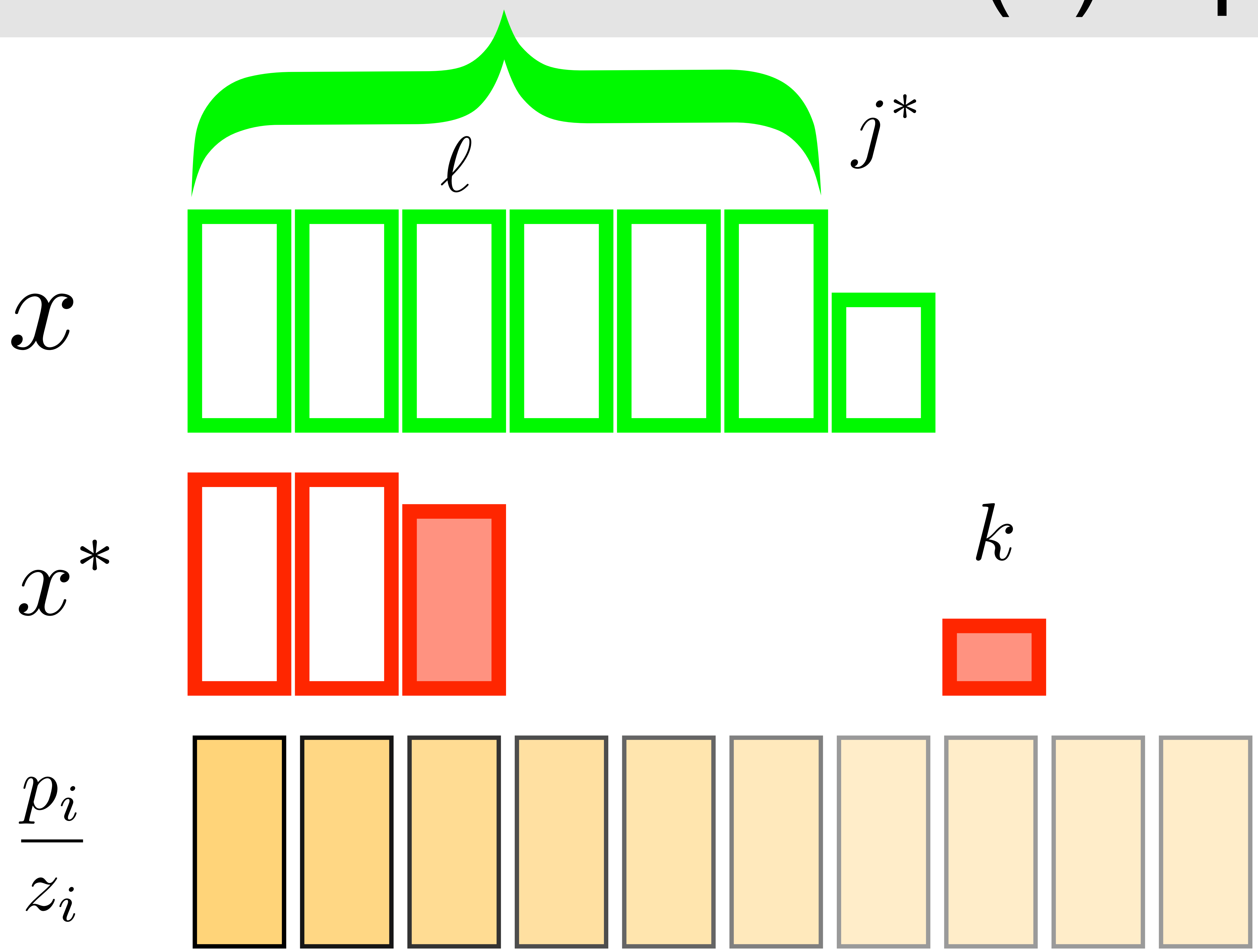

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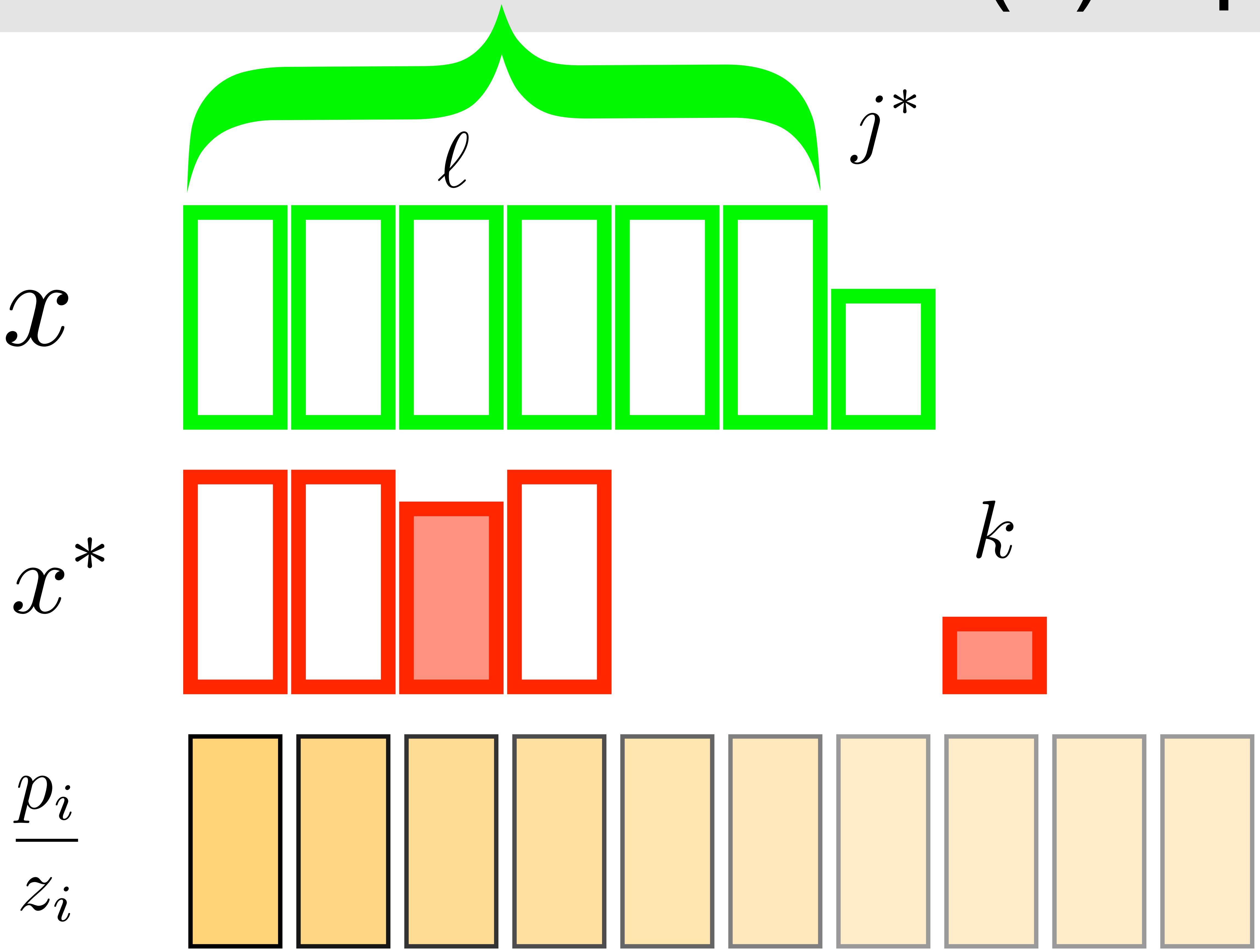

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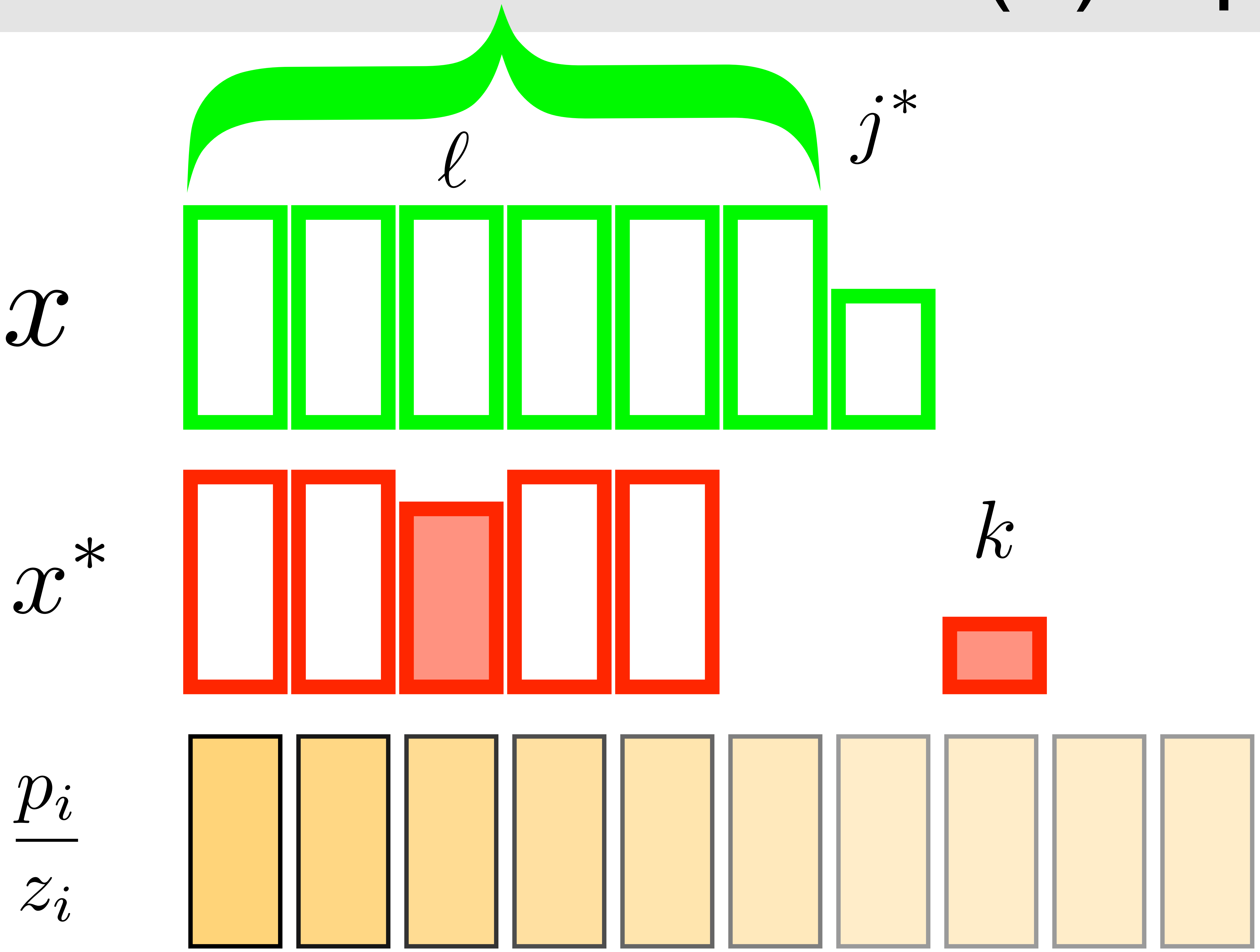

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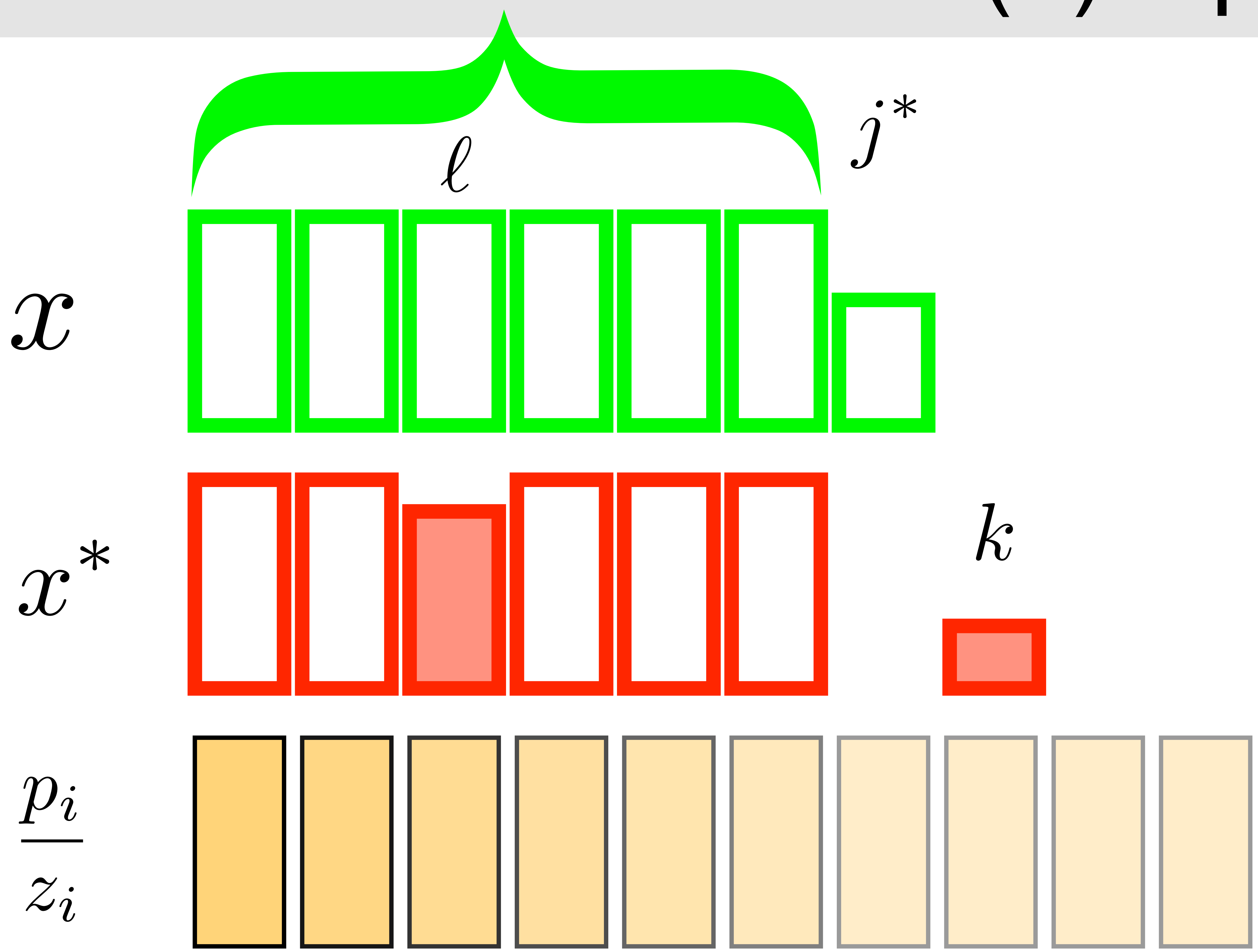

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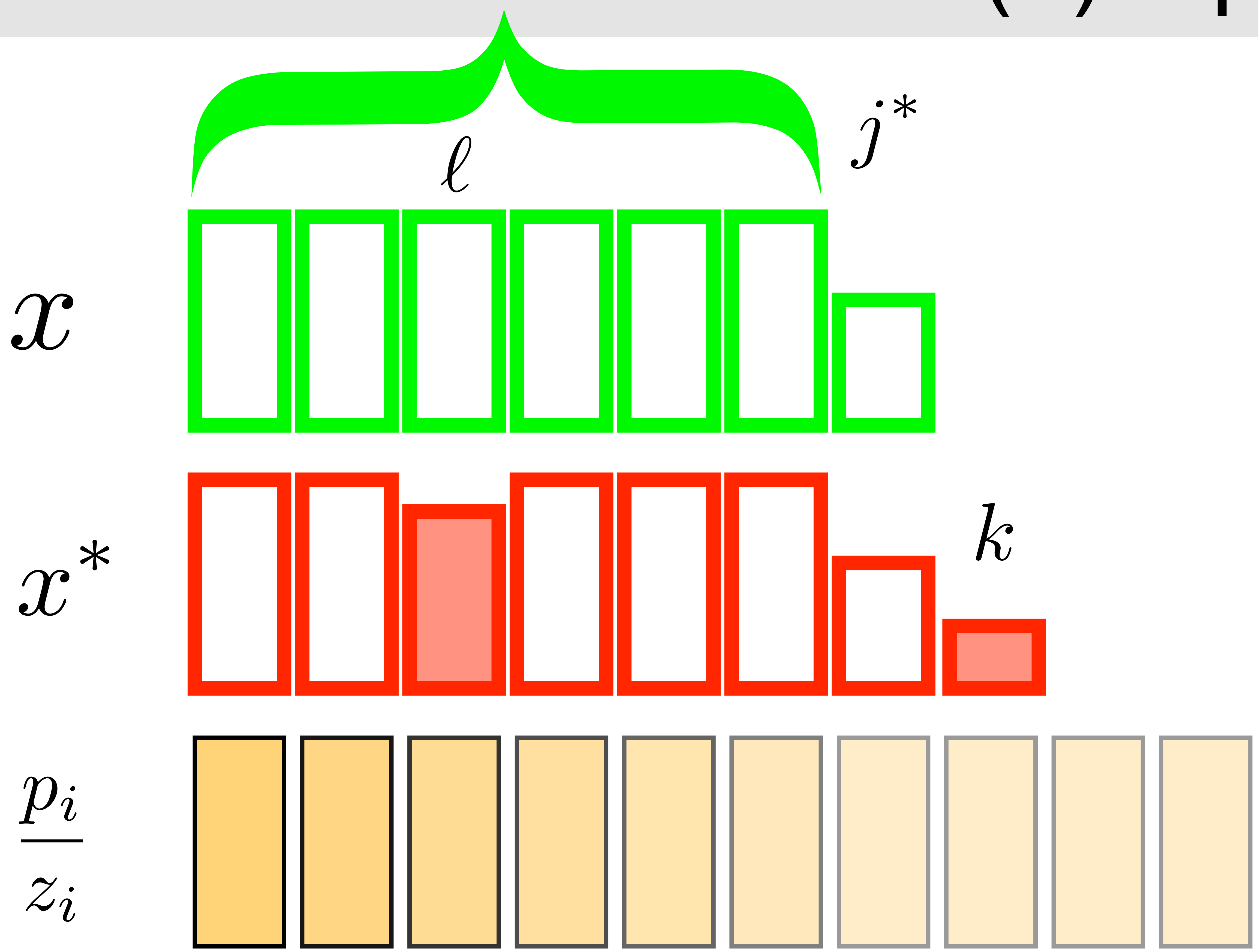

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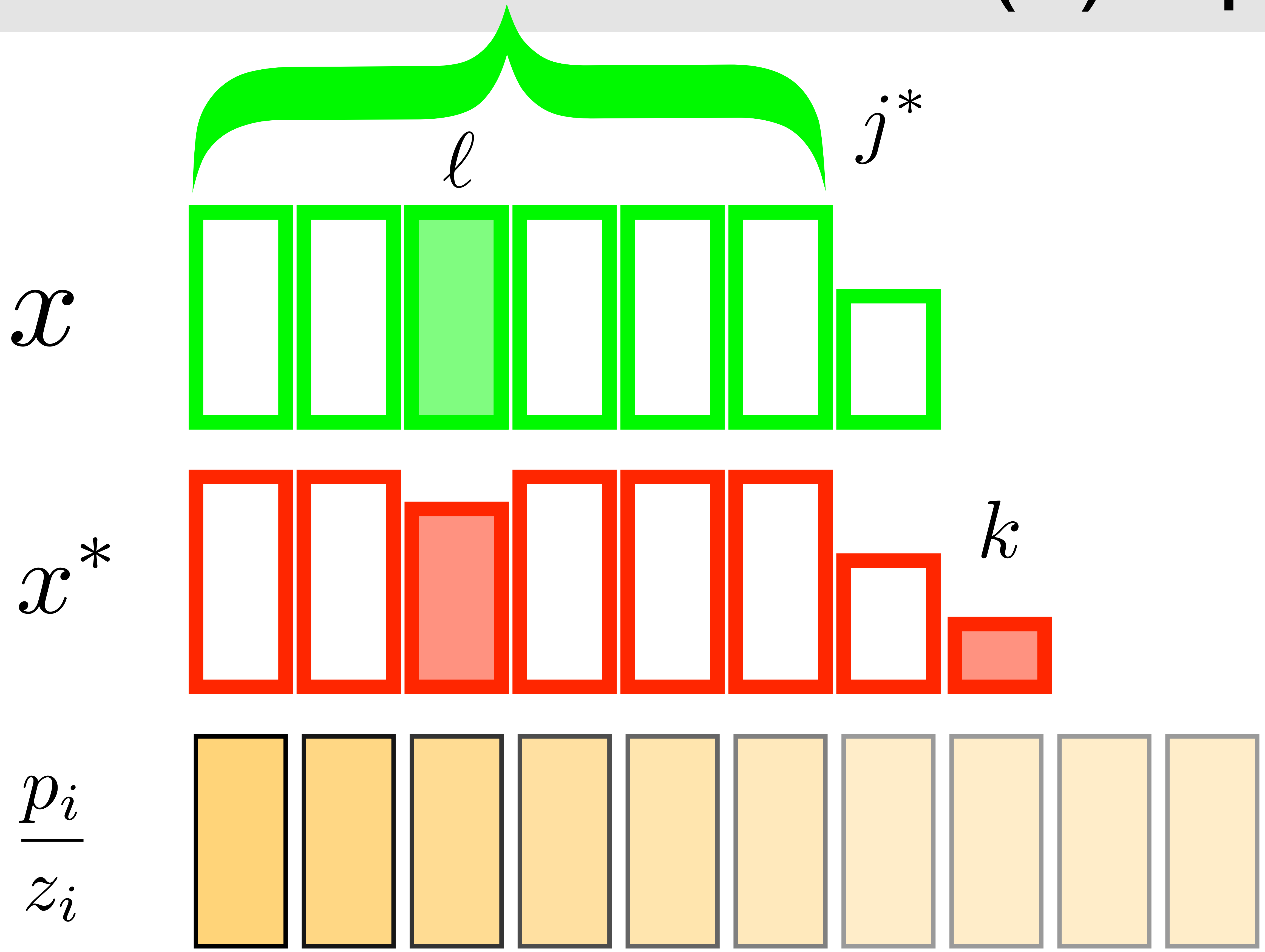

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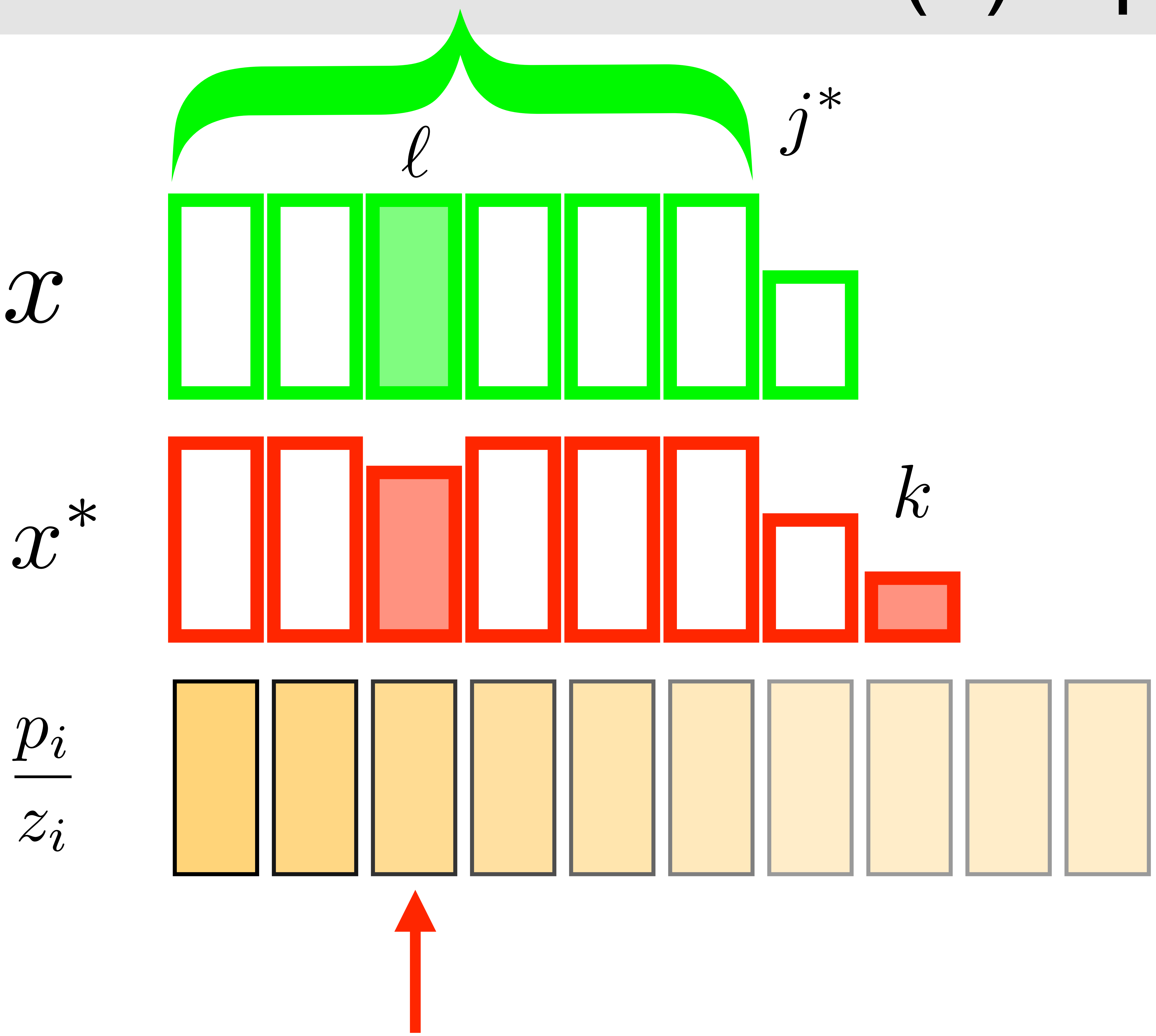

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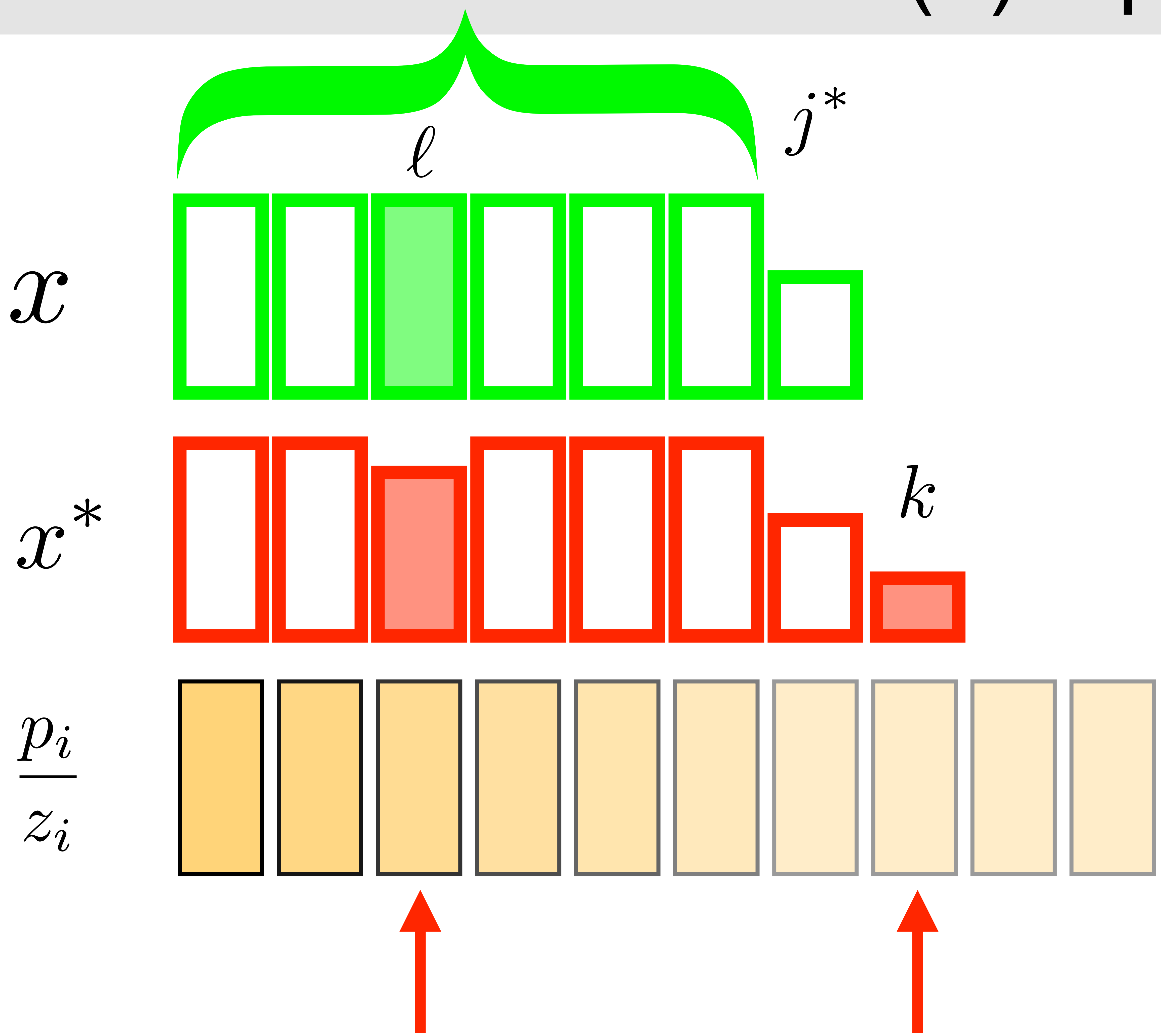

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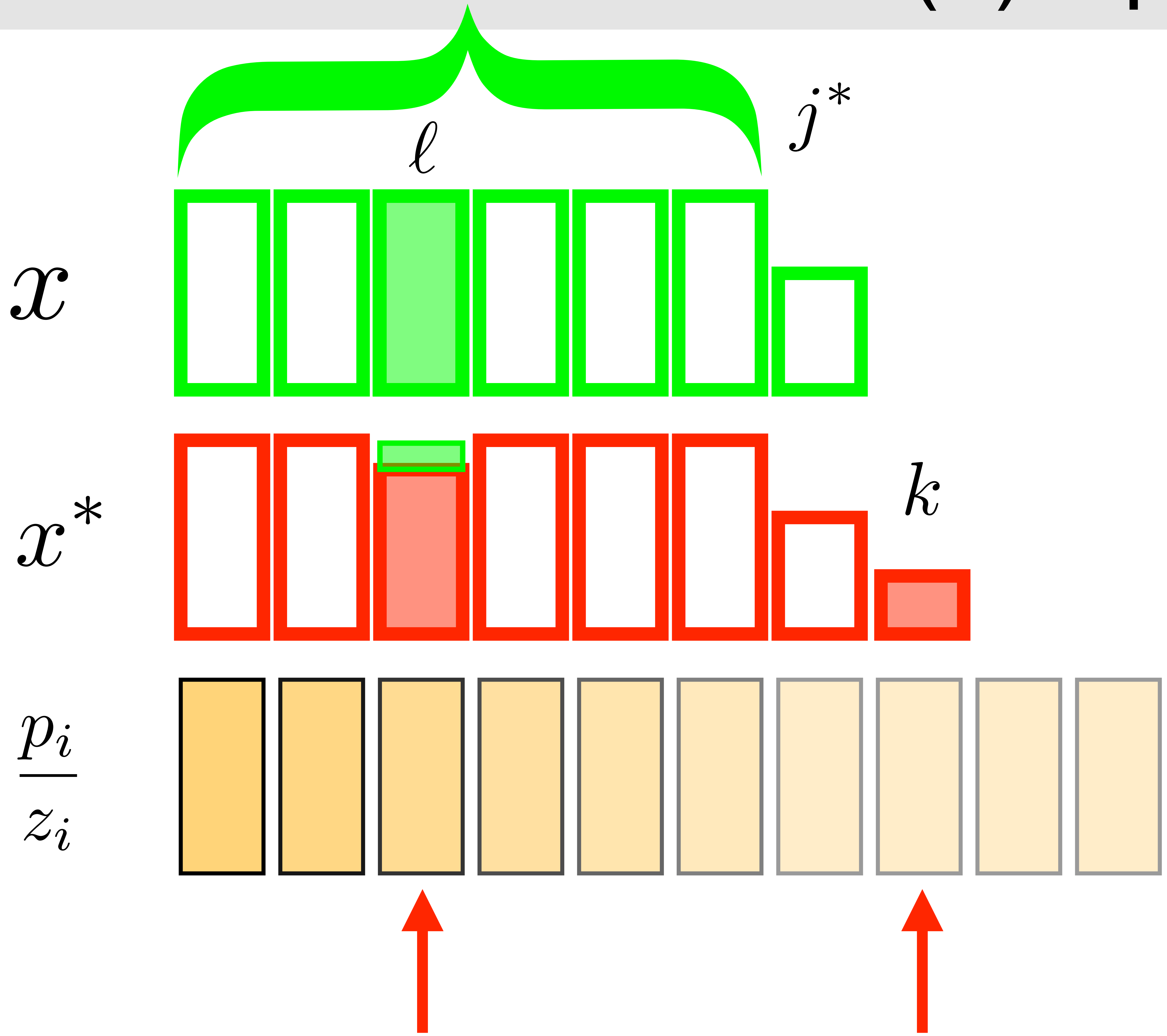

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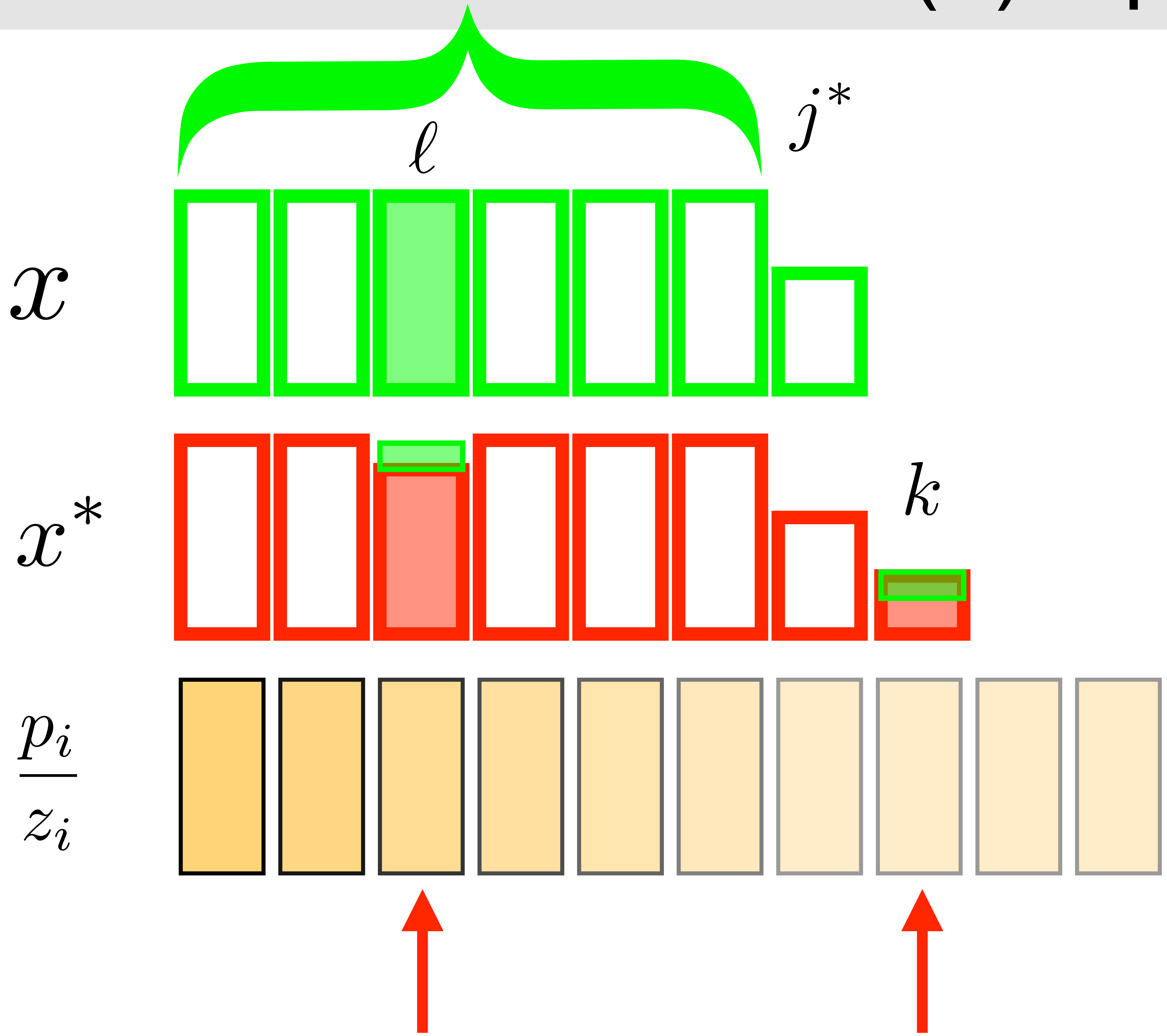

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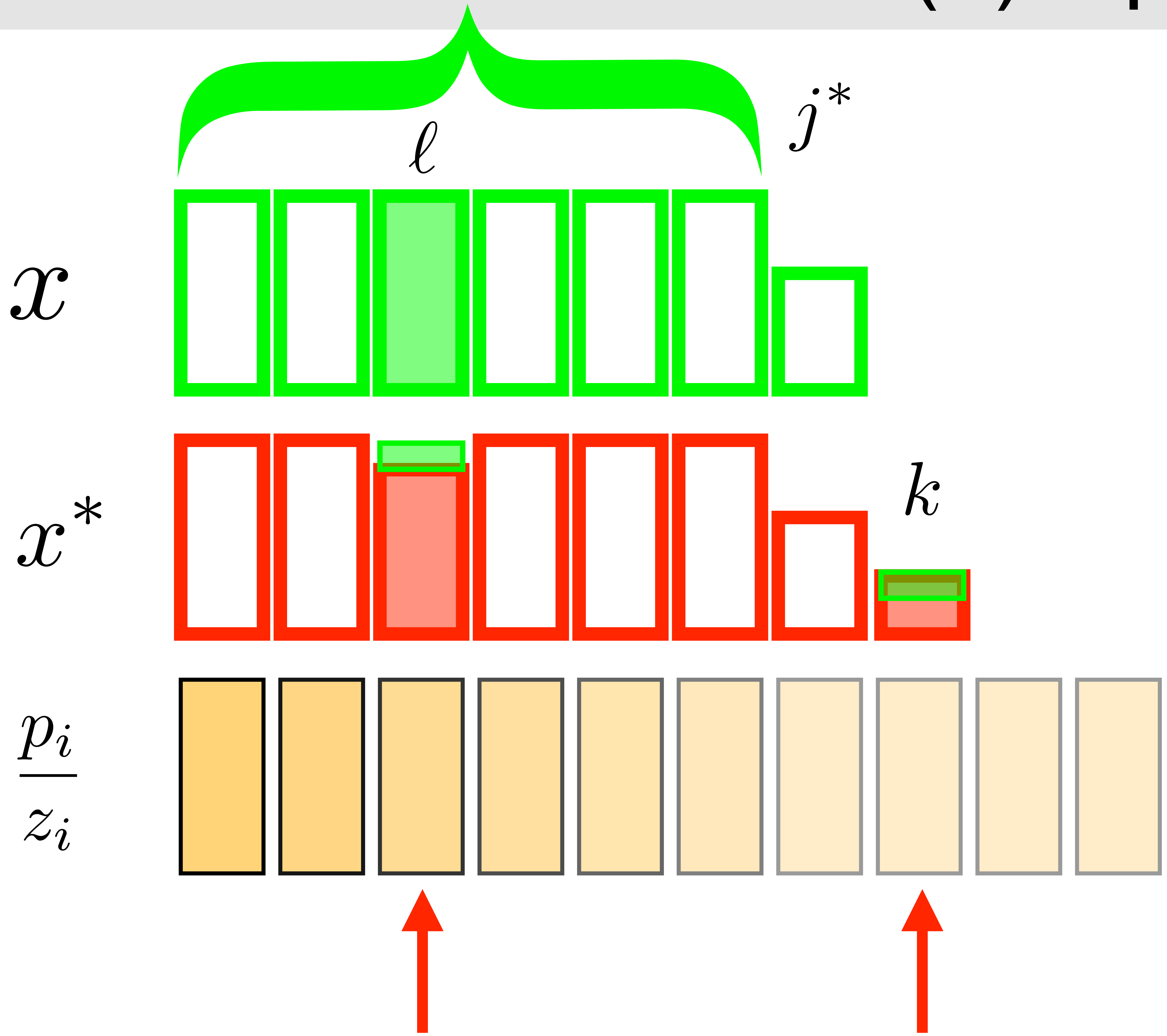

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x^* nicht optimal!

Verallgemeinerung: Mehrfachverwendung

Problem 1.6 (INTEGER KNAPSACK).

Gegeben:

- *n Objekte* $1, \ldots, n$ *mit jeweils Größe* z_i *Gewinn* p_i
- *Größenschranke* Z

Gesucht:

Für jedes Objekt ein Wert

$$\boxed{x_i \in \mathbb{N}}$$

mit

$$\sum_{i=1}^{n} z_i x_i \leq Z$$

und

$$\sum_{i=1}^{n} p_i x_i = Maximal$$

Spezialfall: Alle Gewinndichten gleich

Problem 1.7.
Gegeben:

- n Objekte $1, \ldots, n$ mit jeweils Größe z_i
- Größenschranke Z

Gesucht:
Eine Menge

$$S \subseteq \{1, \ldots, n\}$$

mit

$$\sum_{i \in S} z_i \leq Z$$

und

$$\sum_{i \in S} z_i = Maximal$$

Spezialfall des Spezialfalls: Größenschranke treffen

Problem 1.8 (Subset Sum).

Gegeben:

- n *Objekte* $1, \ldots, n$ *mit jeweils Größe* z_i
- *Zielgröße* Z

Gesucht:

Eine Menge

$$S \subseteq \{1, \ldots, n\}$$

mit

$$\sum_{i \in S} z_i = Z$$

Spezialfall³: Partition

Problem 1.9 (PARTITION).

Gegeben:

- *n Objekte $1, \ldots, n$ mit jeweils Größe z_i*

Gesucht:

Eine Menge

$$S \subseteq \{1, \ldots, n\}$$

mit

$$\sum_{i \in S} z_i = \sum_{i \notin S} z_i$$

Beispiel

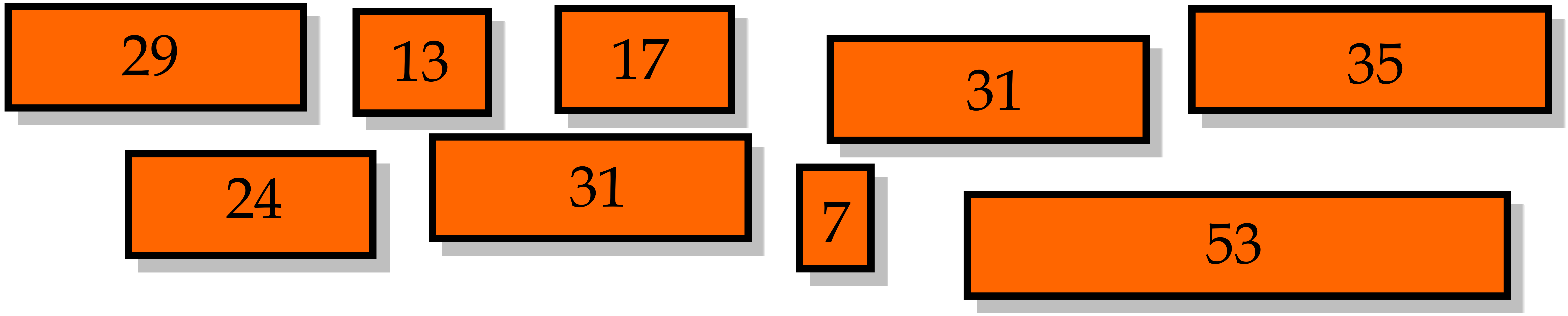
Beispiel 1.10.

Partition für $\{z_1, ..., z_9\} = \{7, 13, 17, 20, 29, 31, 31, 35, 57\}$

Gesamtsumme: 240

Finde Aufteilung in zwei gleiche Teilmengen!

Ein Bild hilft



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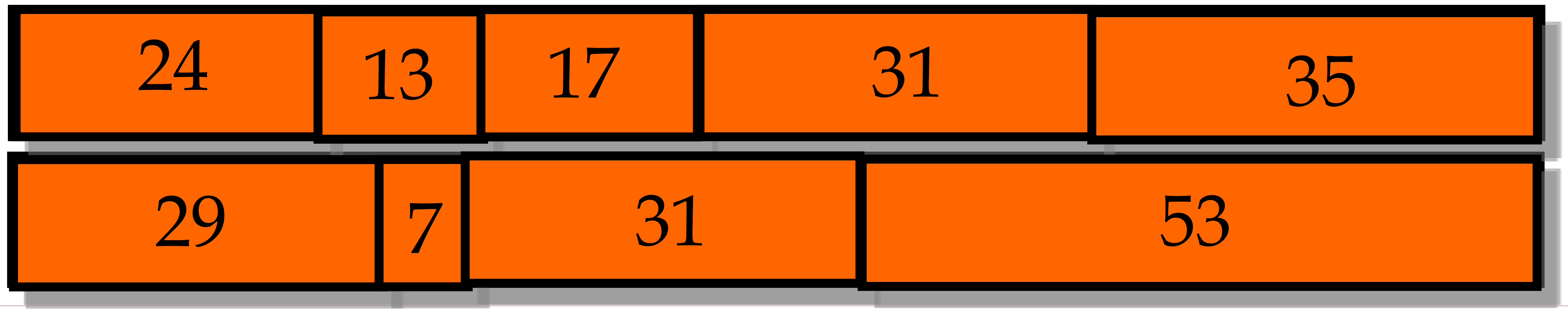

Ein Bild hilft

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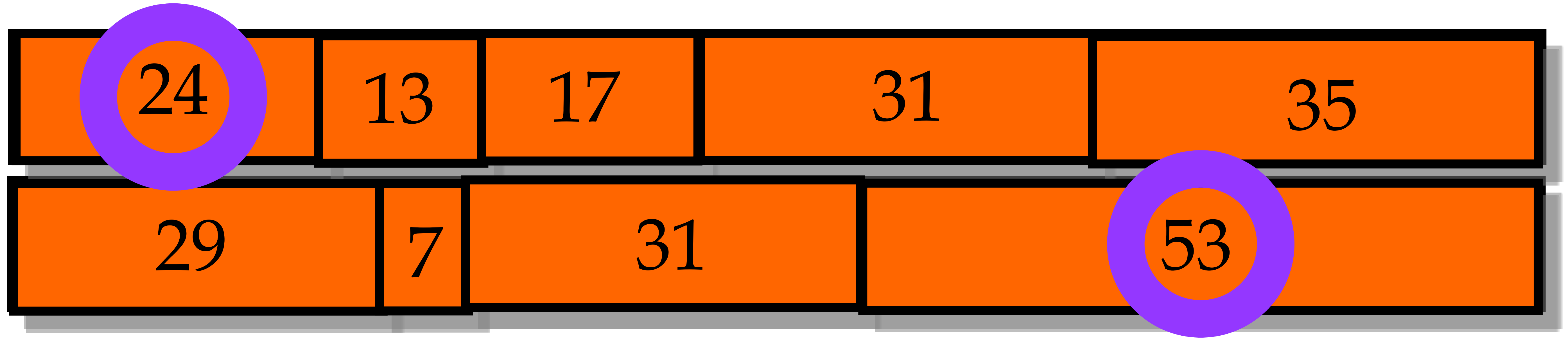
Gesamtsumme: 240



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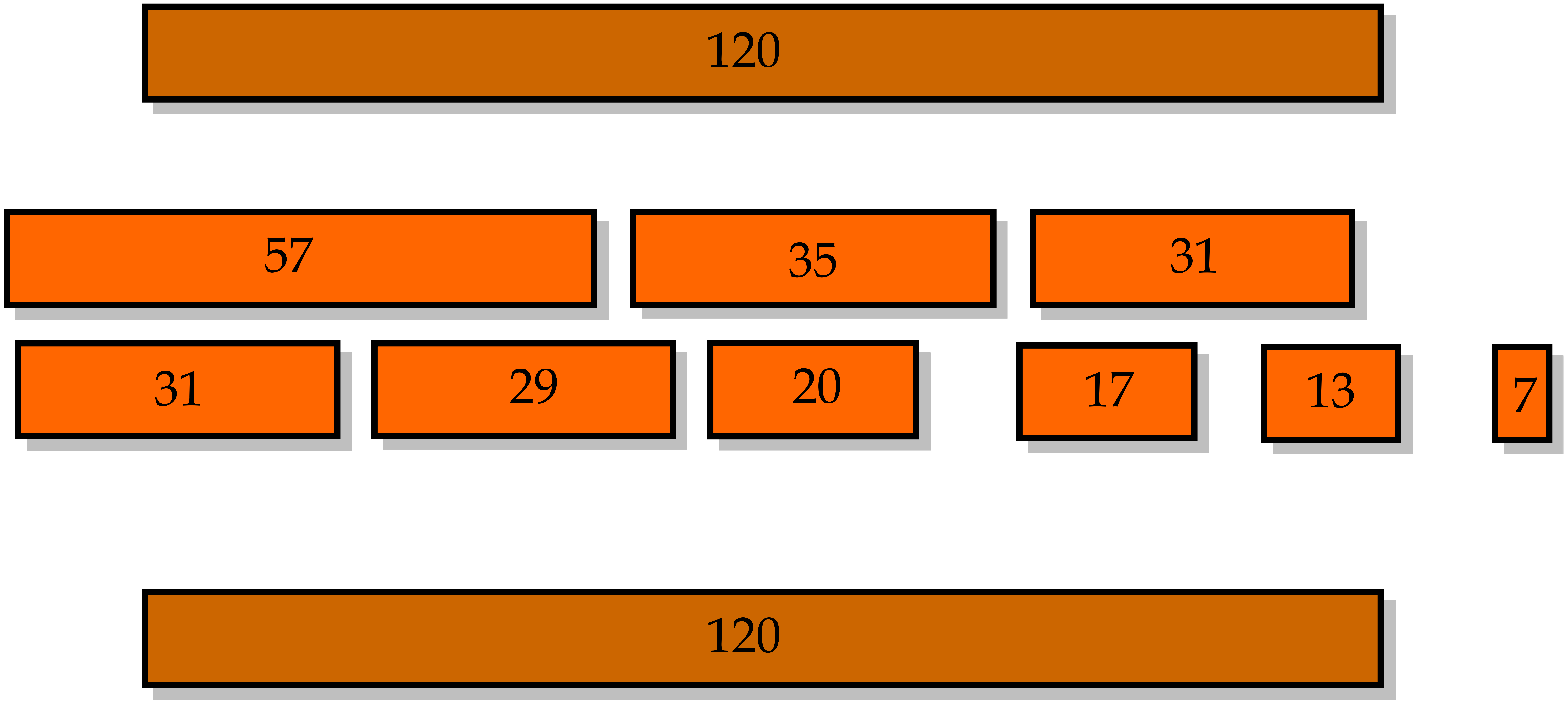

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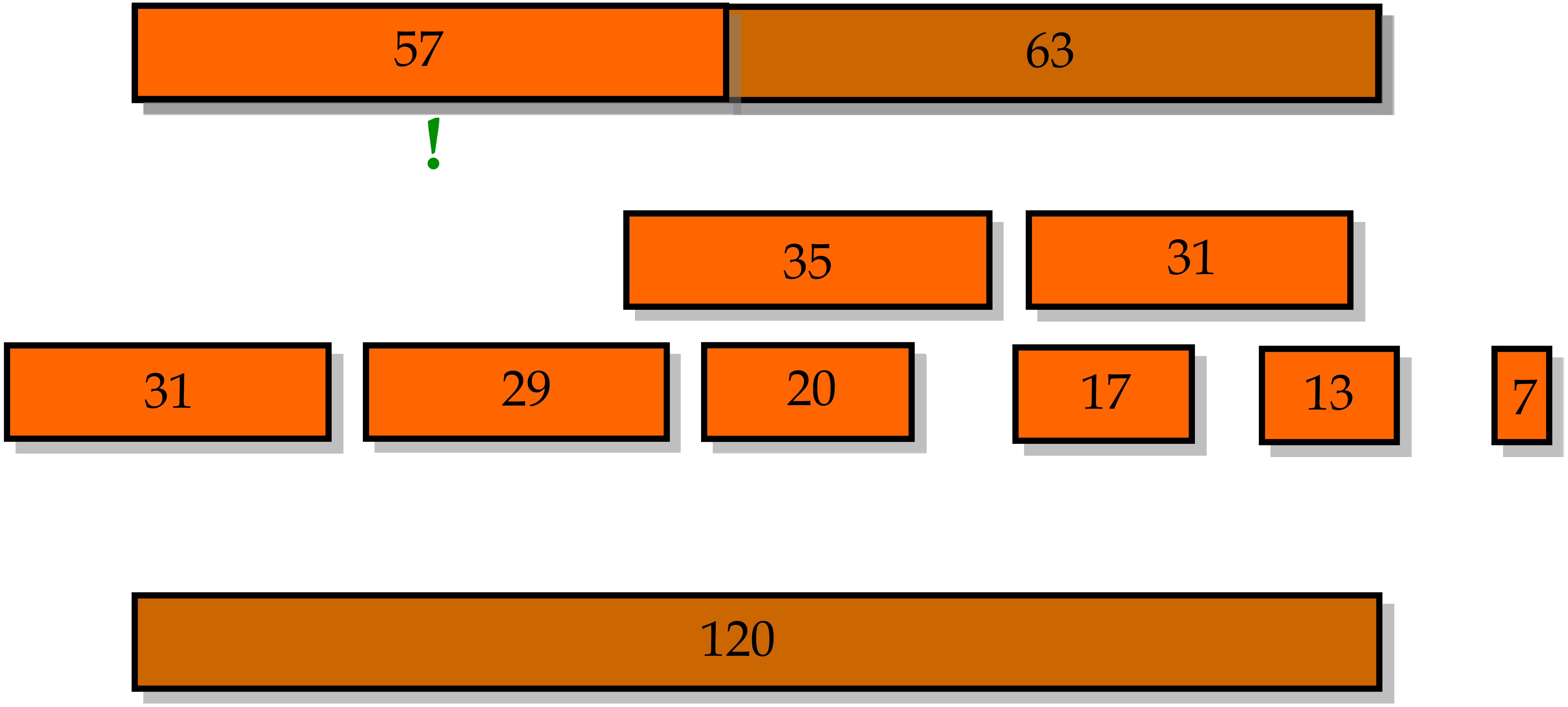

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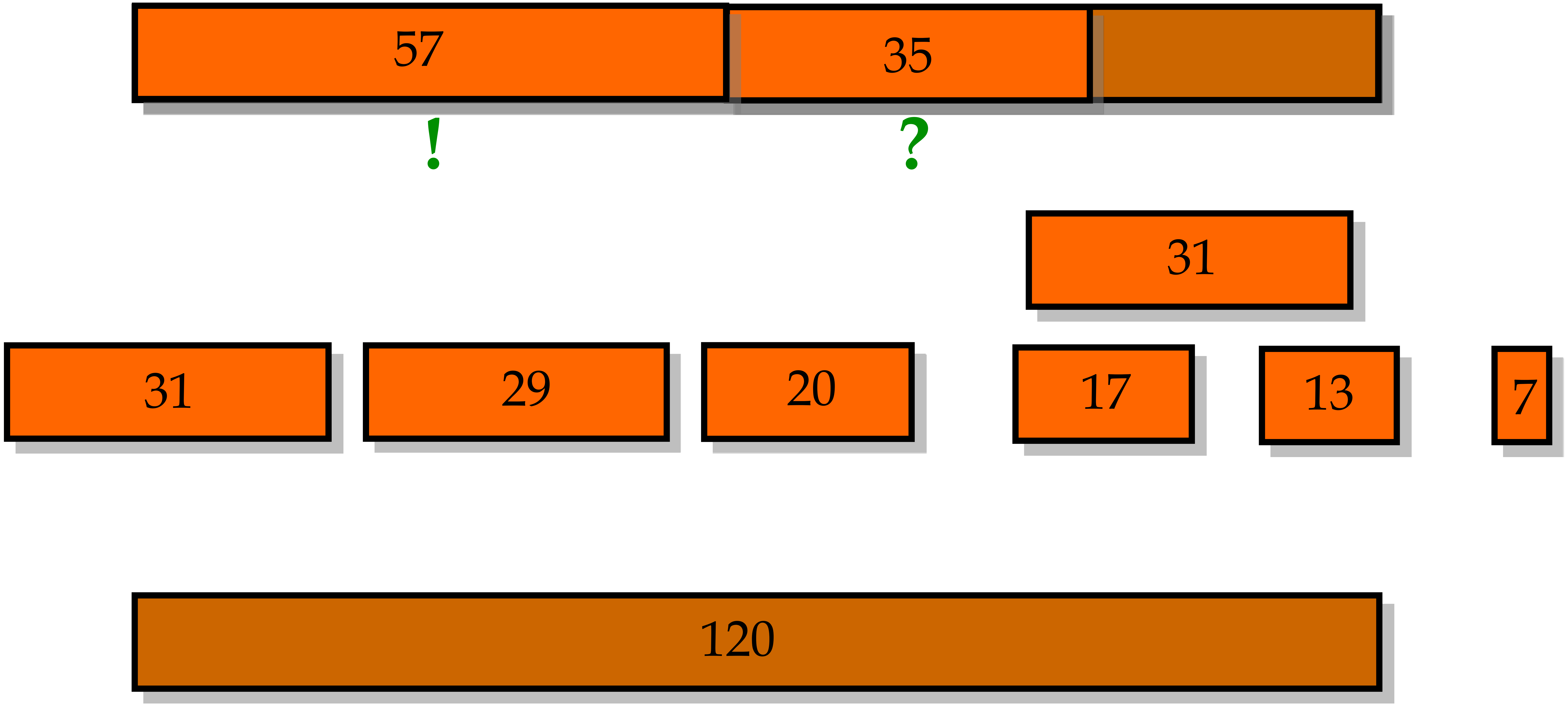

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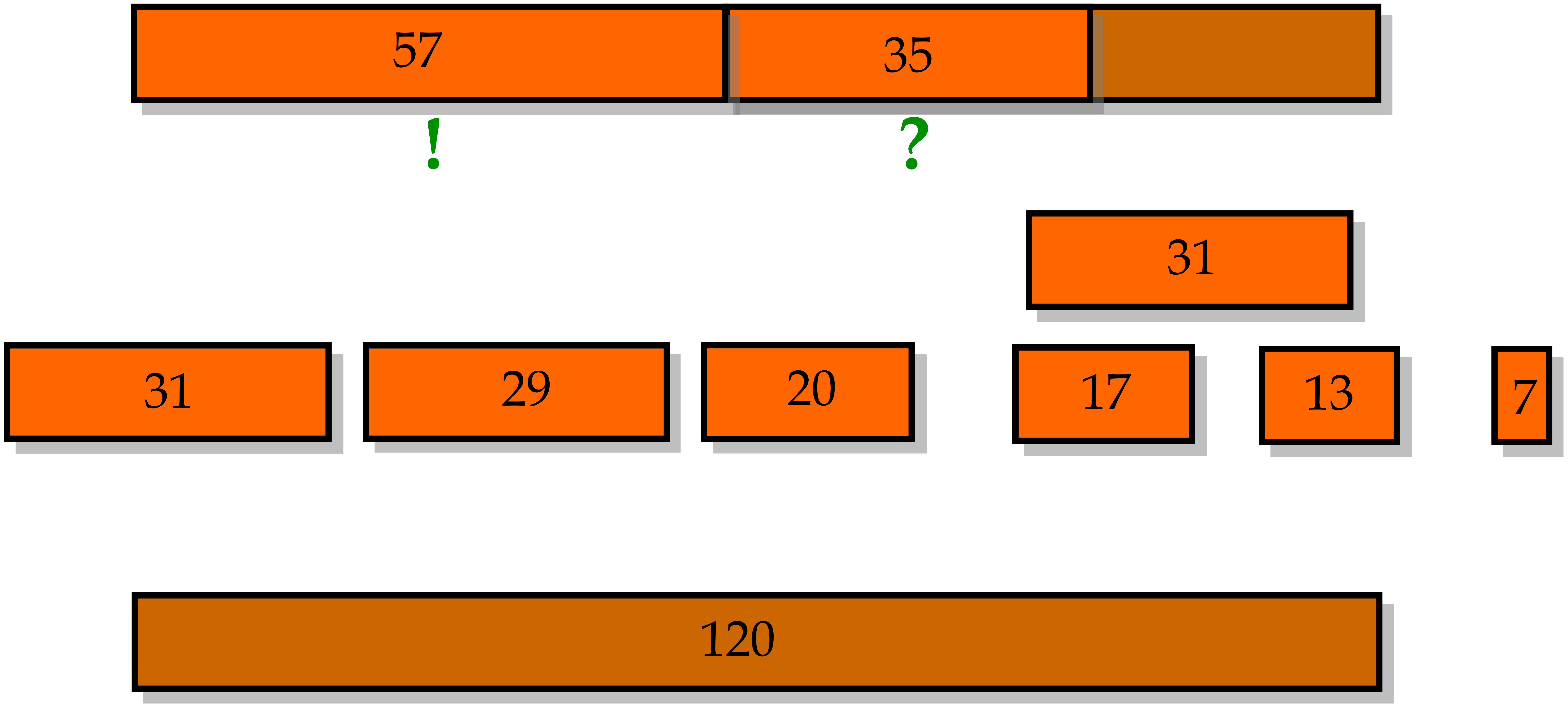

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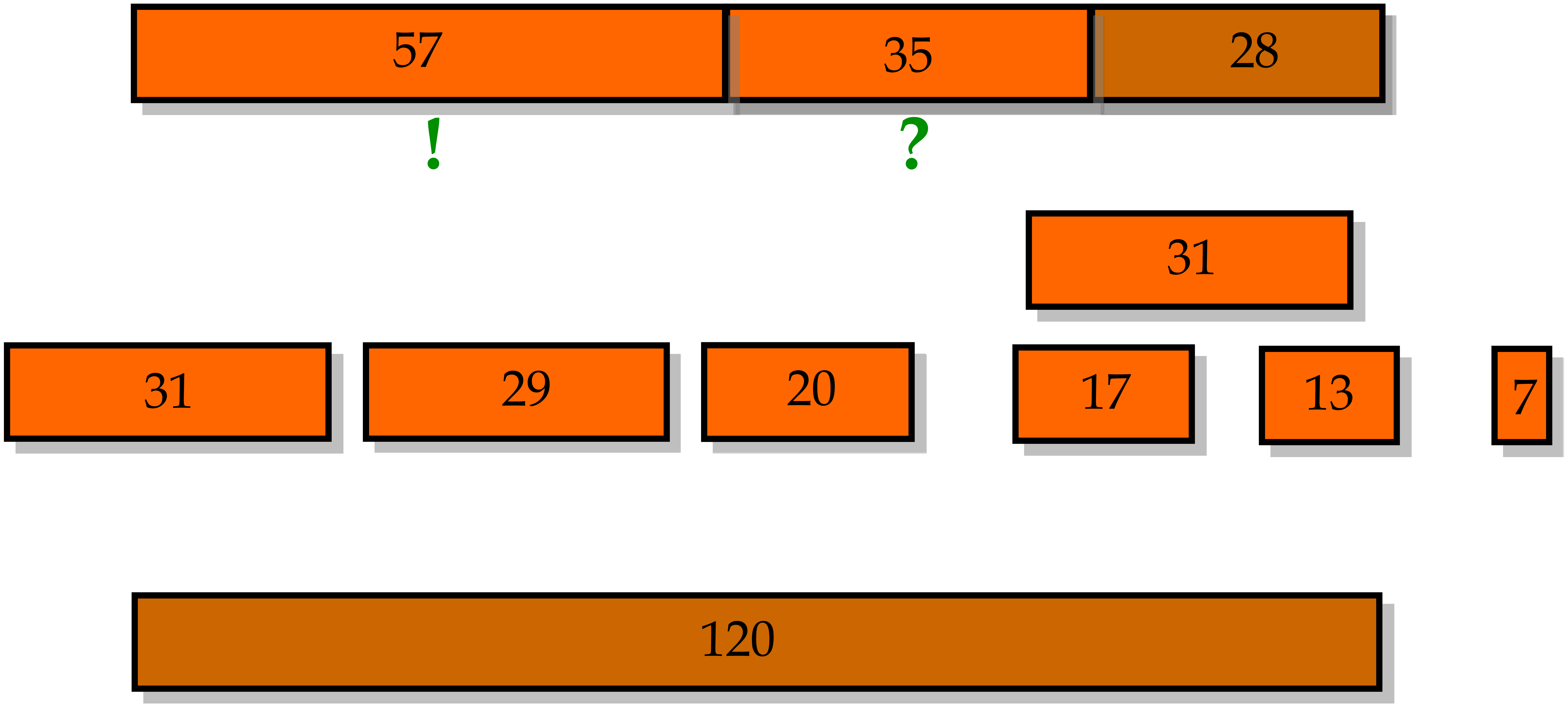

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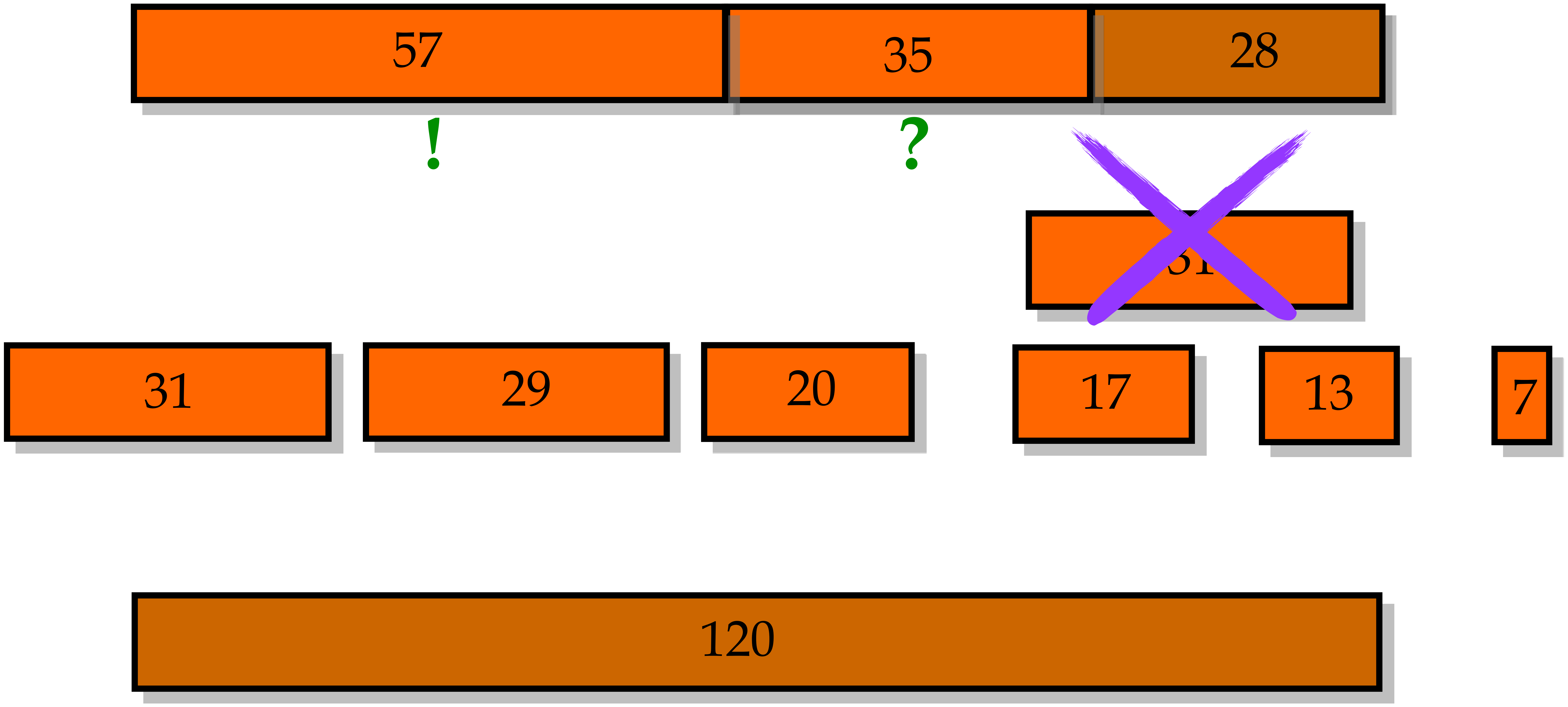

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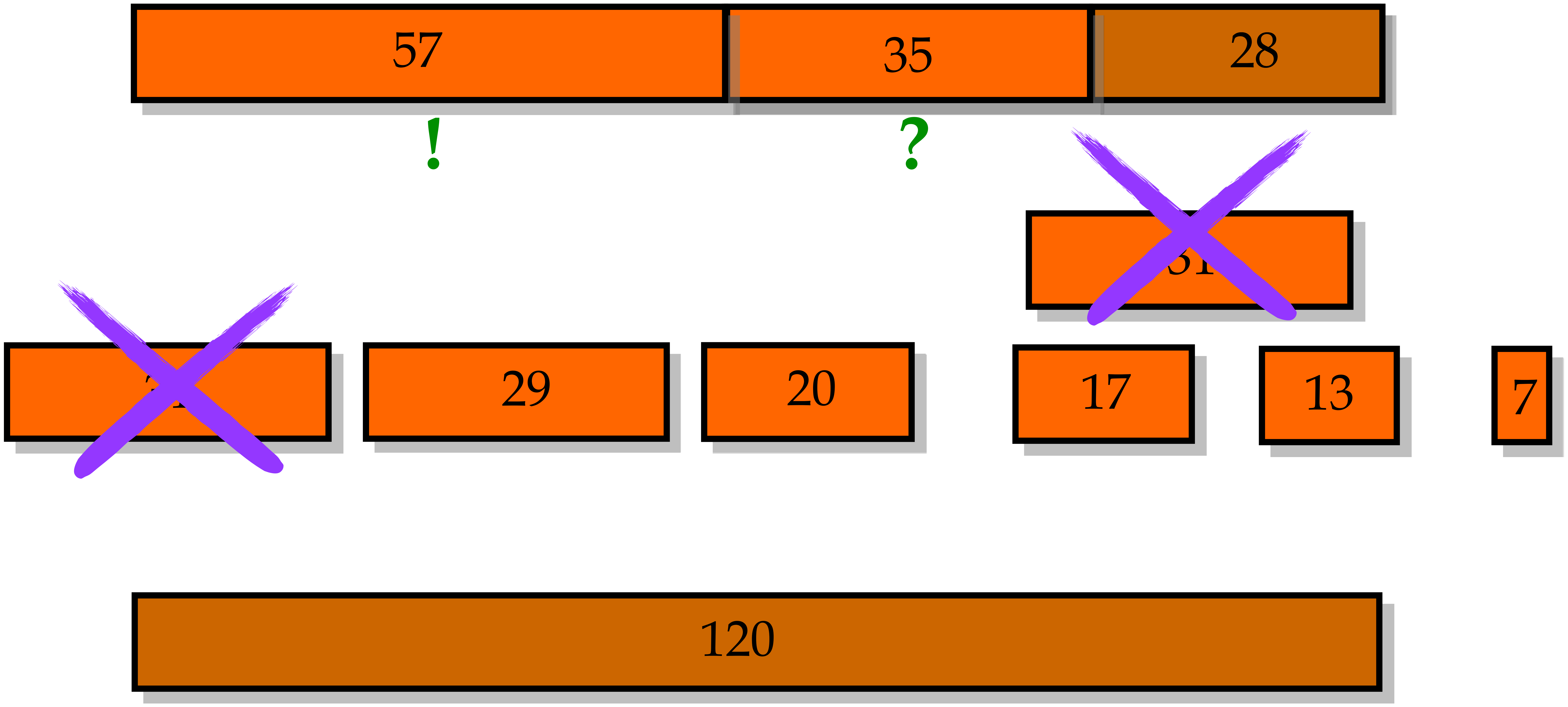

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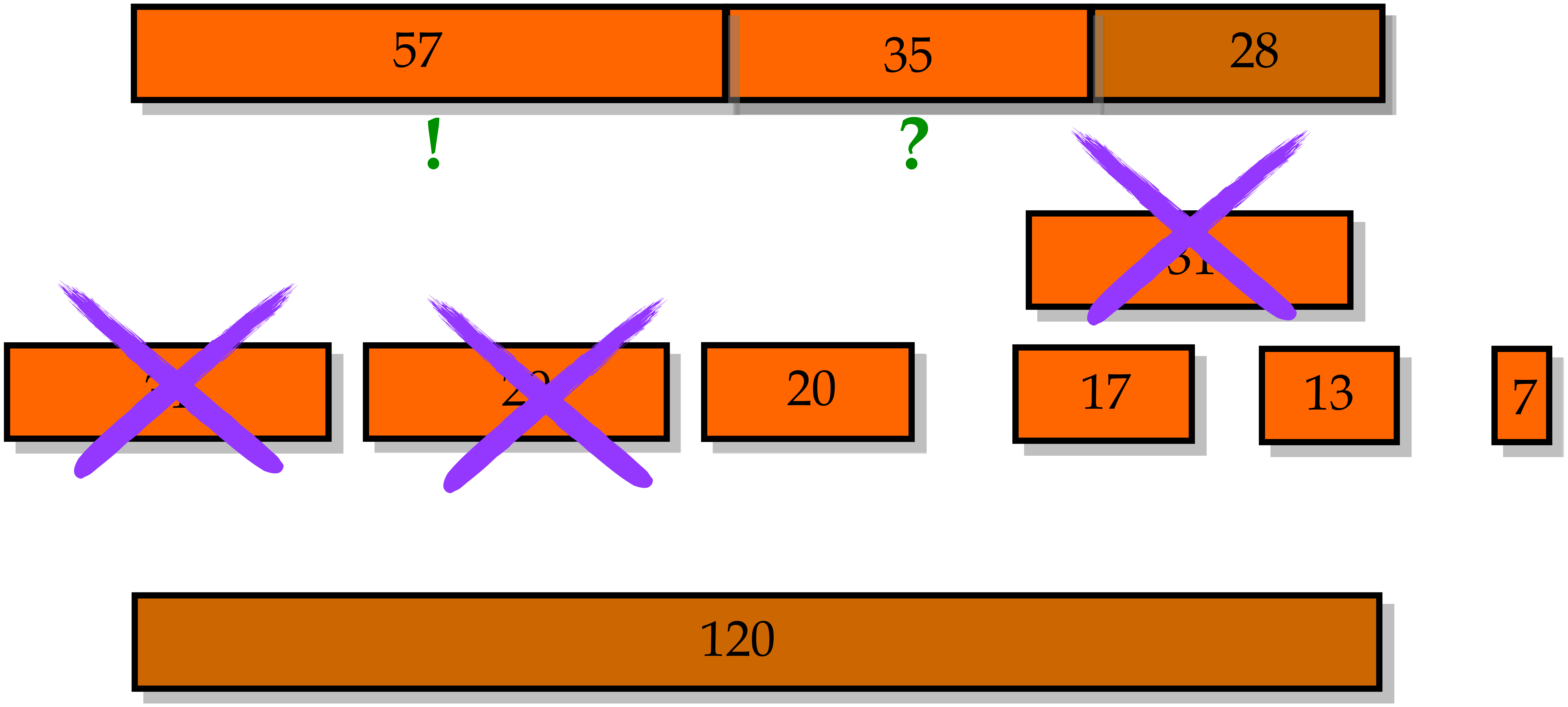

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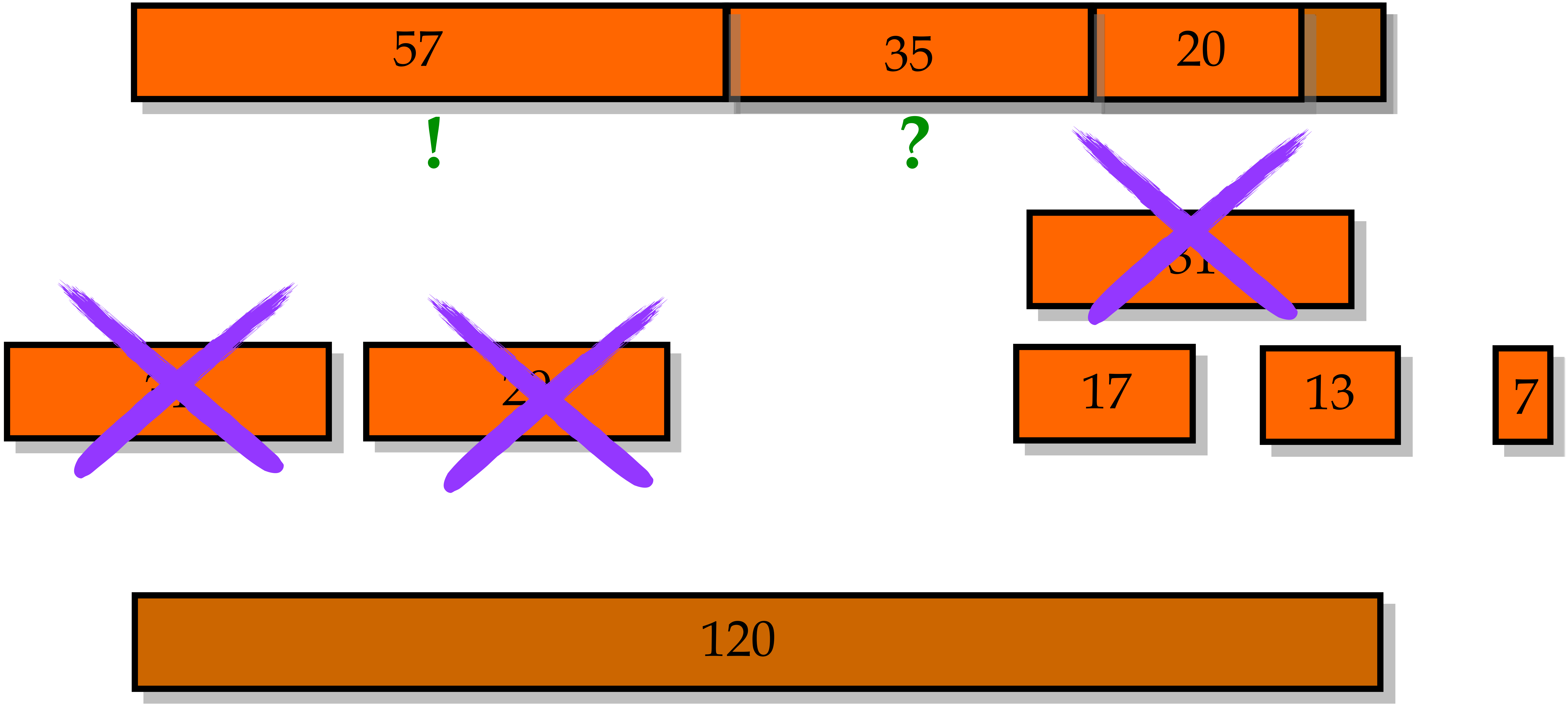

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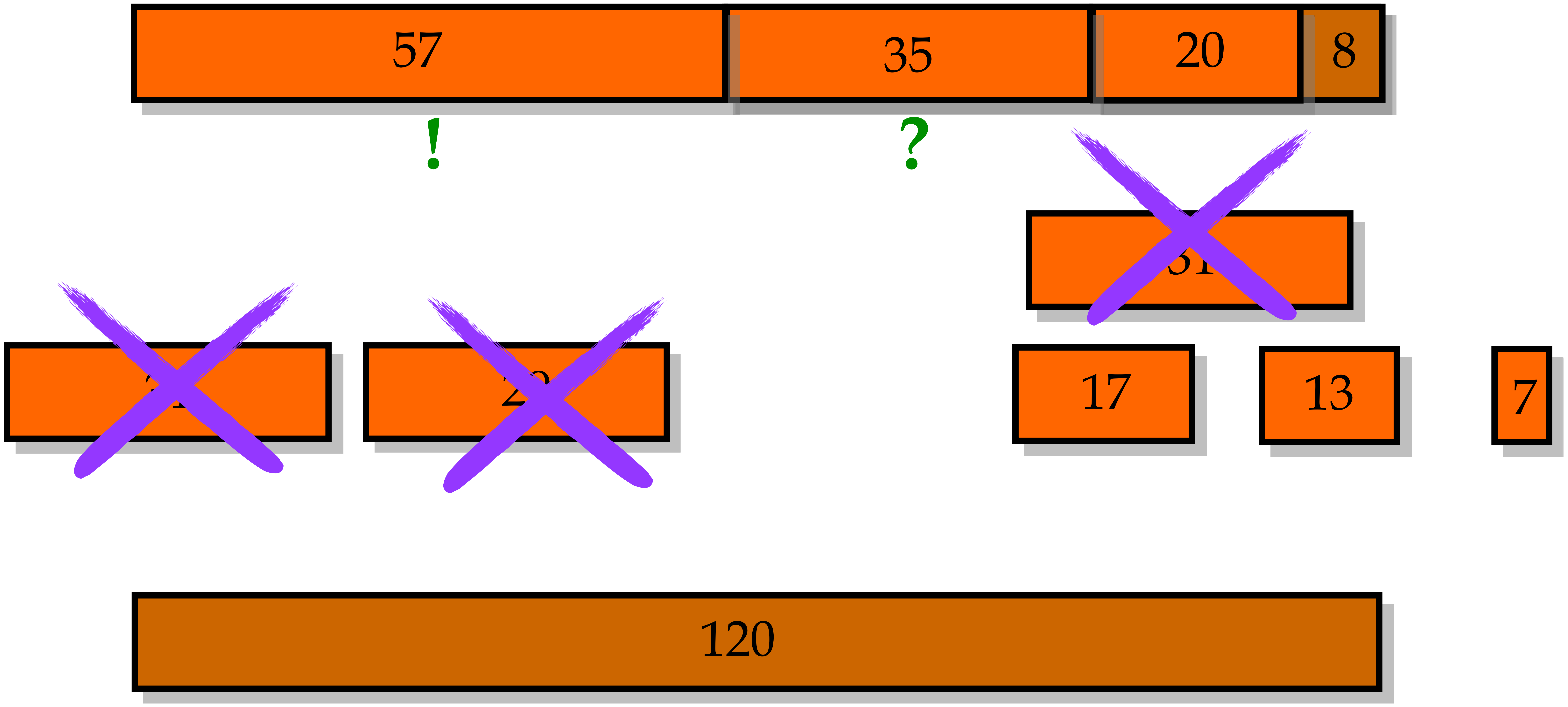

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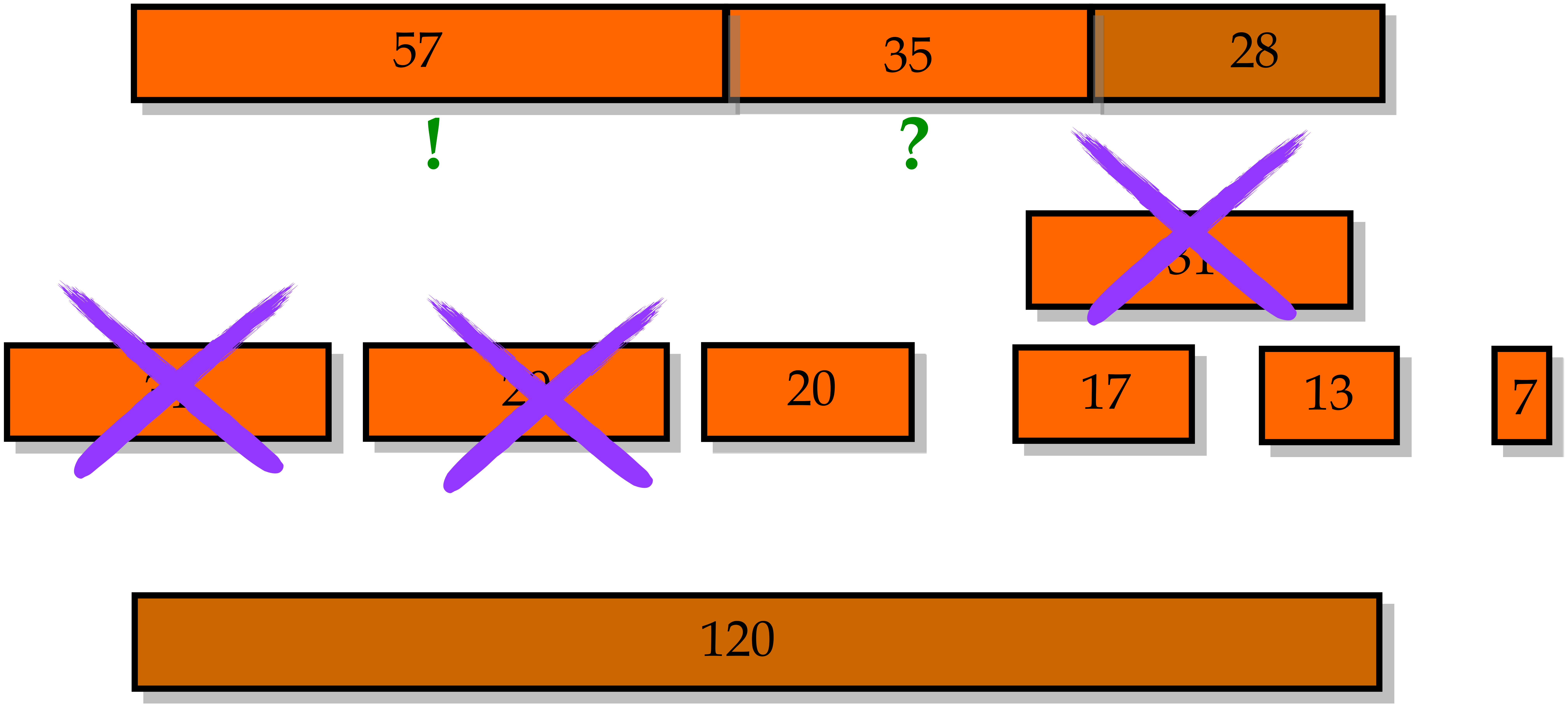

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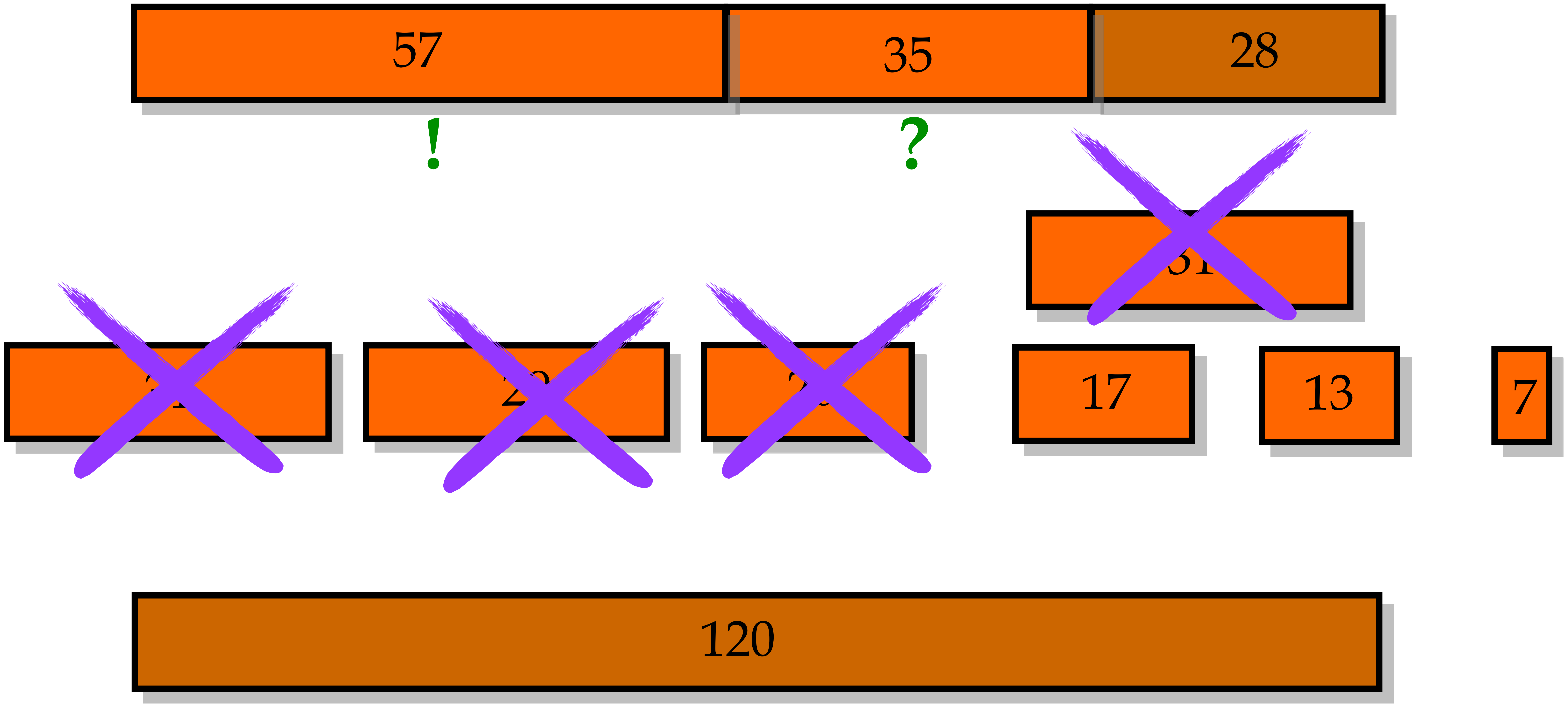

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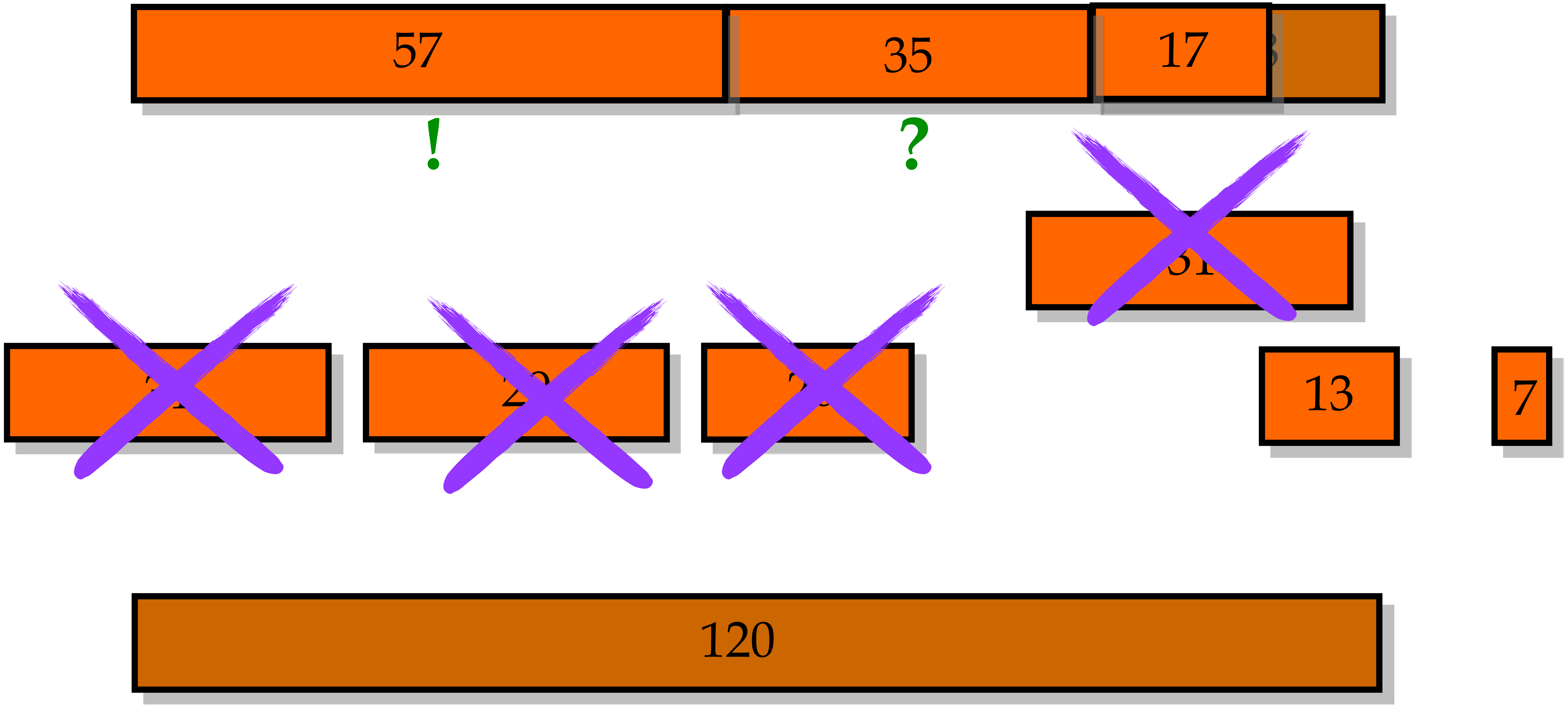

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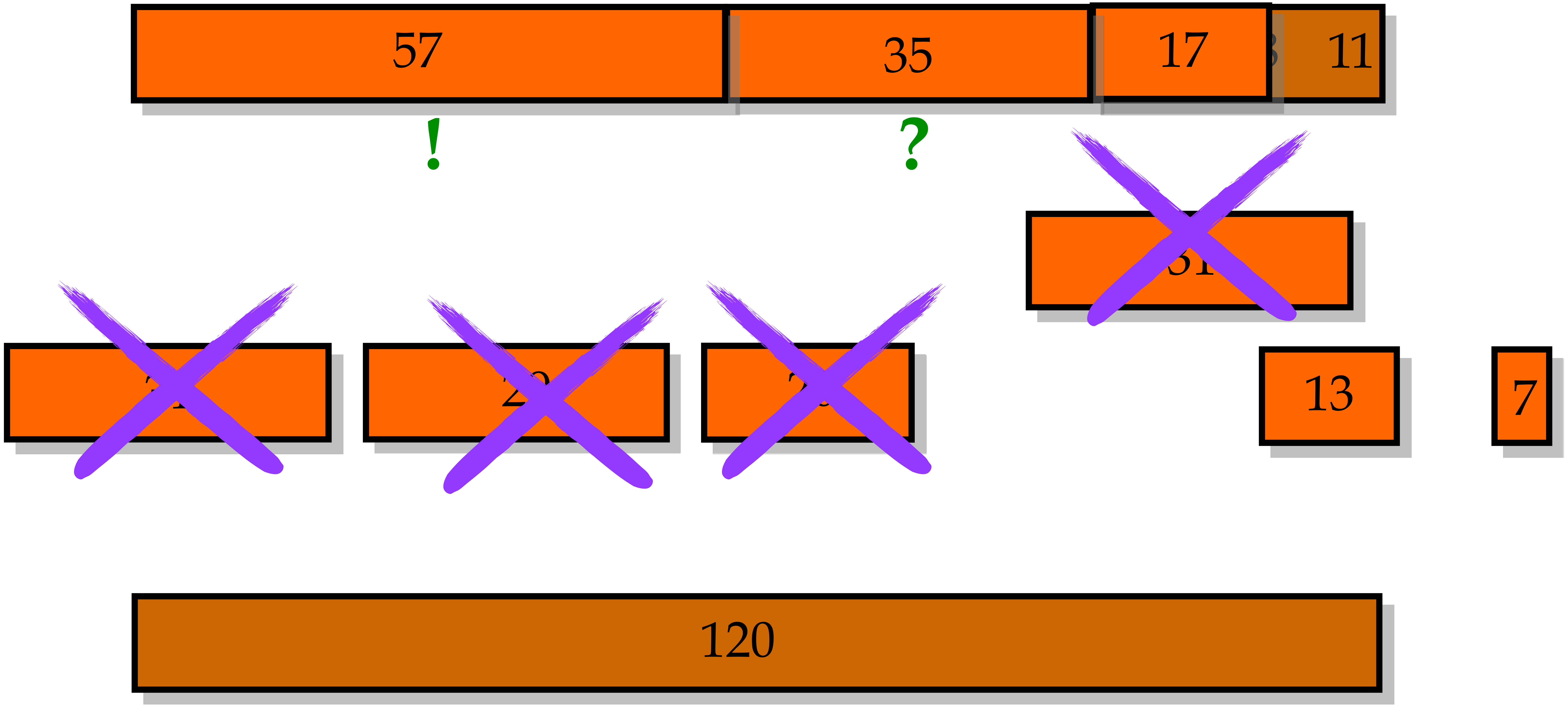

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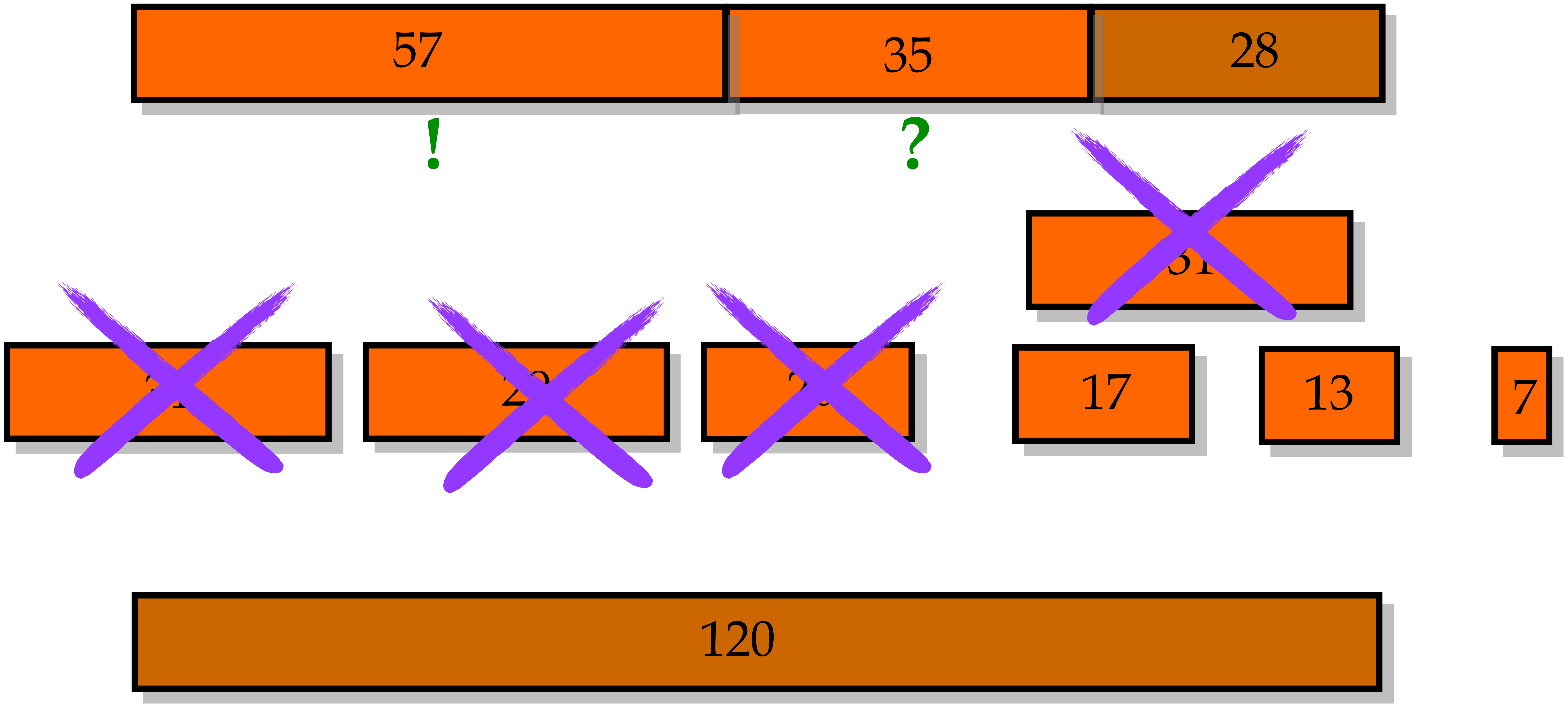

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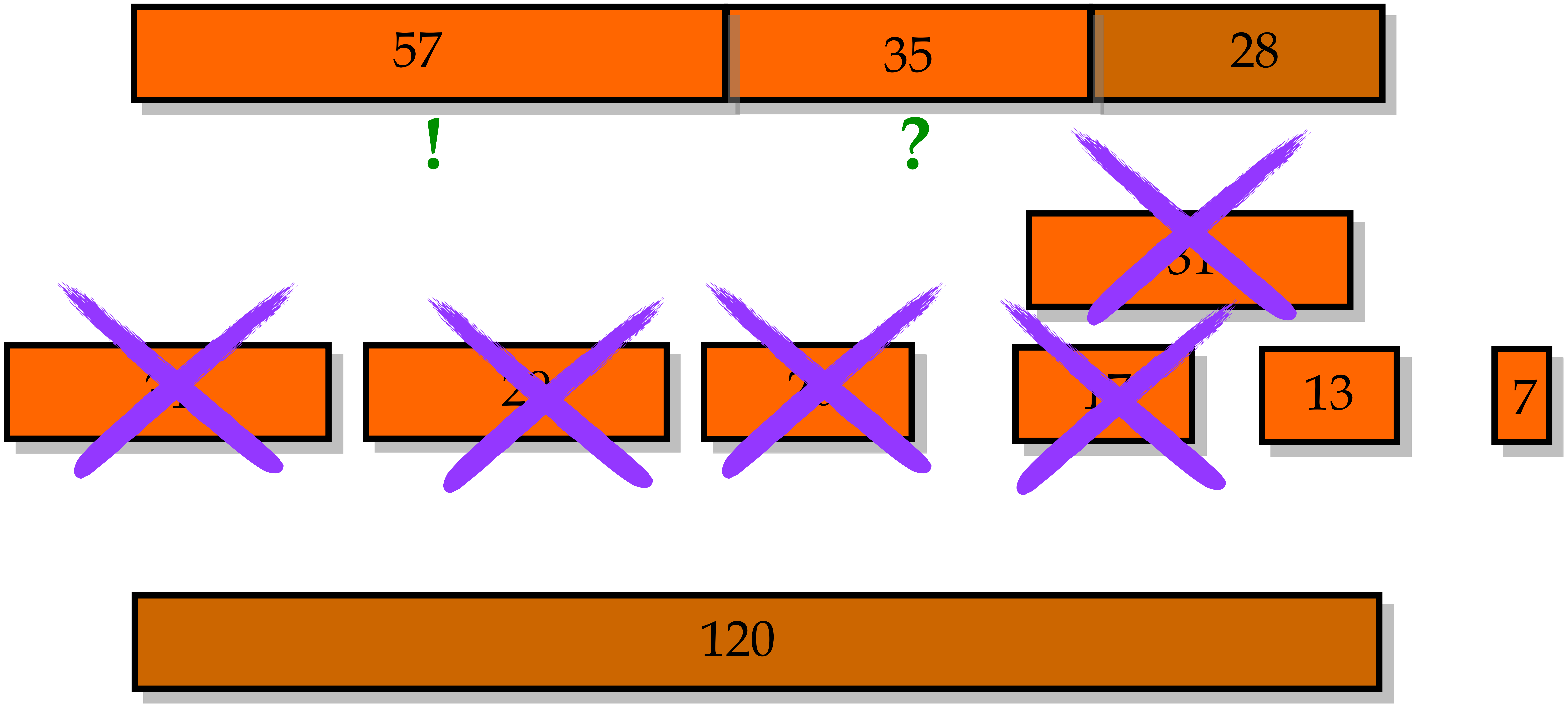

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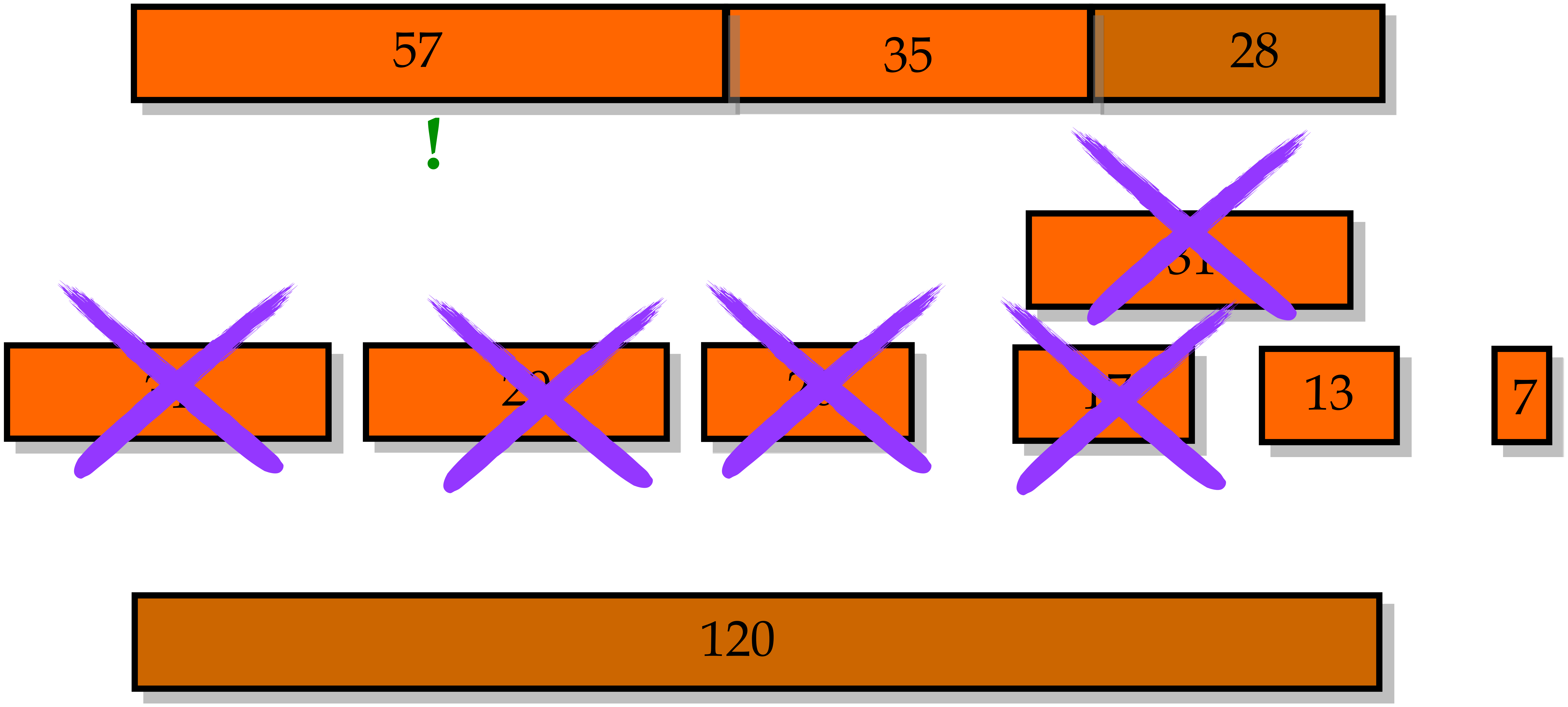

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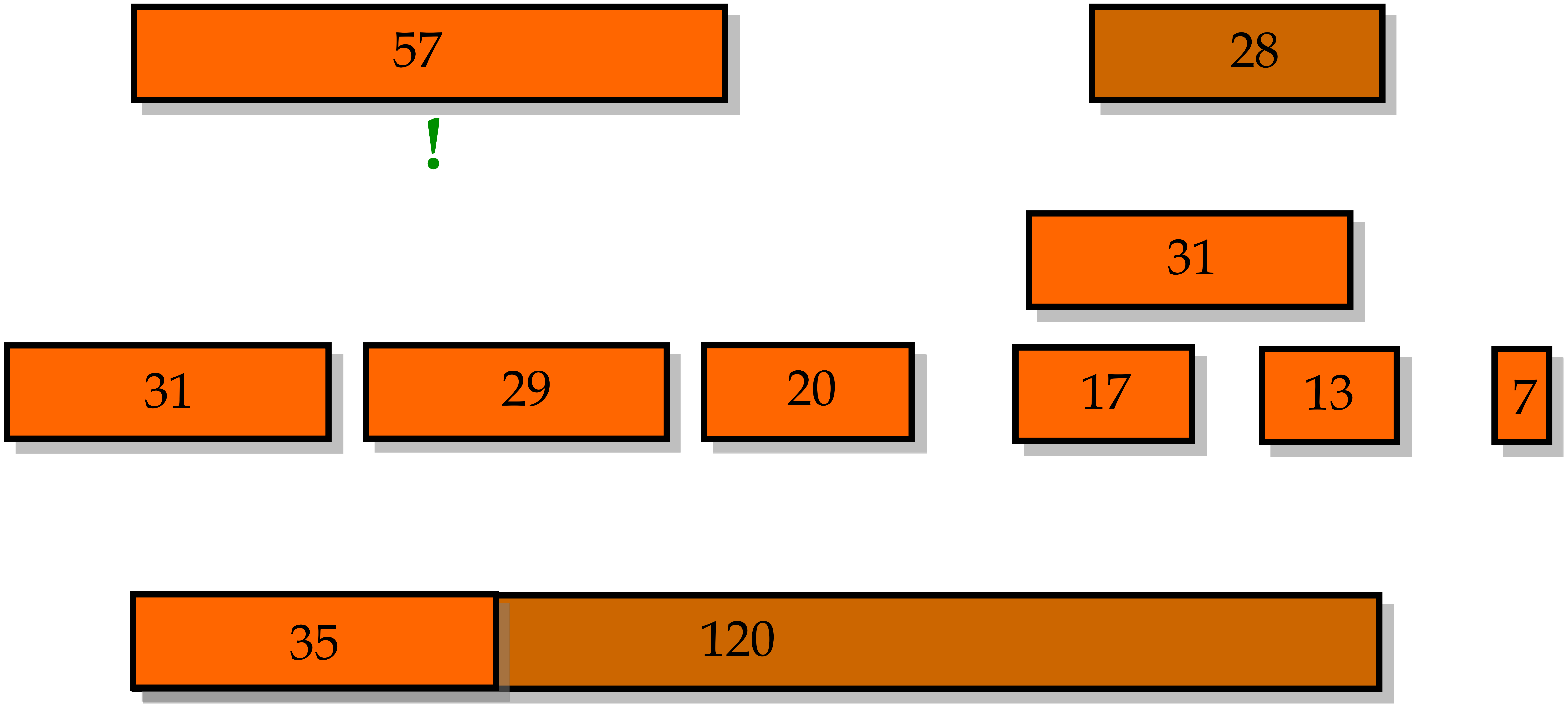

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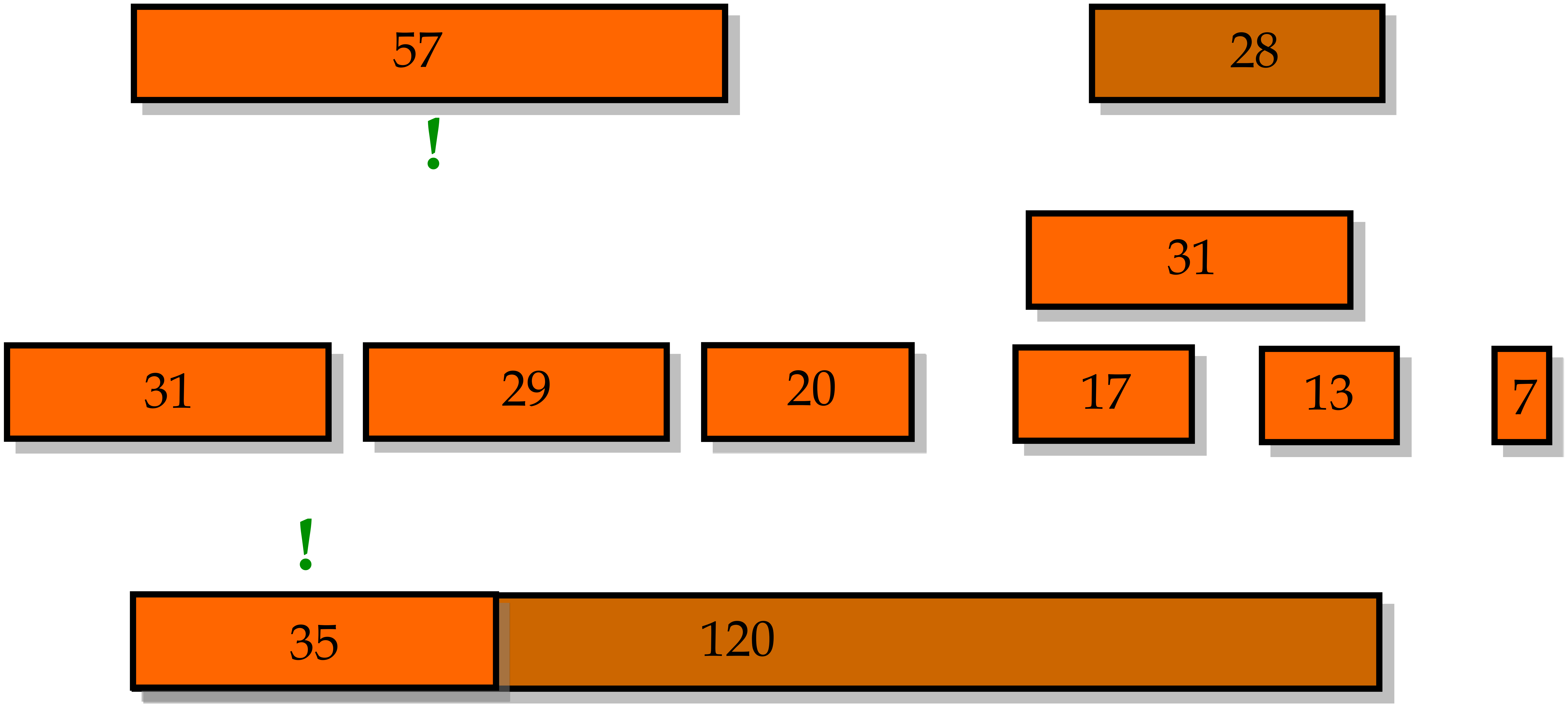

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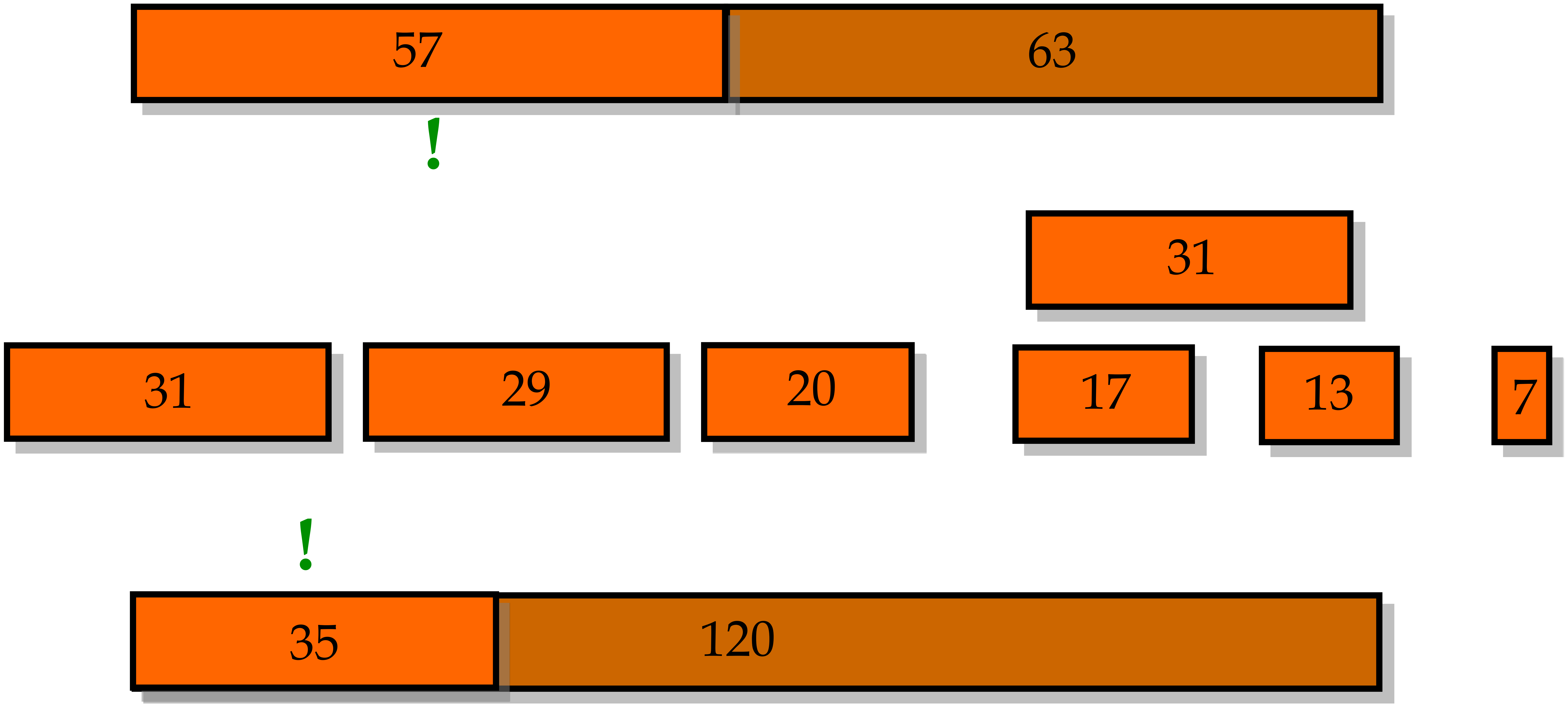

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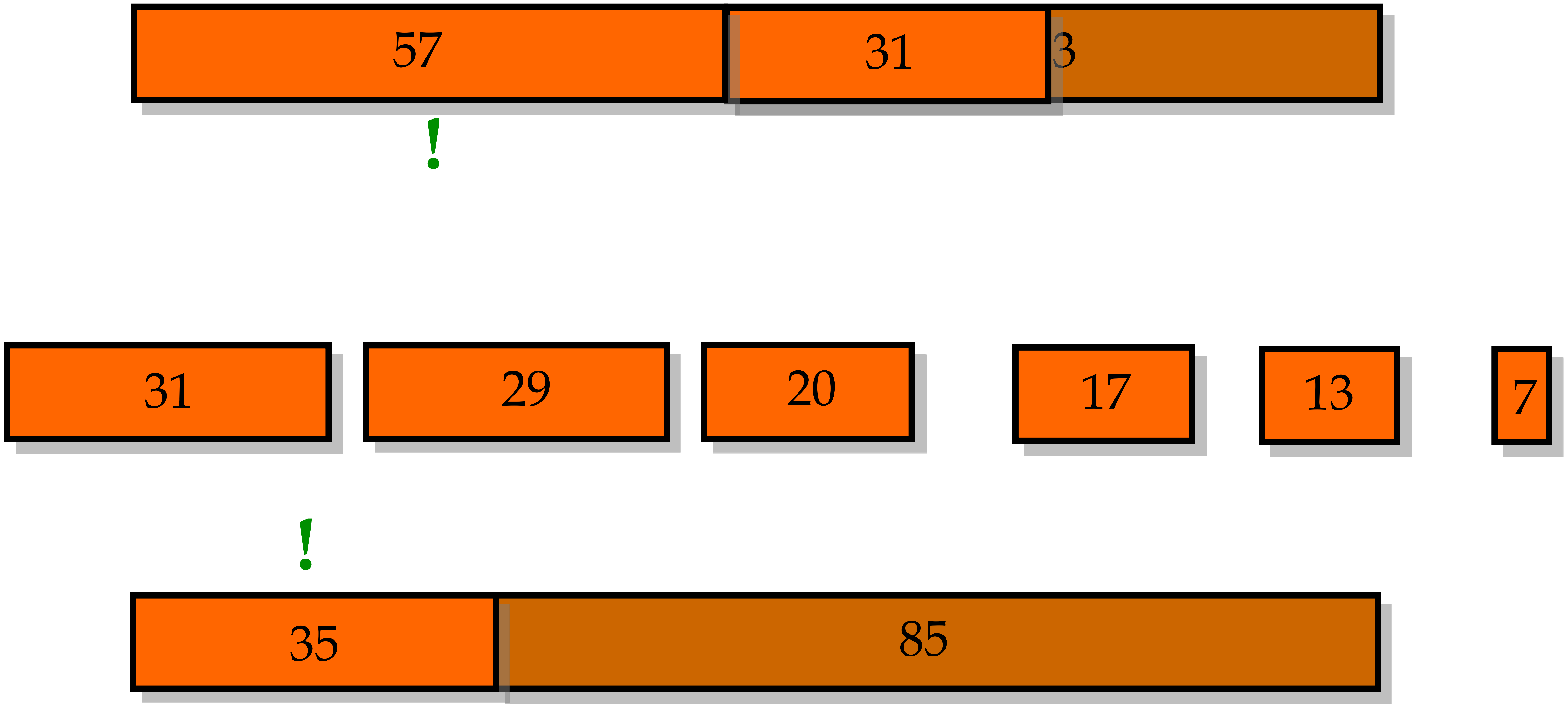

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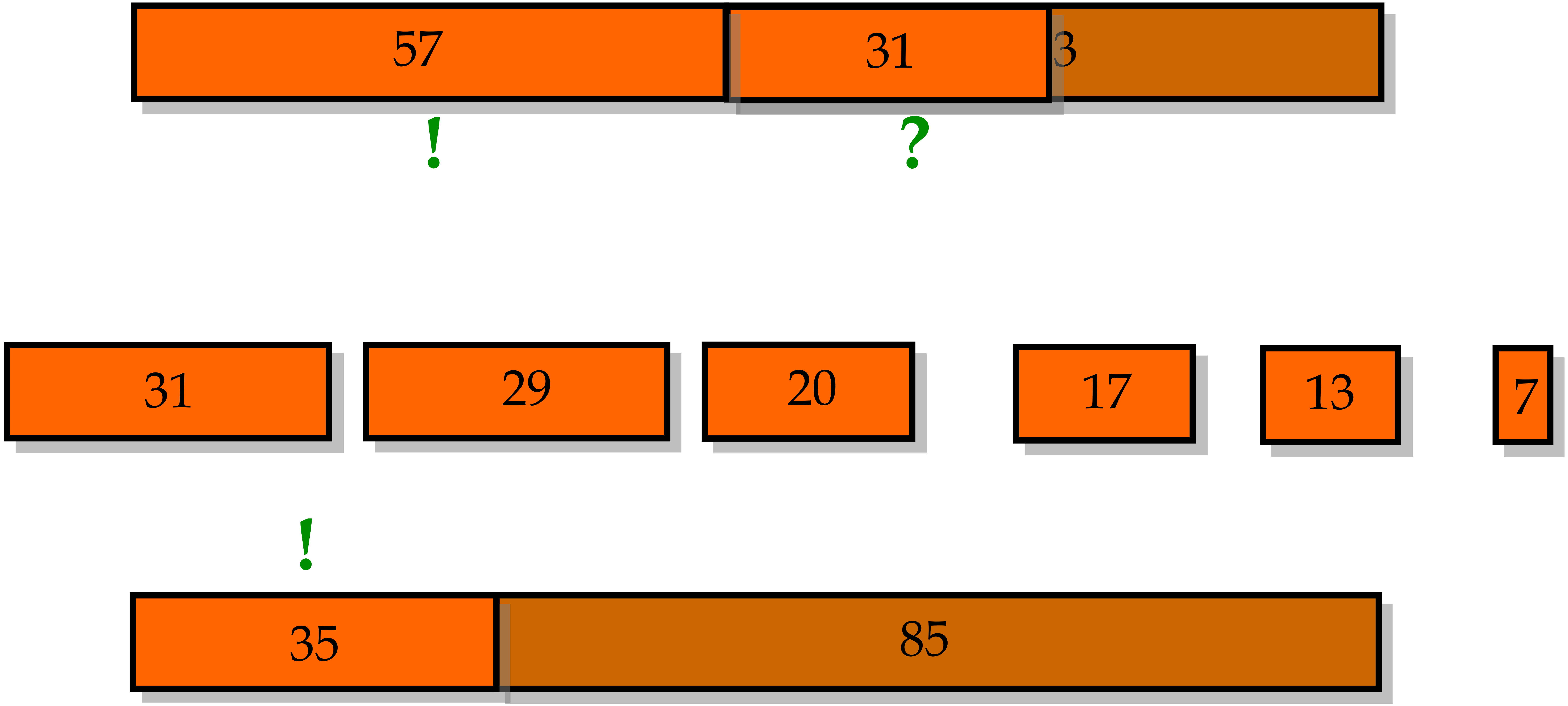

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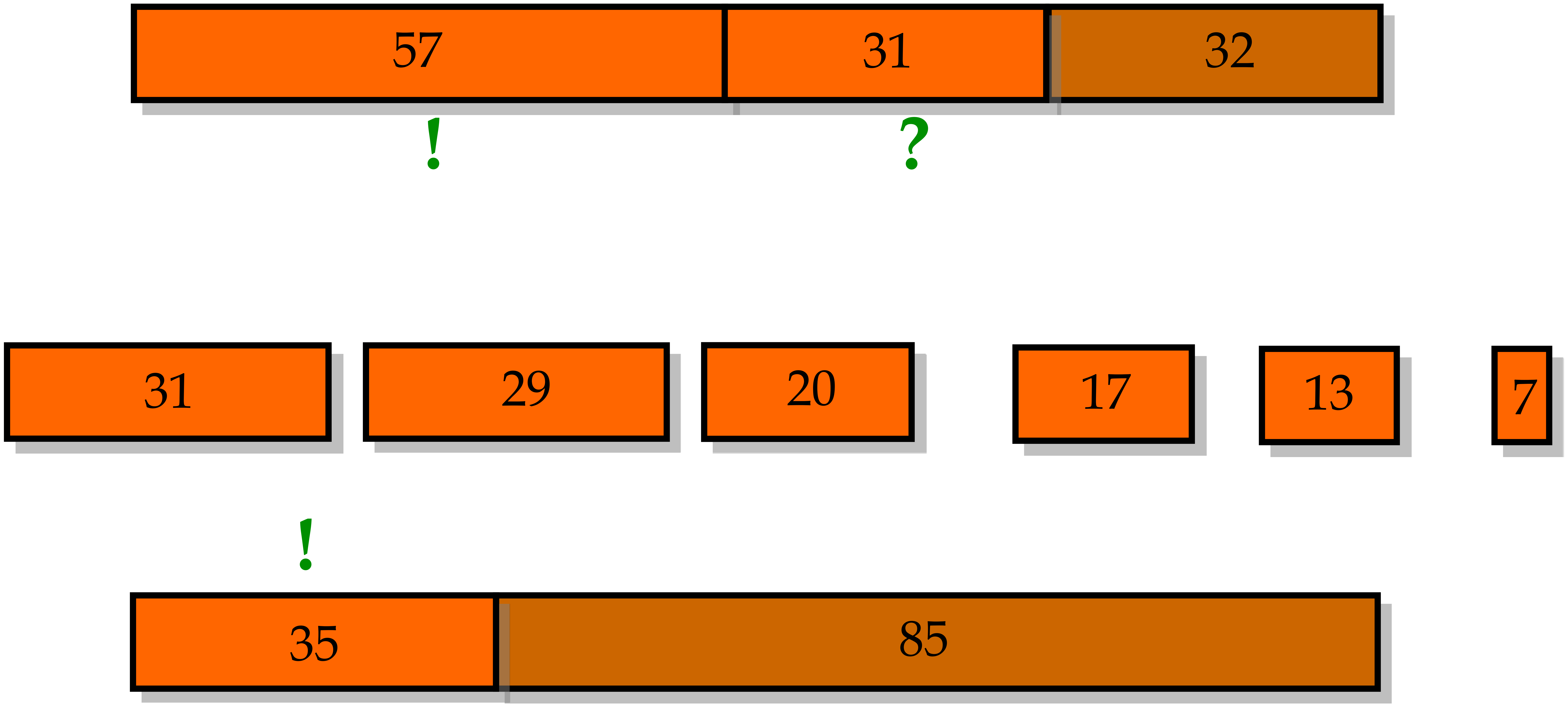

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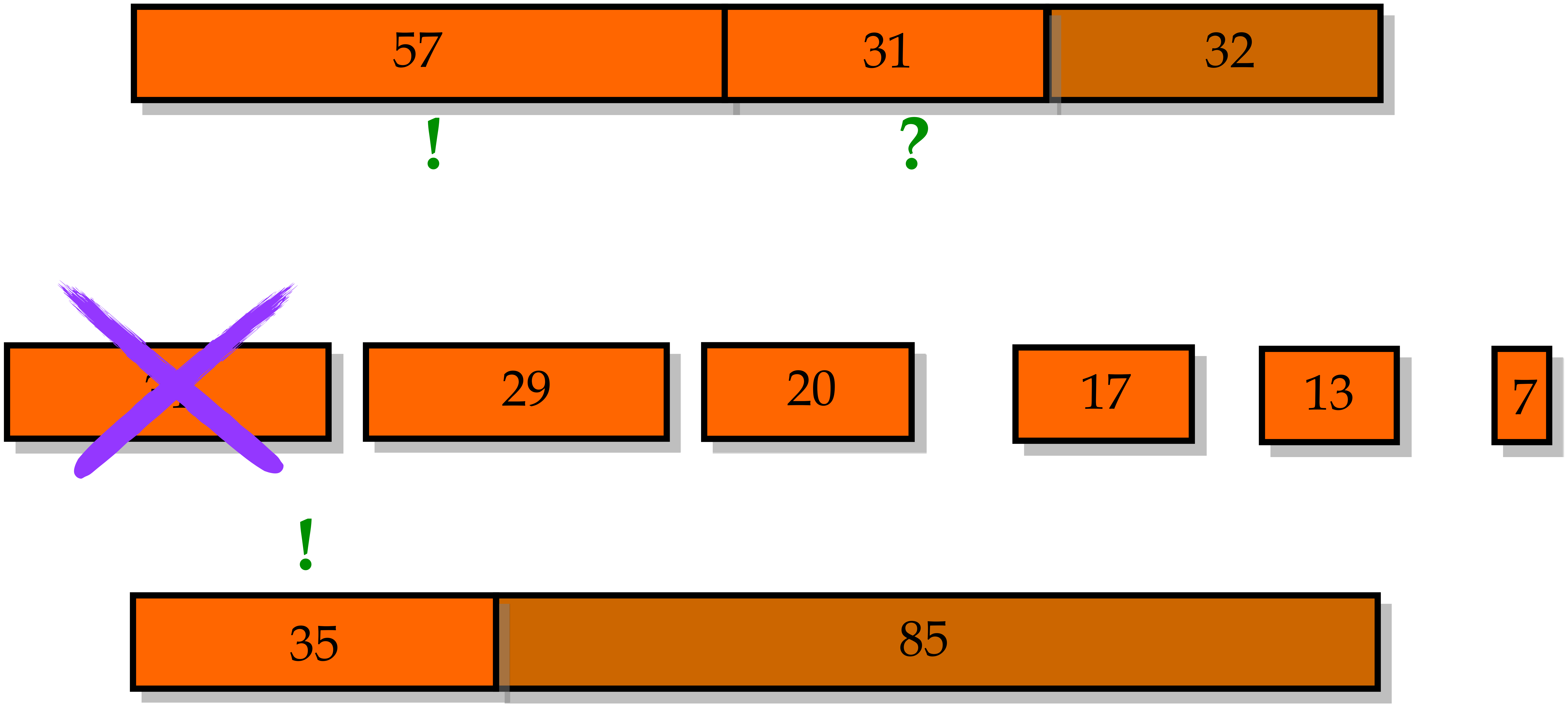

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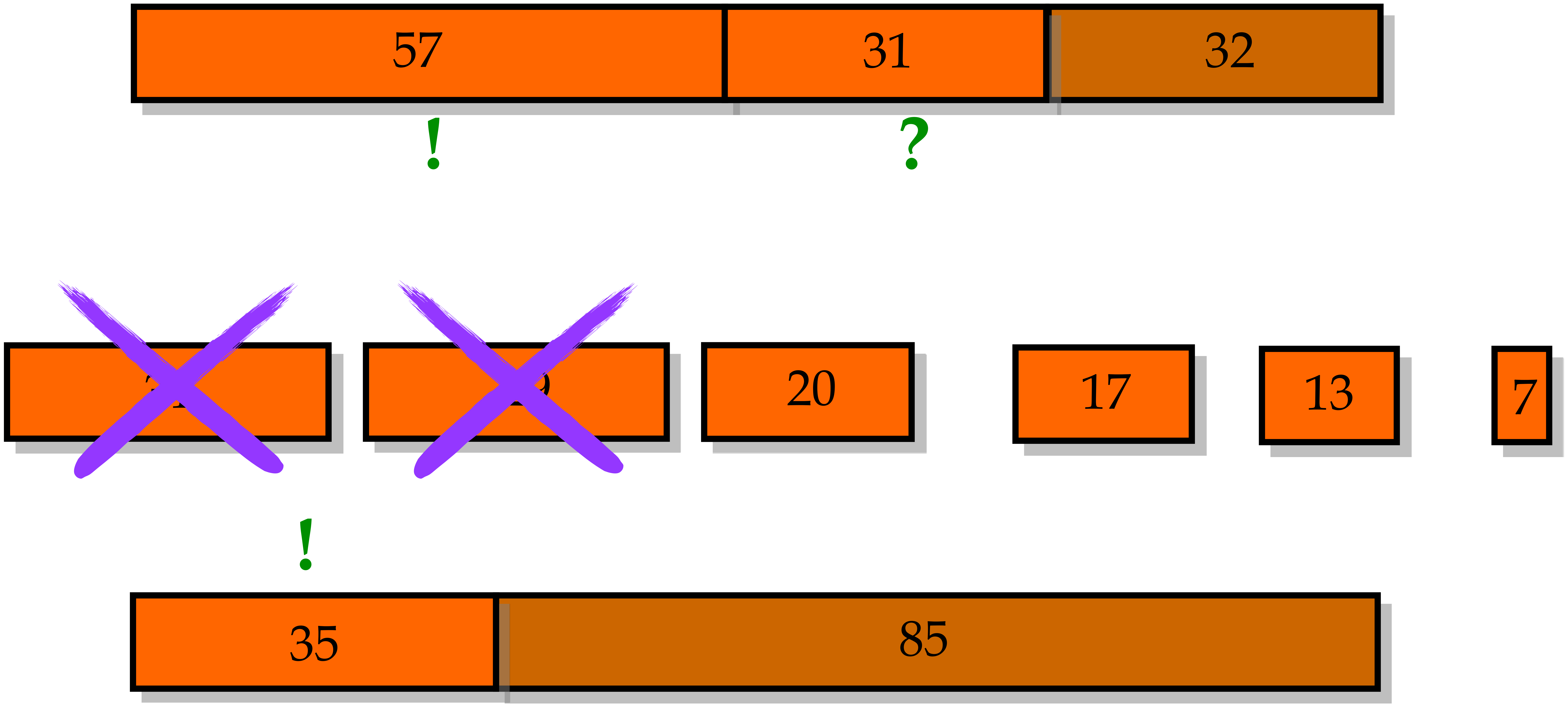

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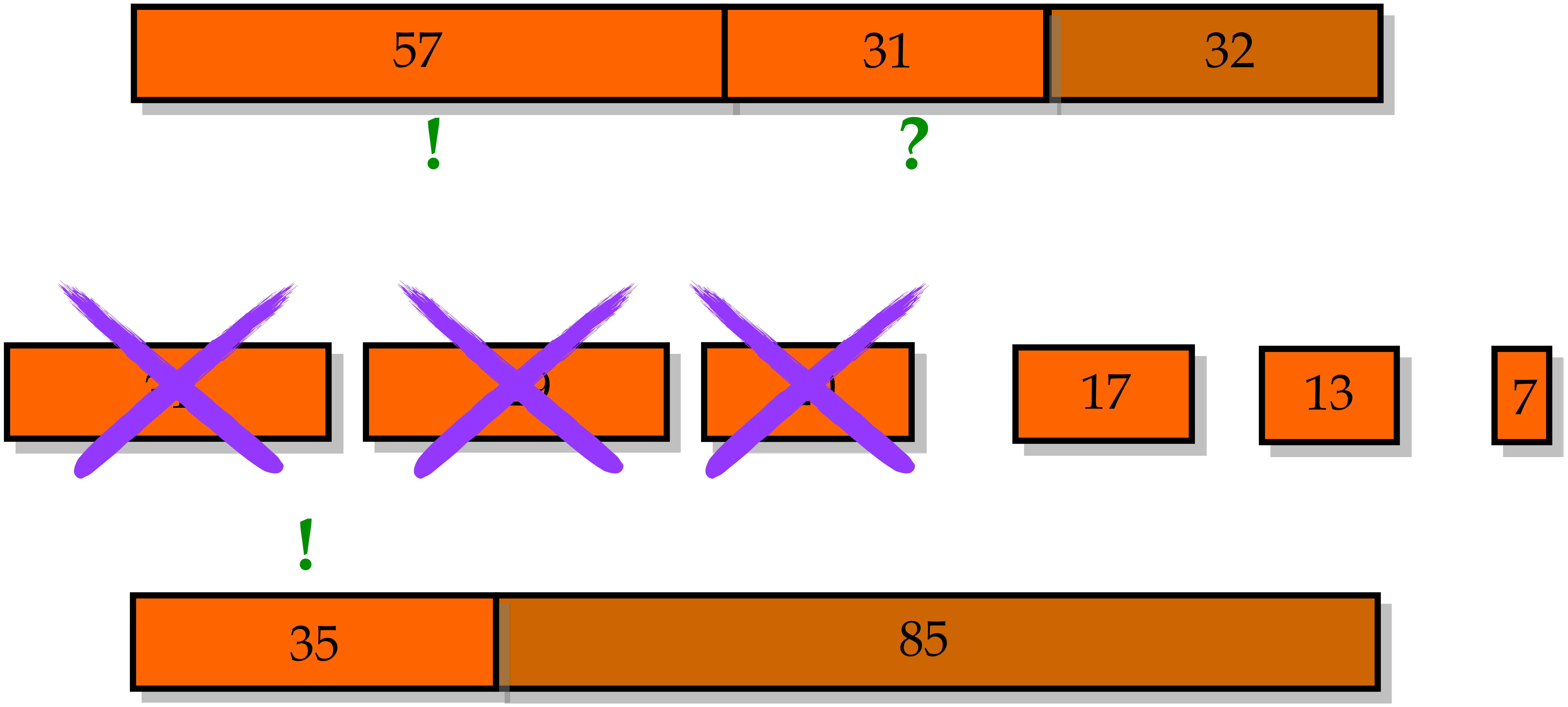

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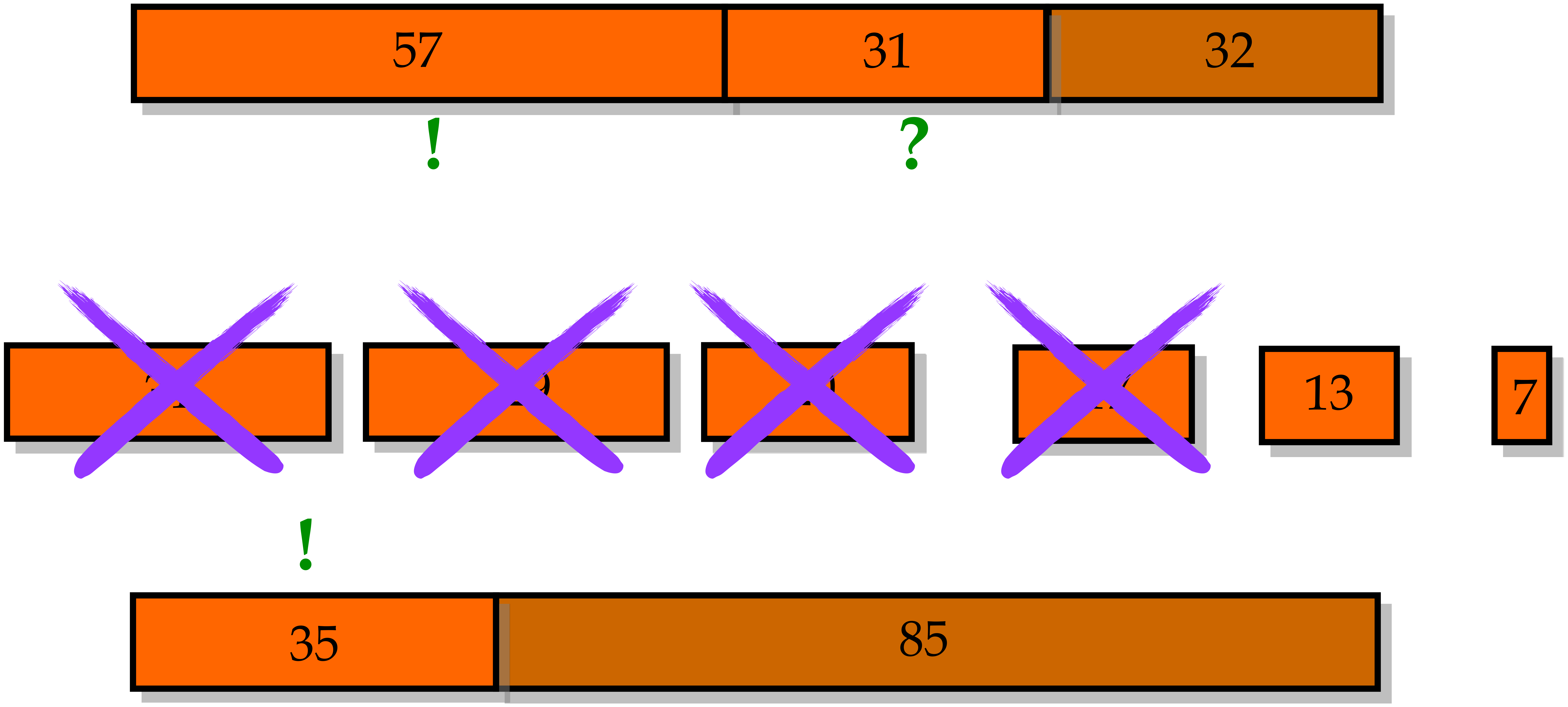

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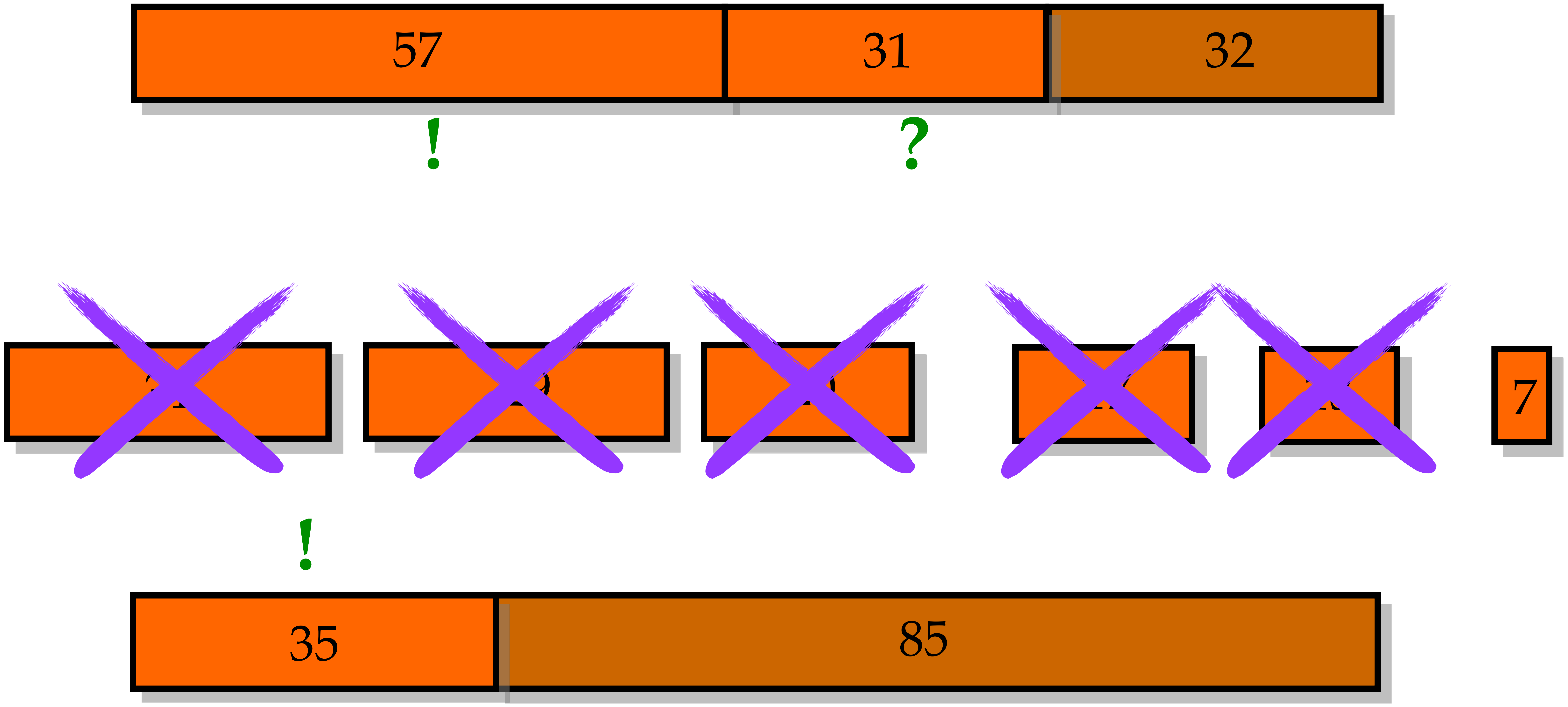

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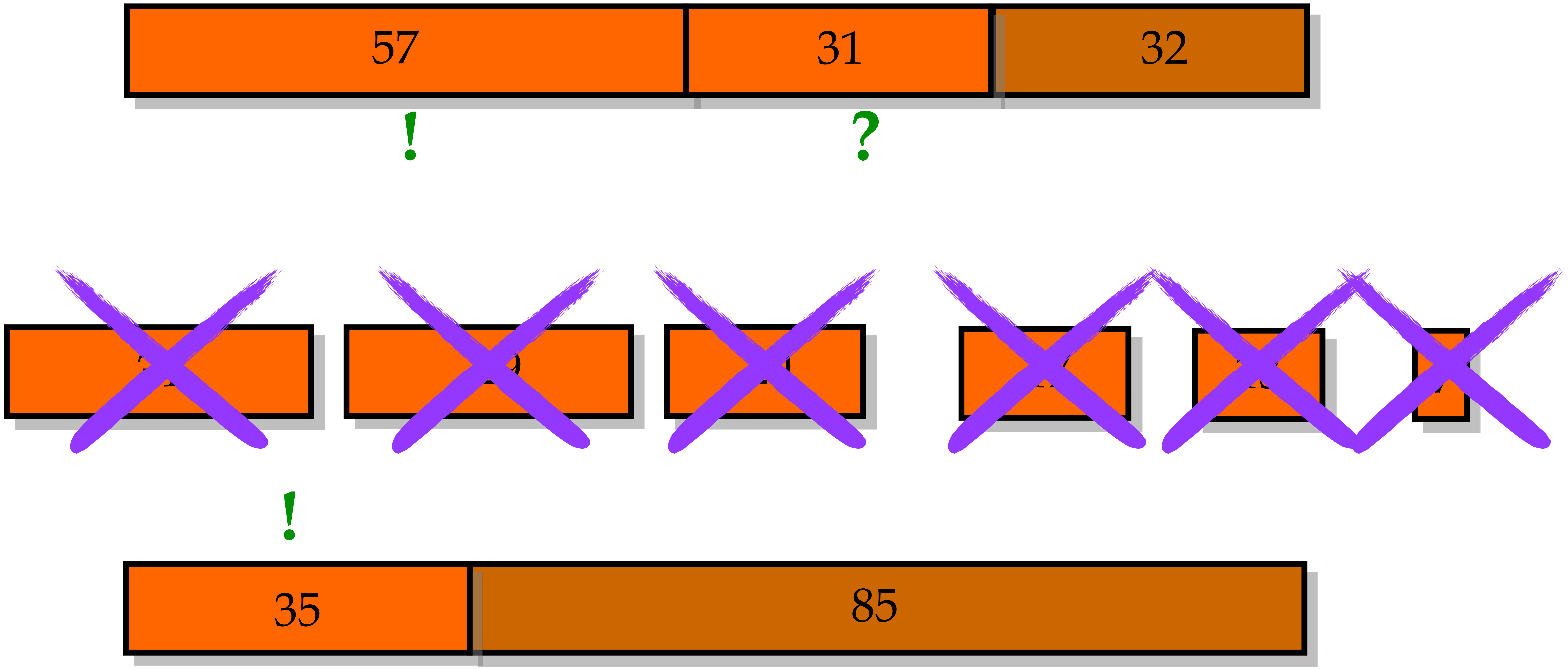

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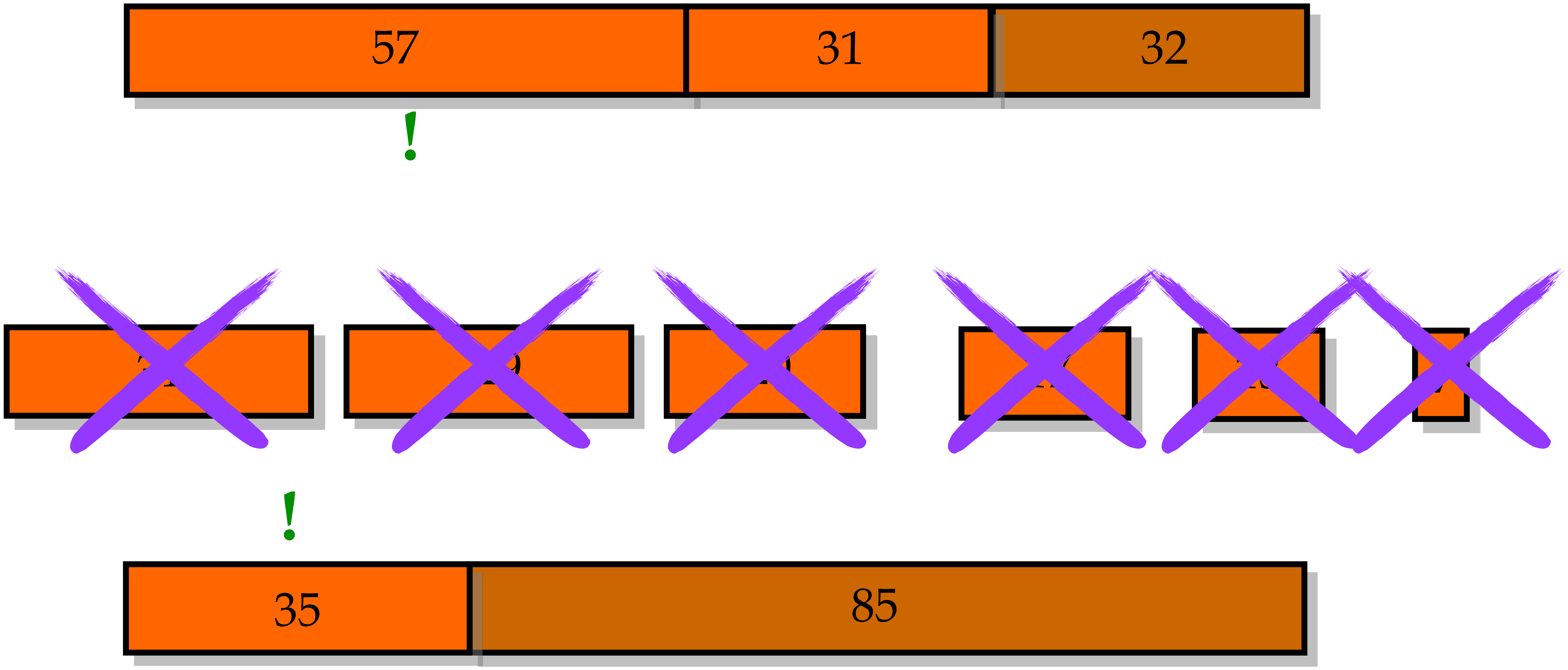

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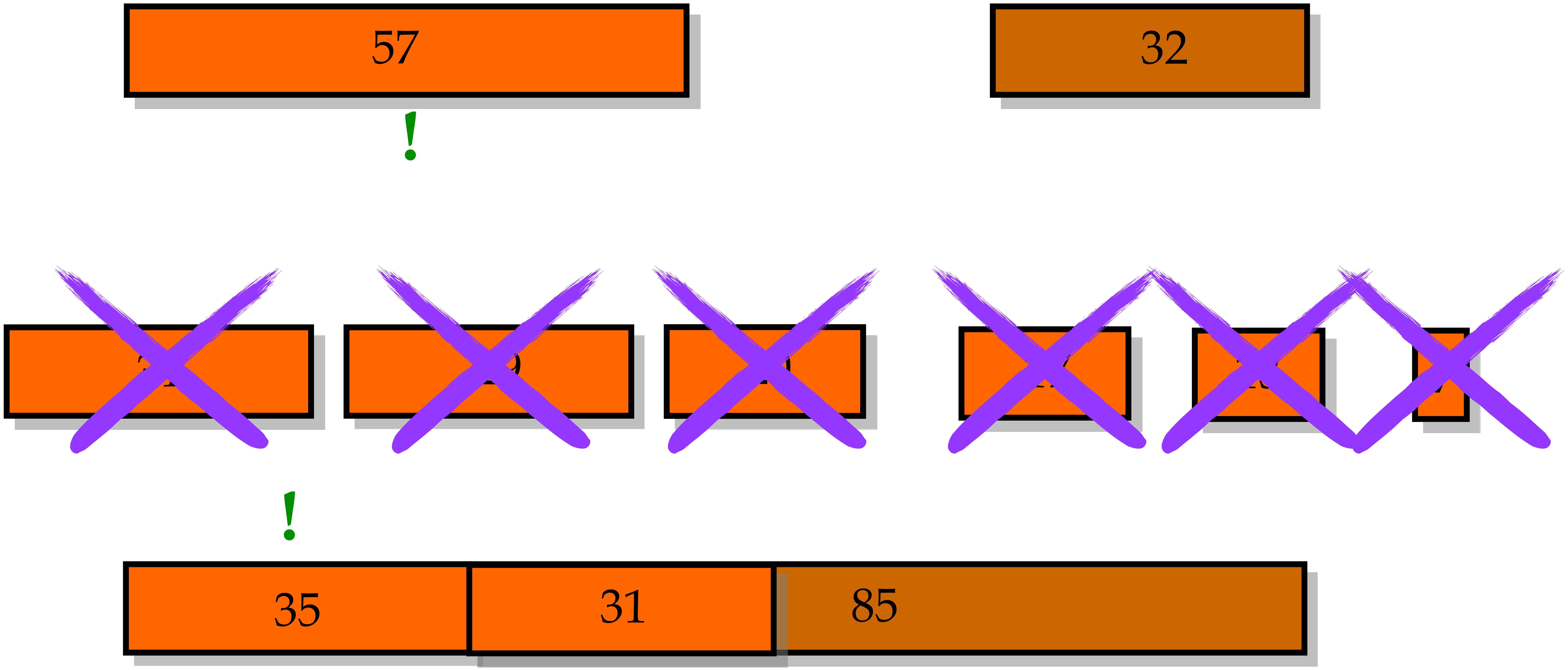

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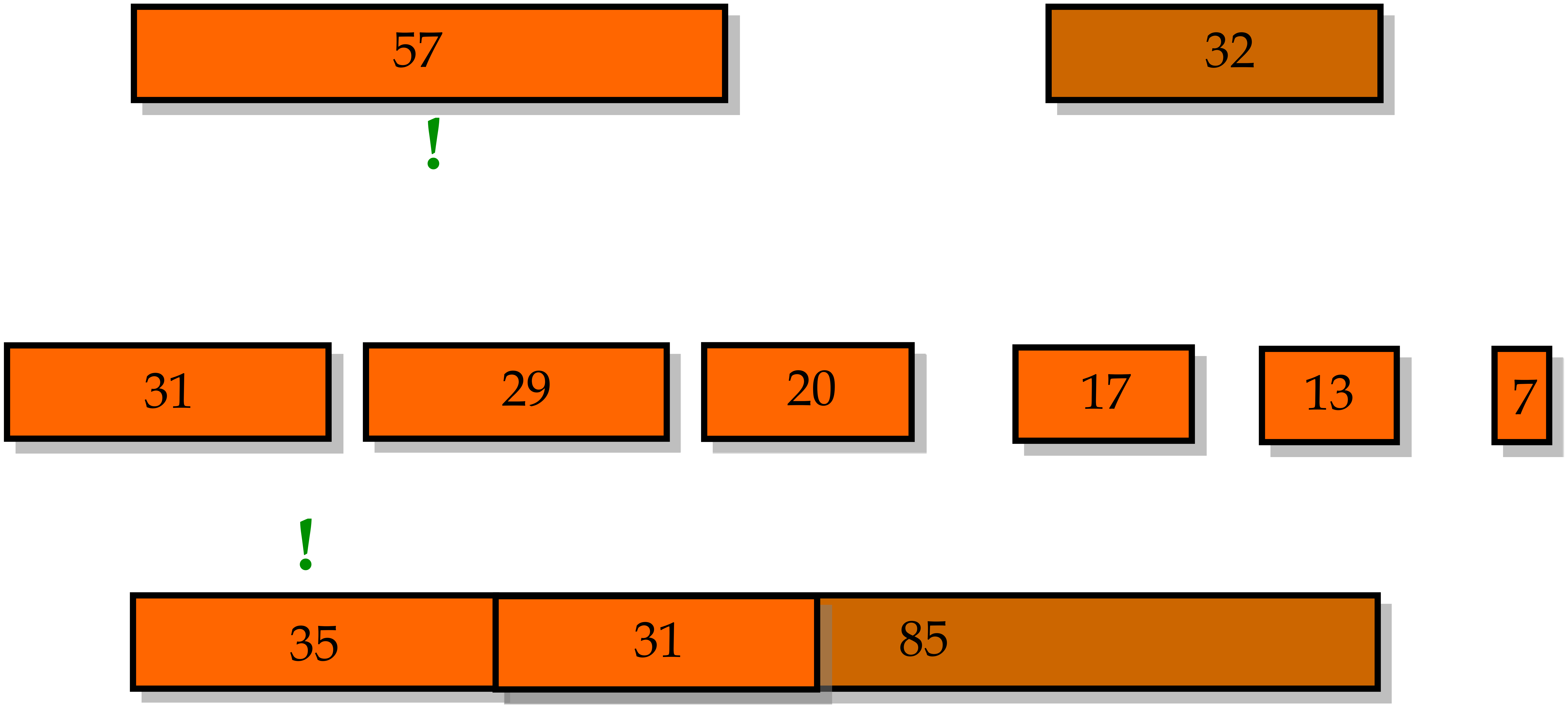

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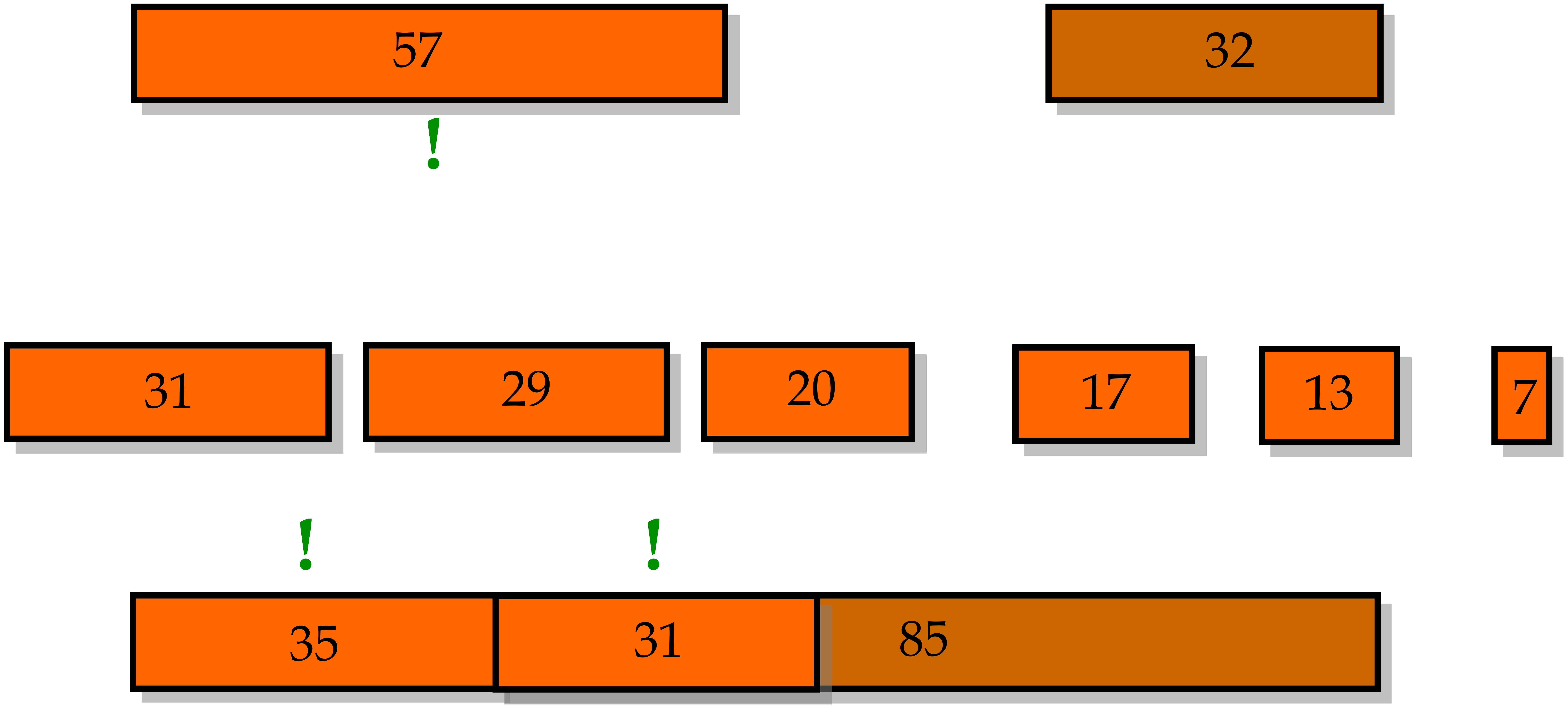

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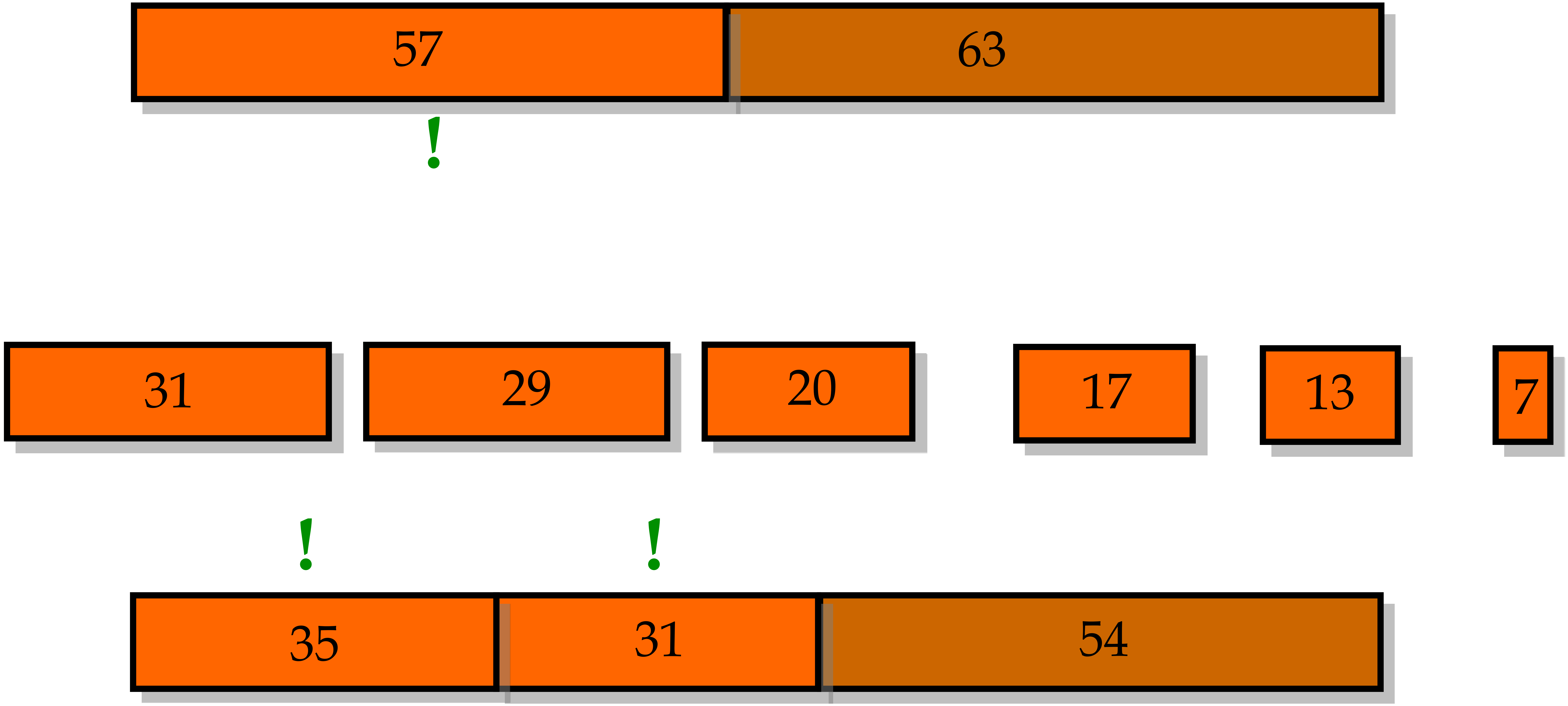
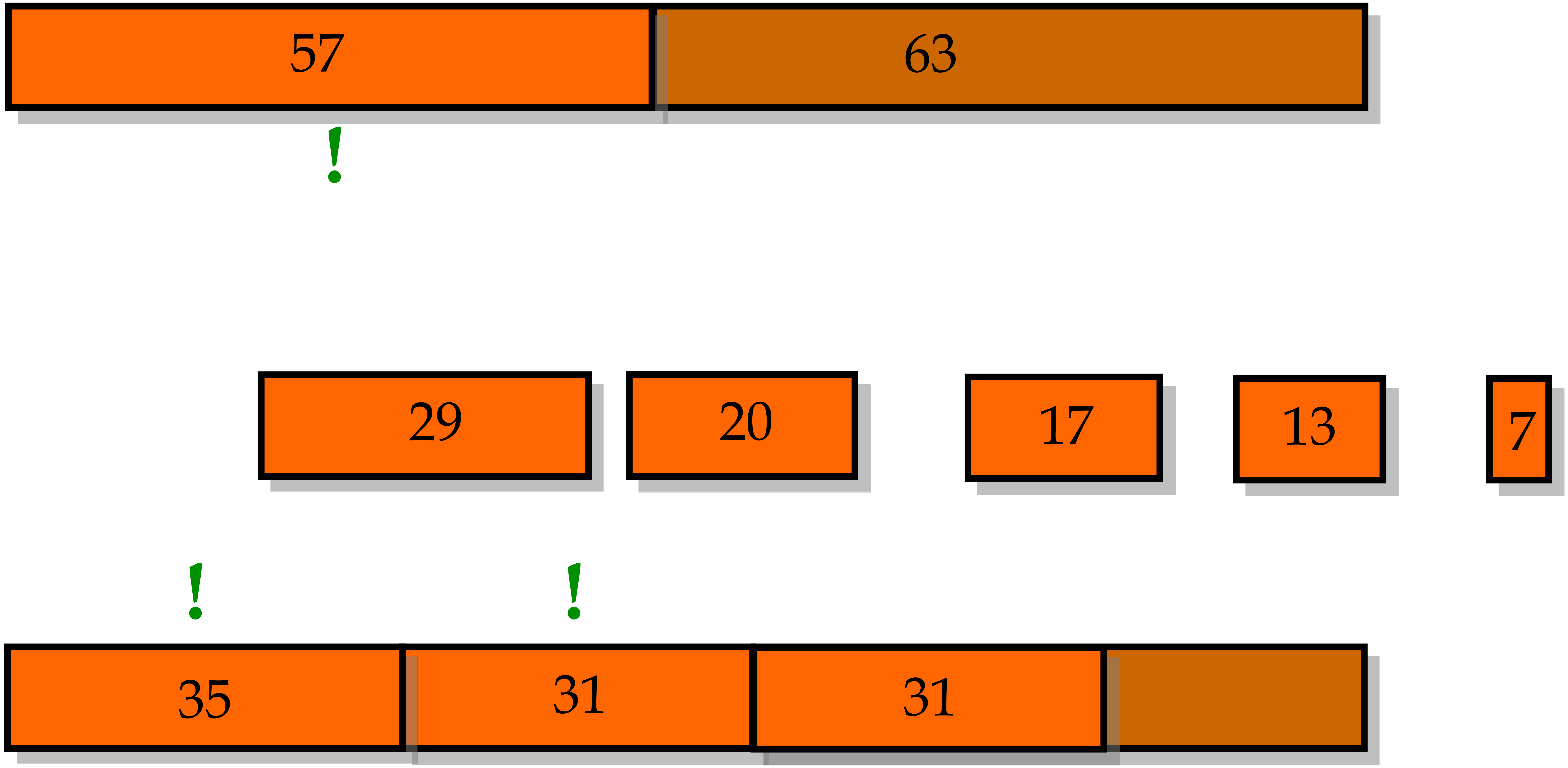


Keine Lösung?!



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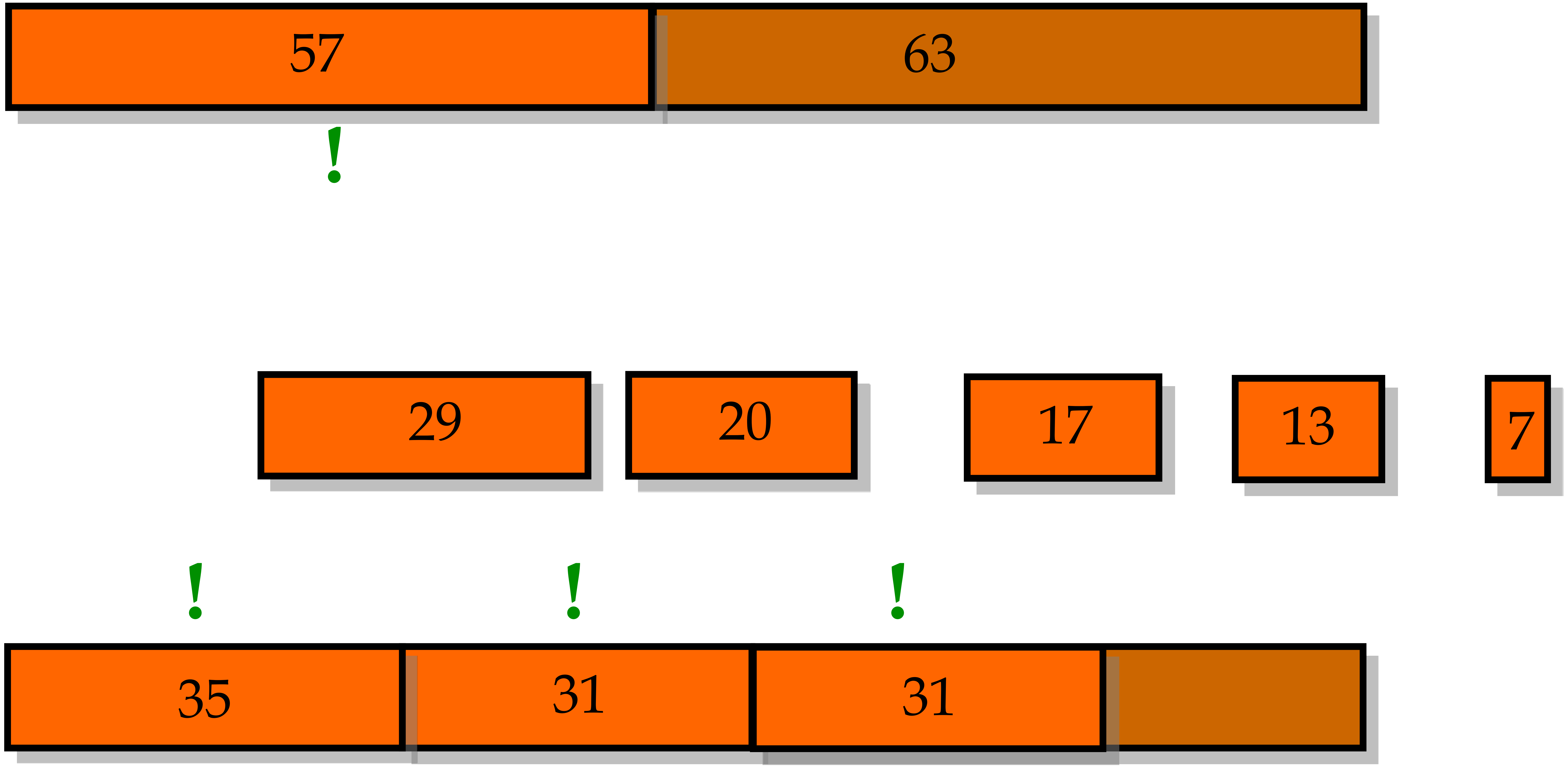

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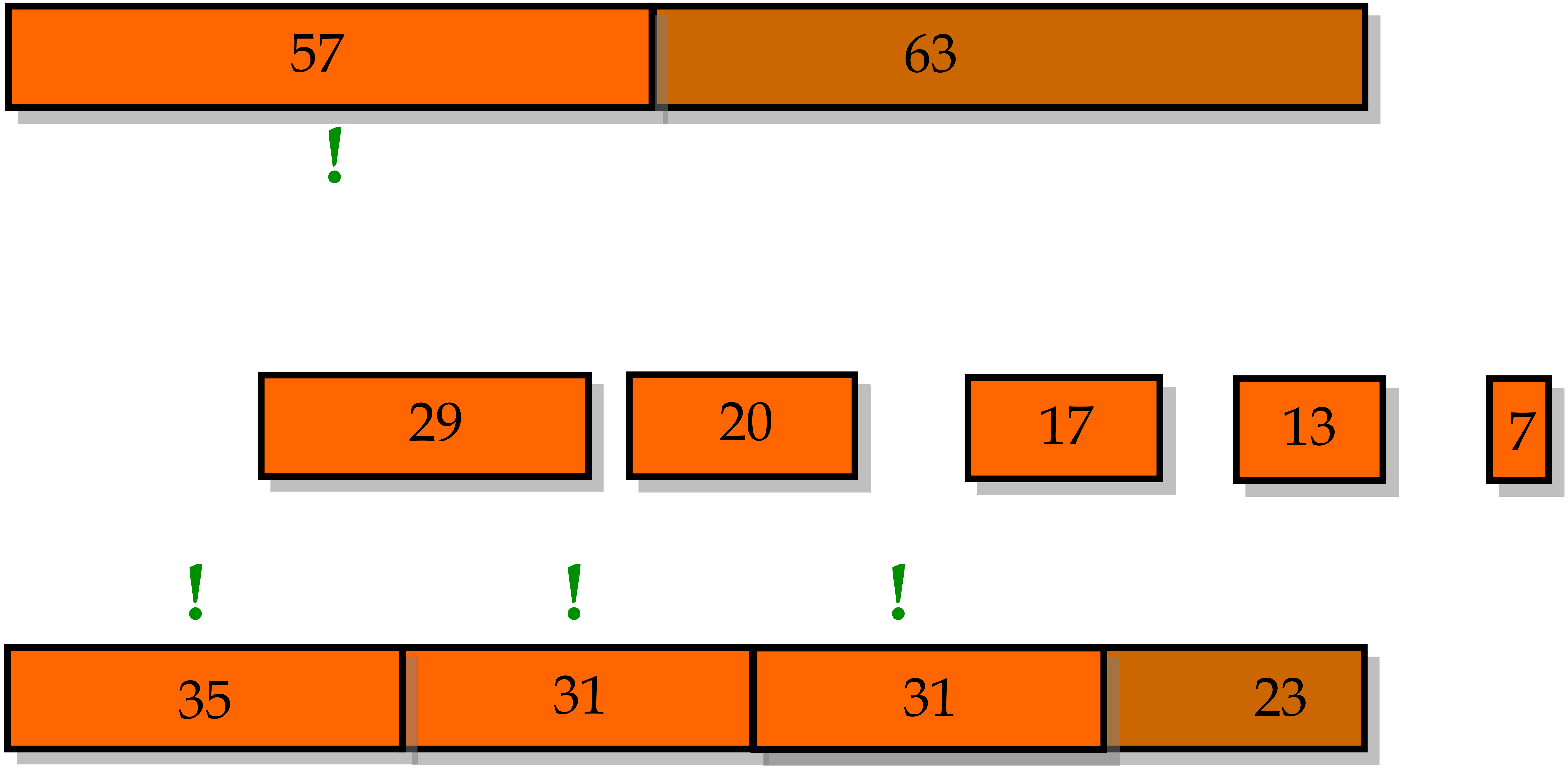

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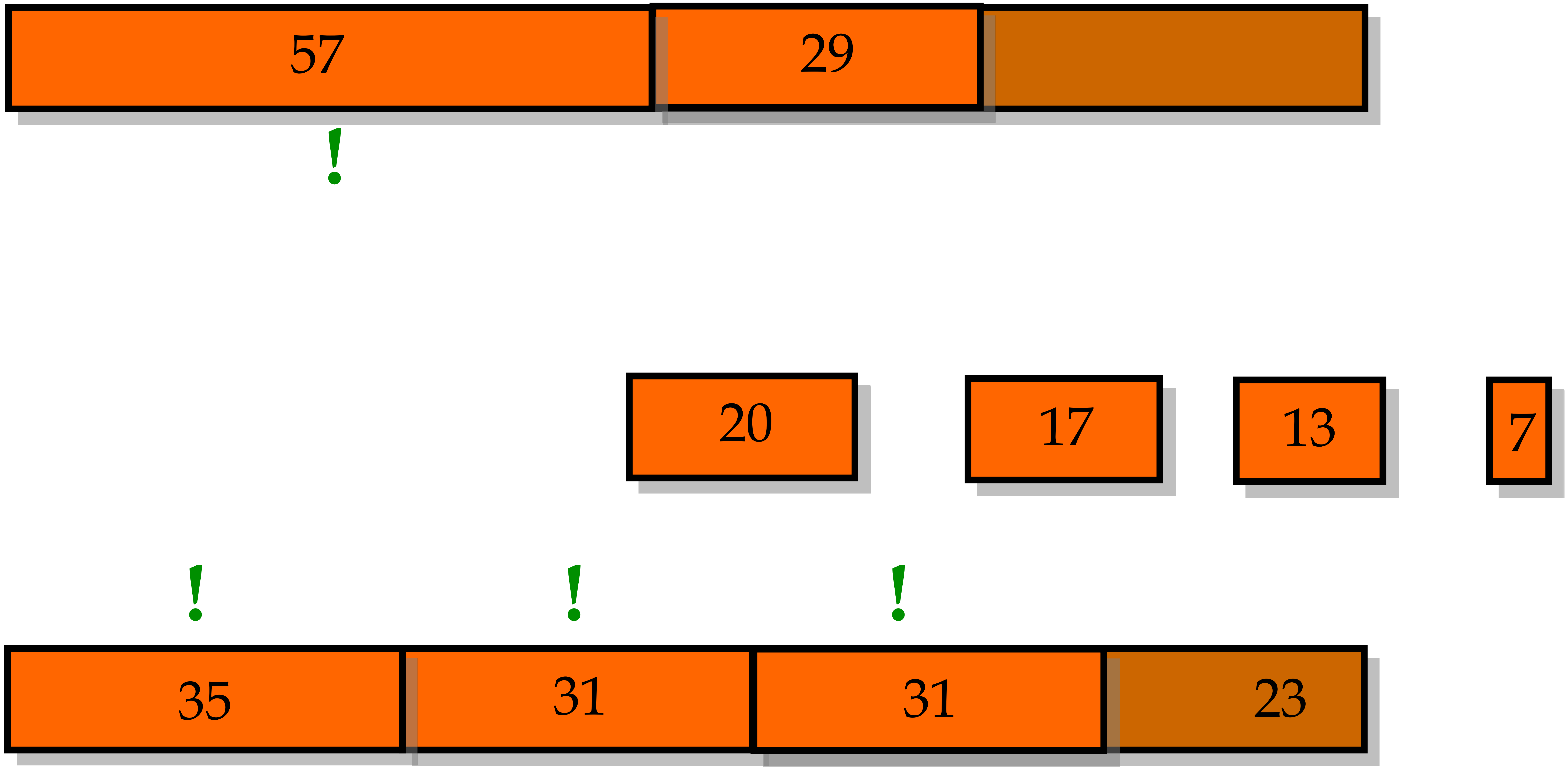

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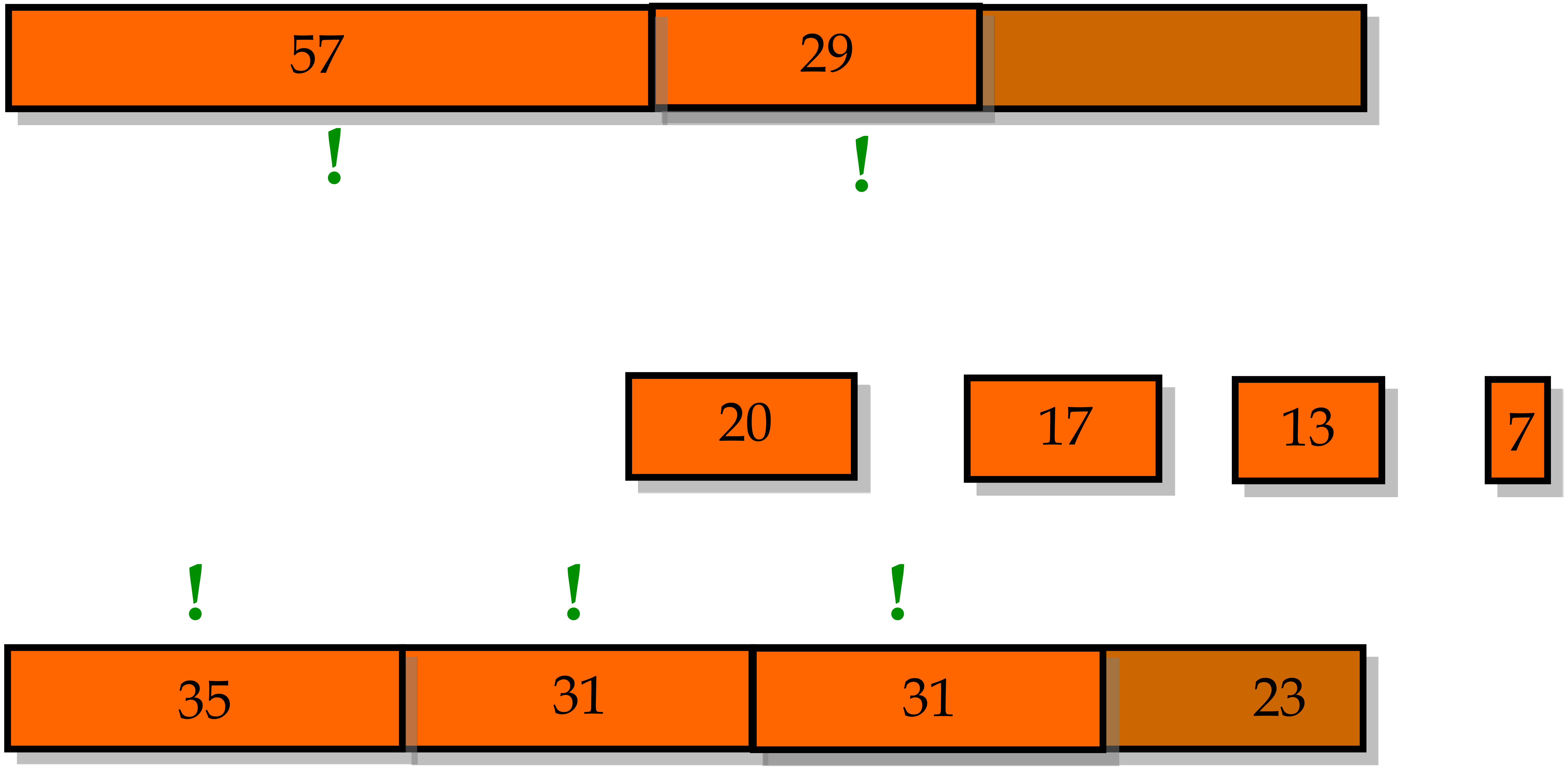

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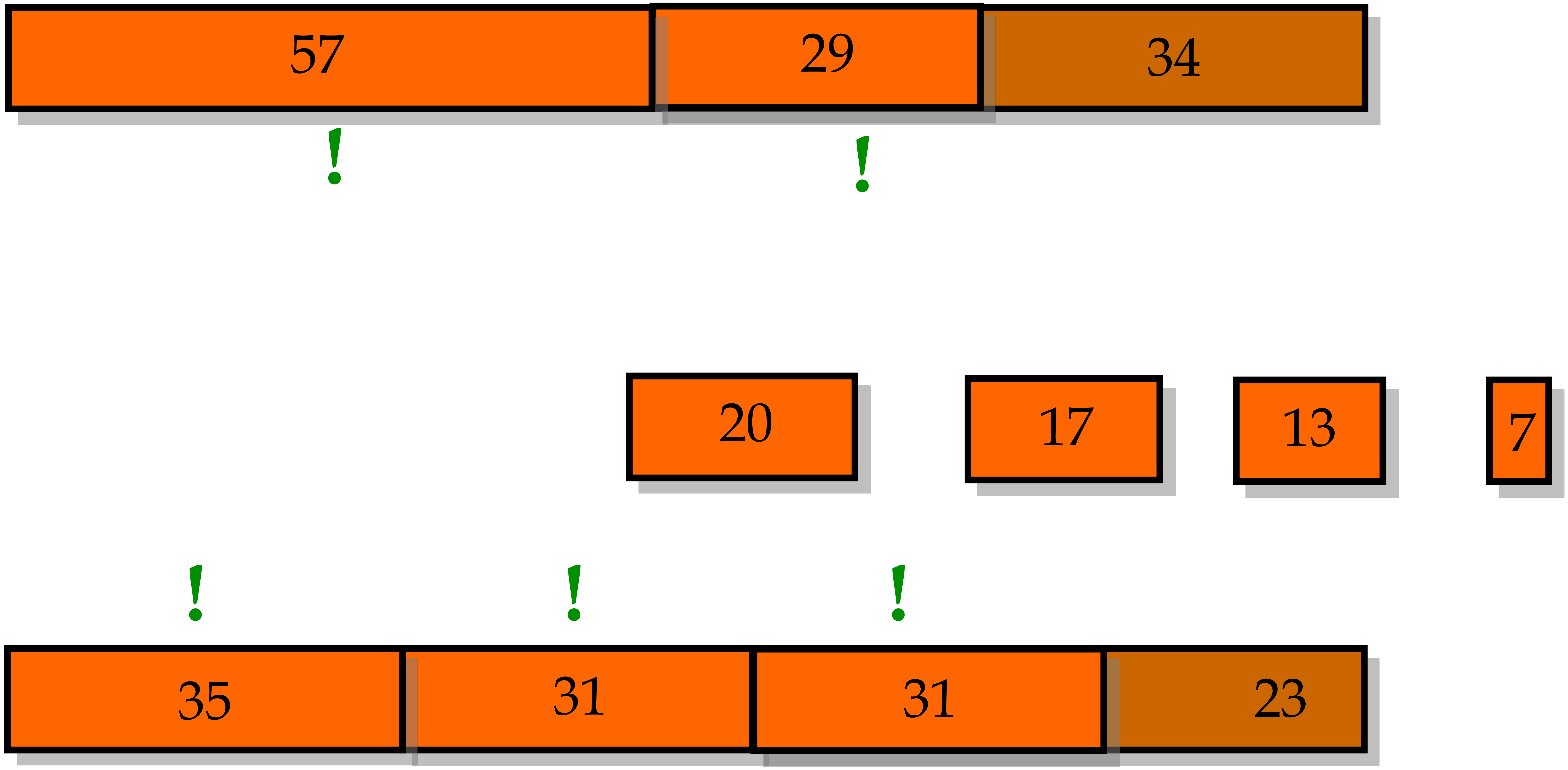

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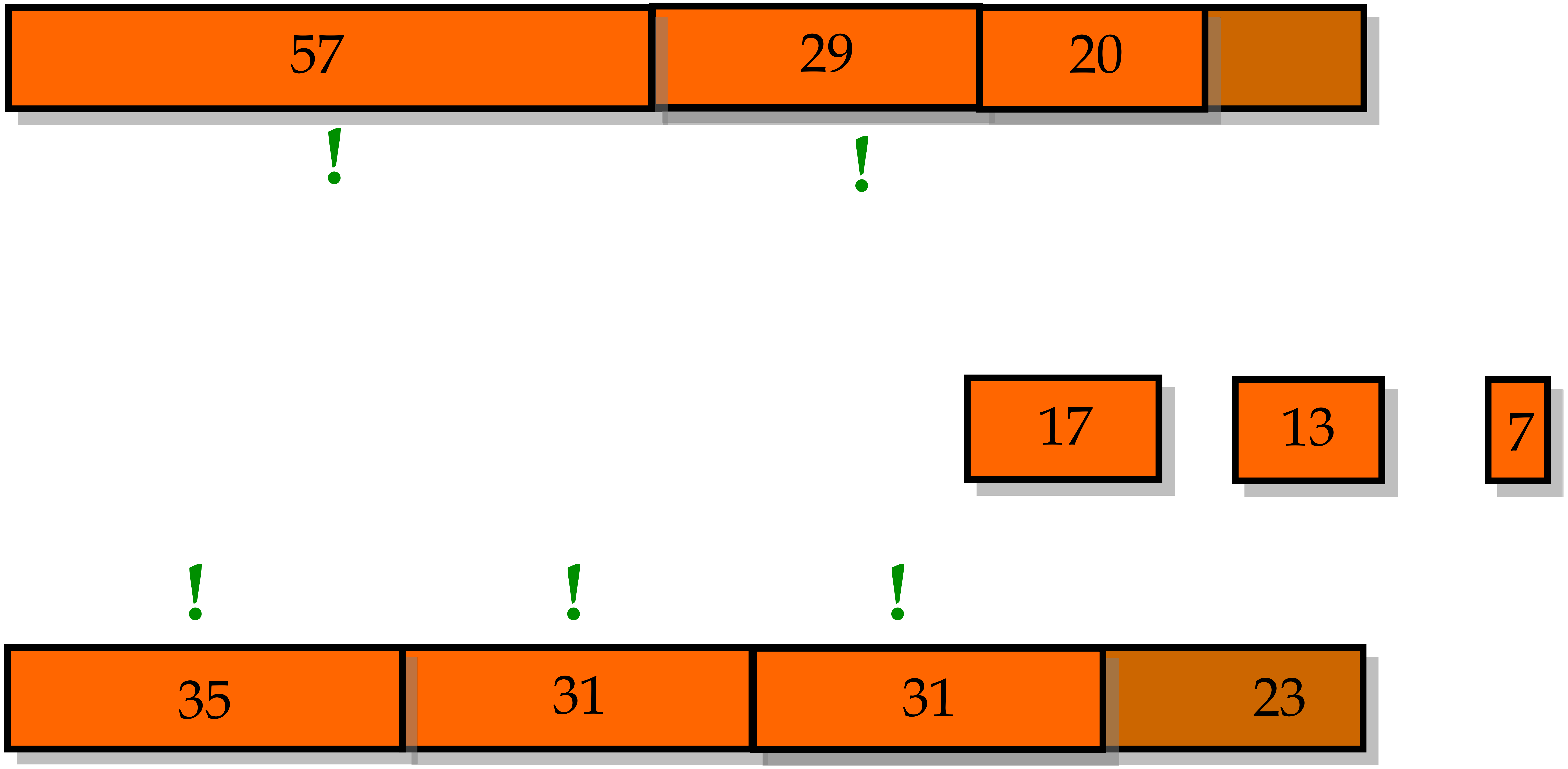

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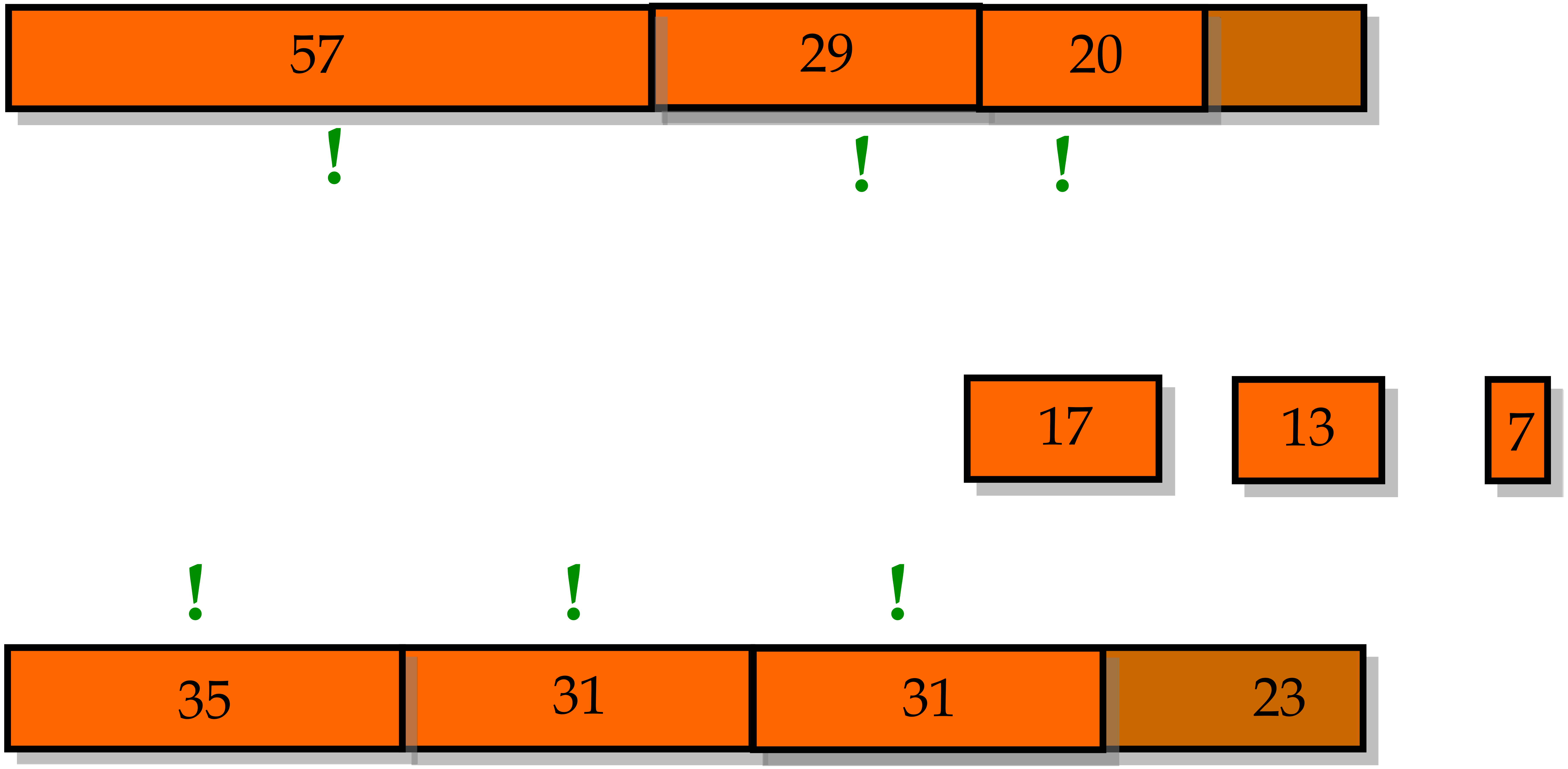

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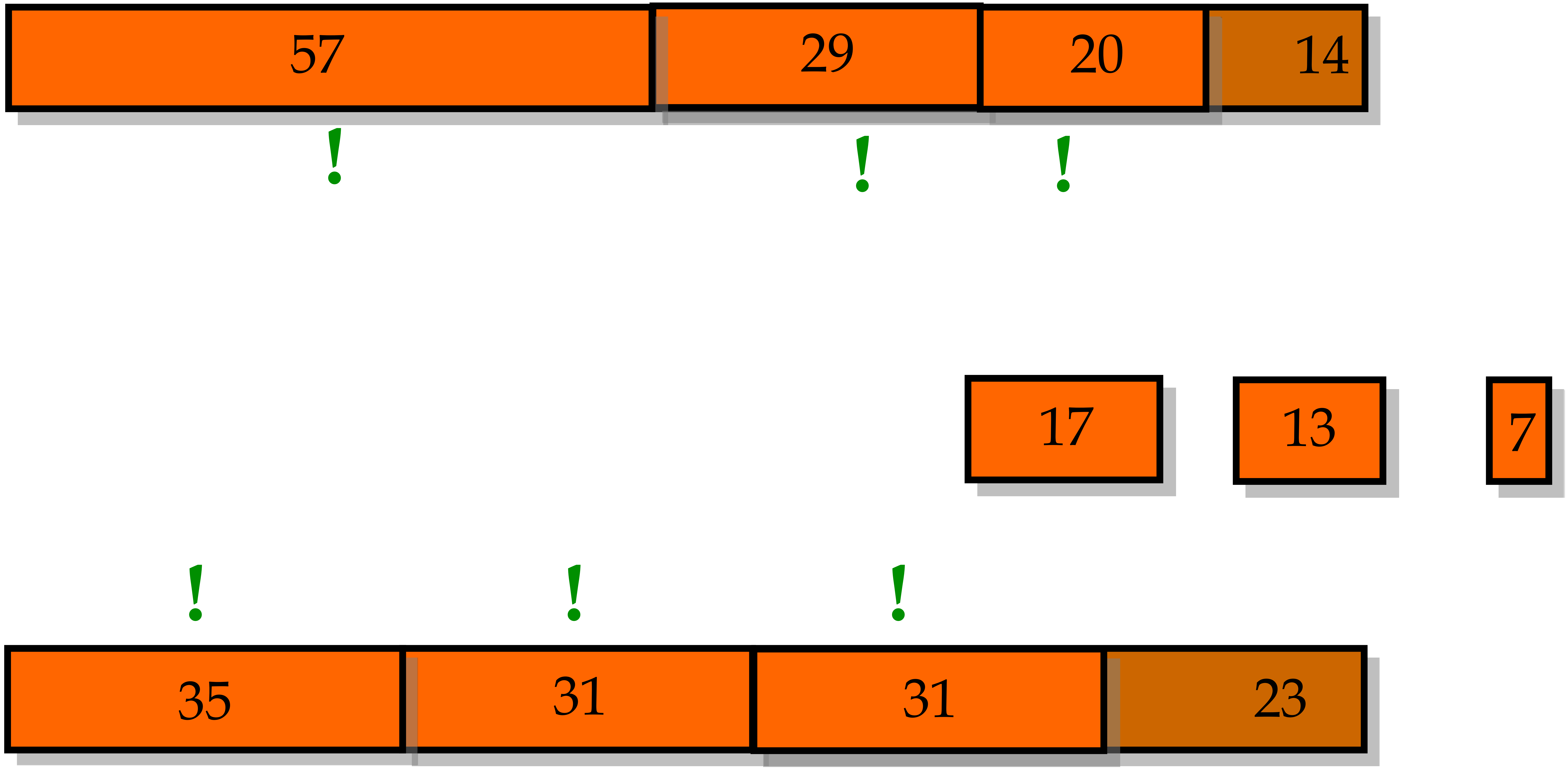

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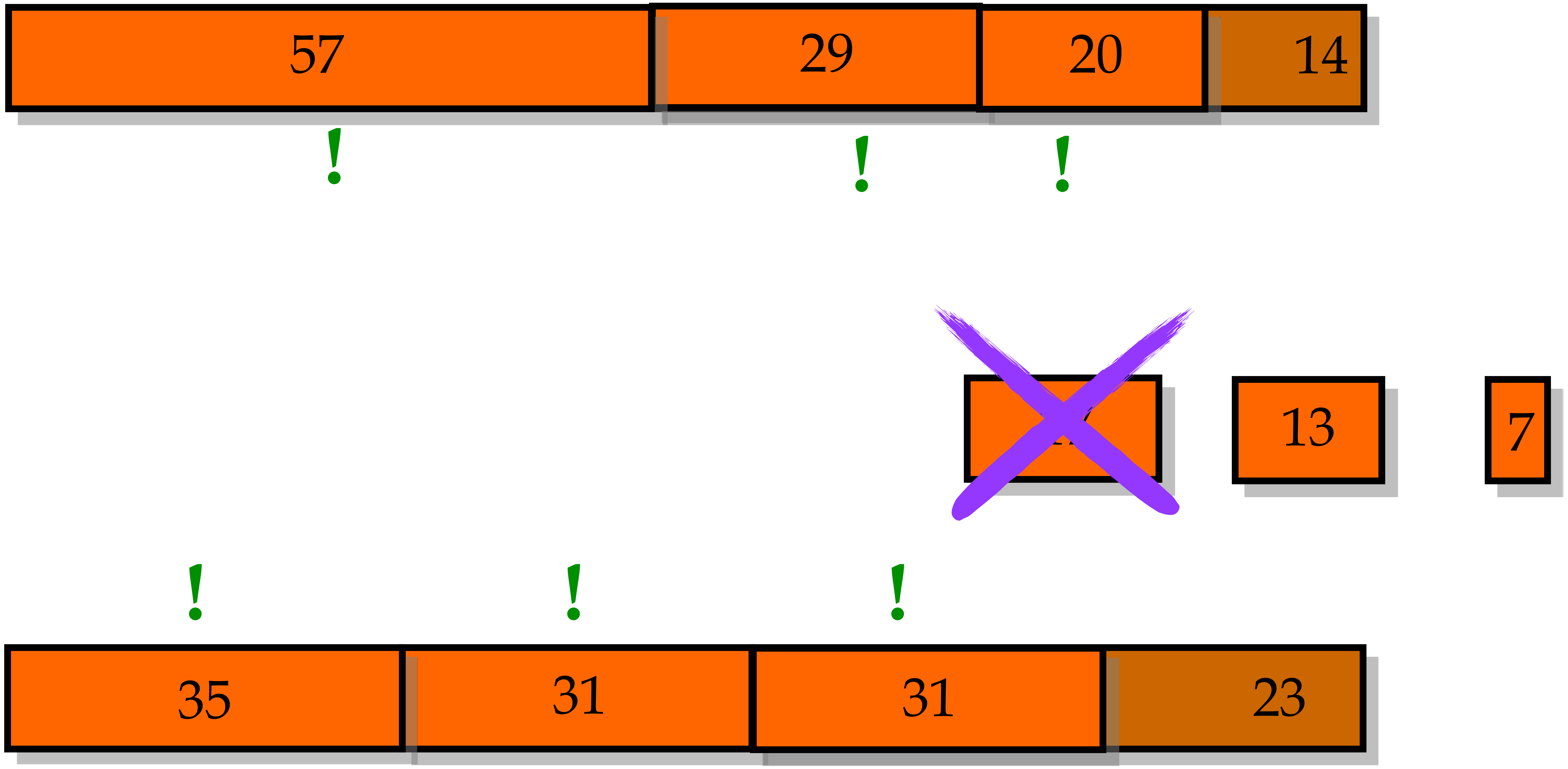

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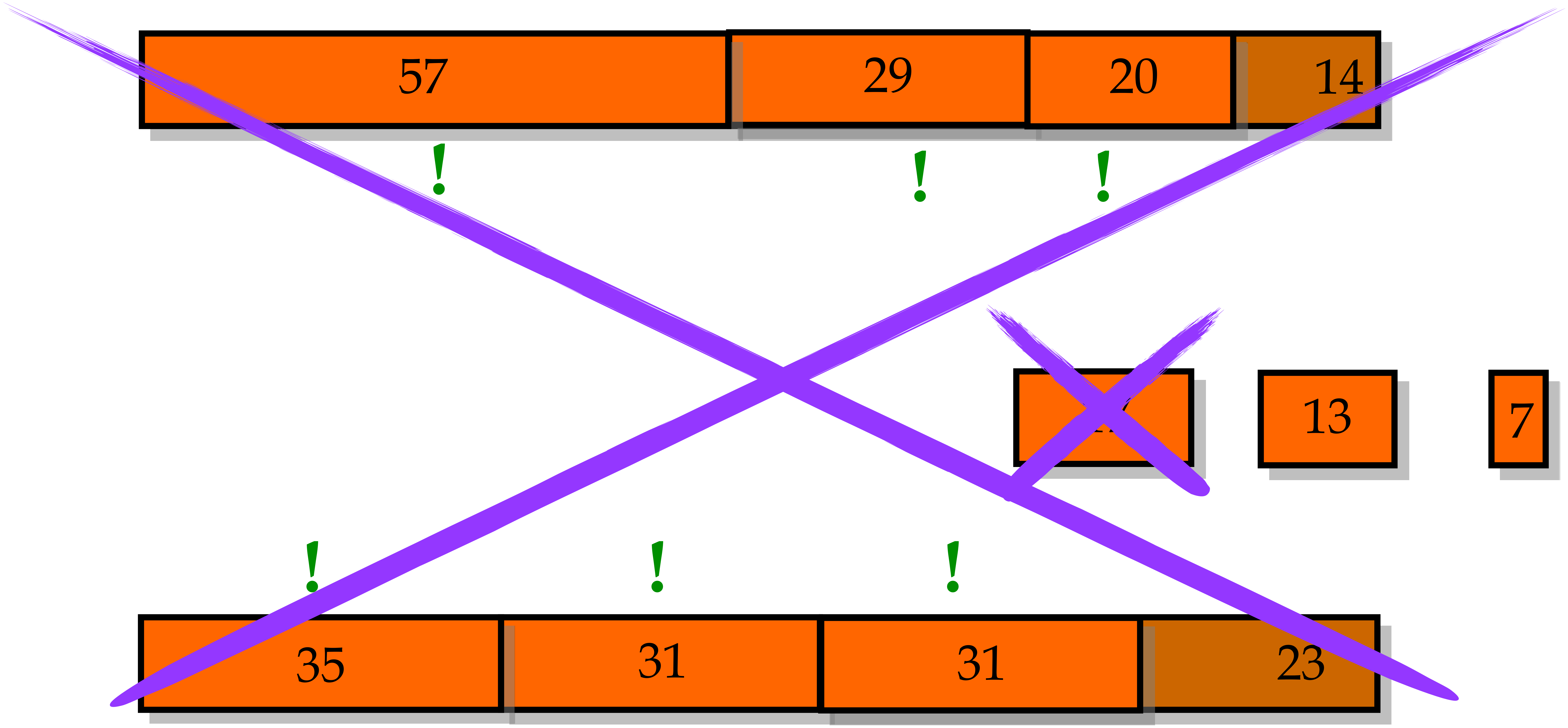
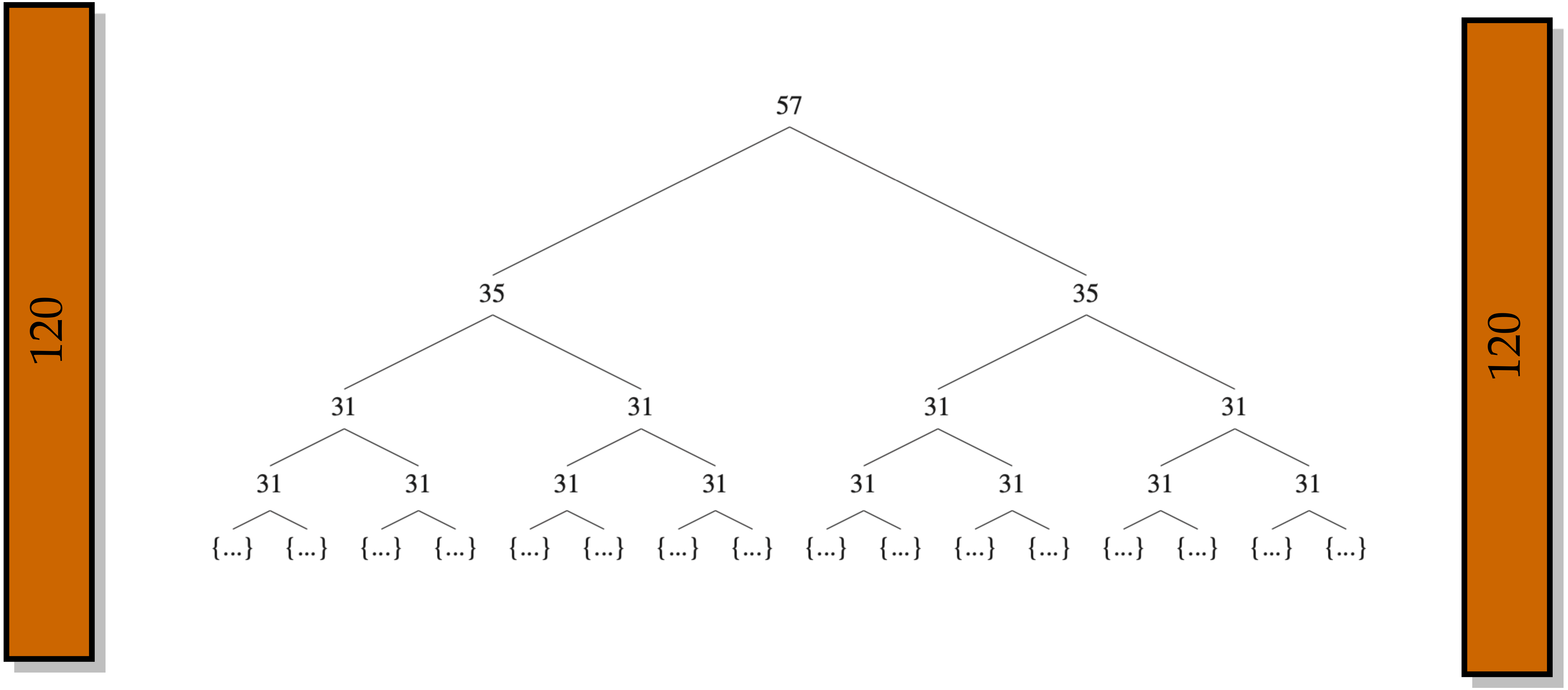
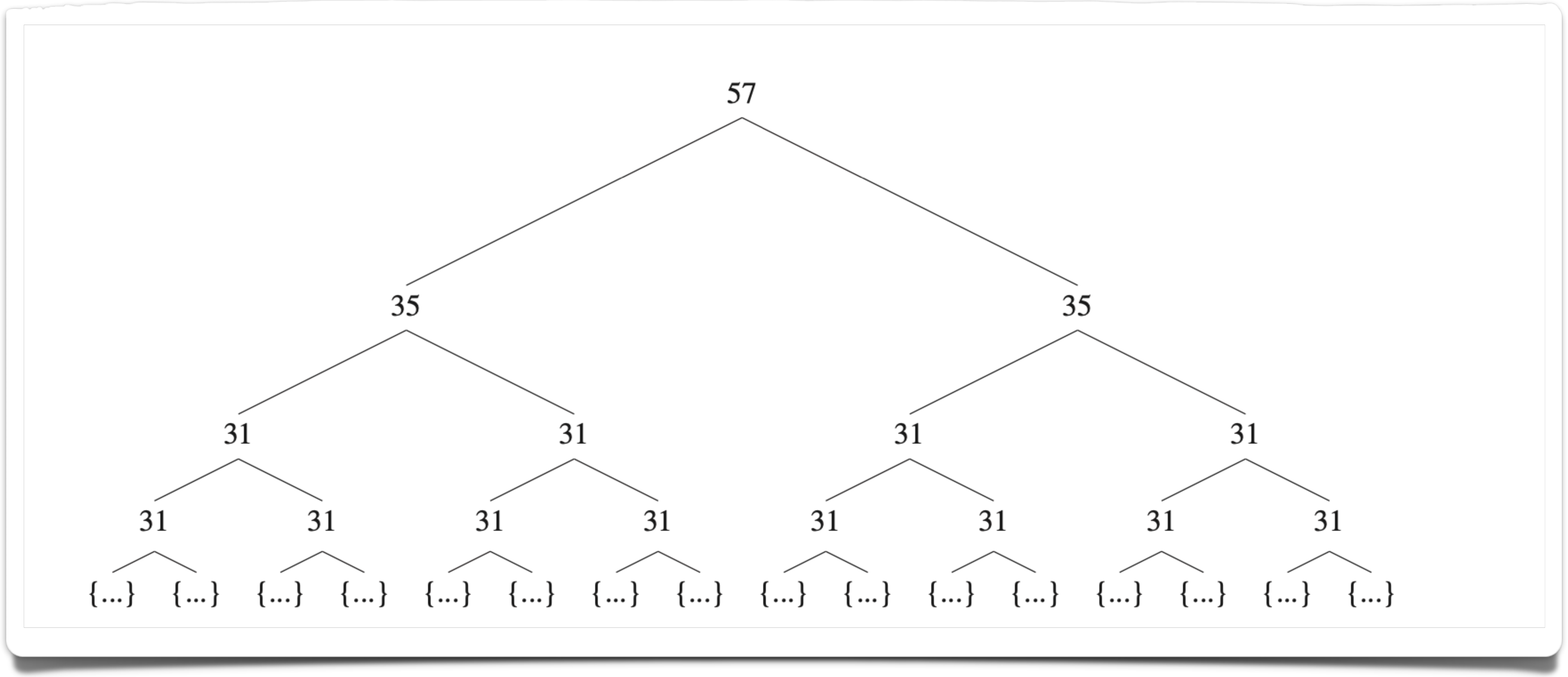


Enumerationsprinzip

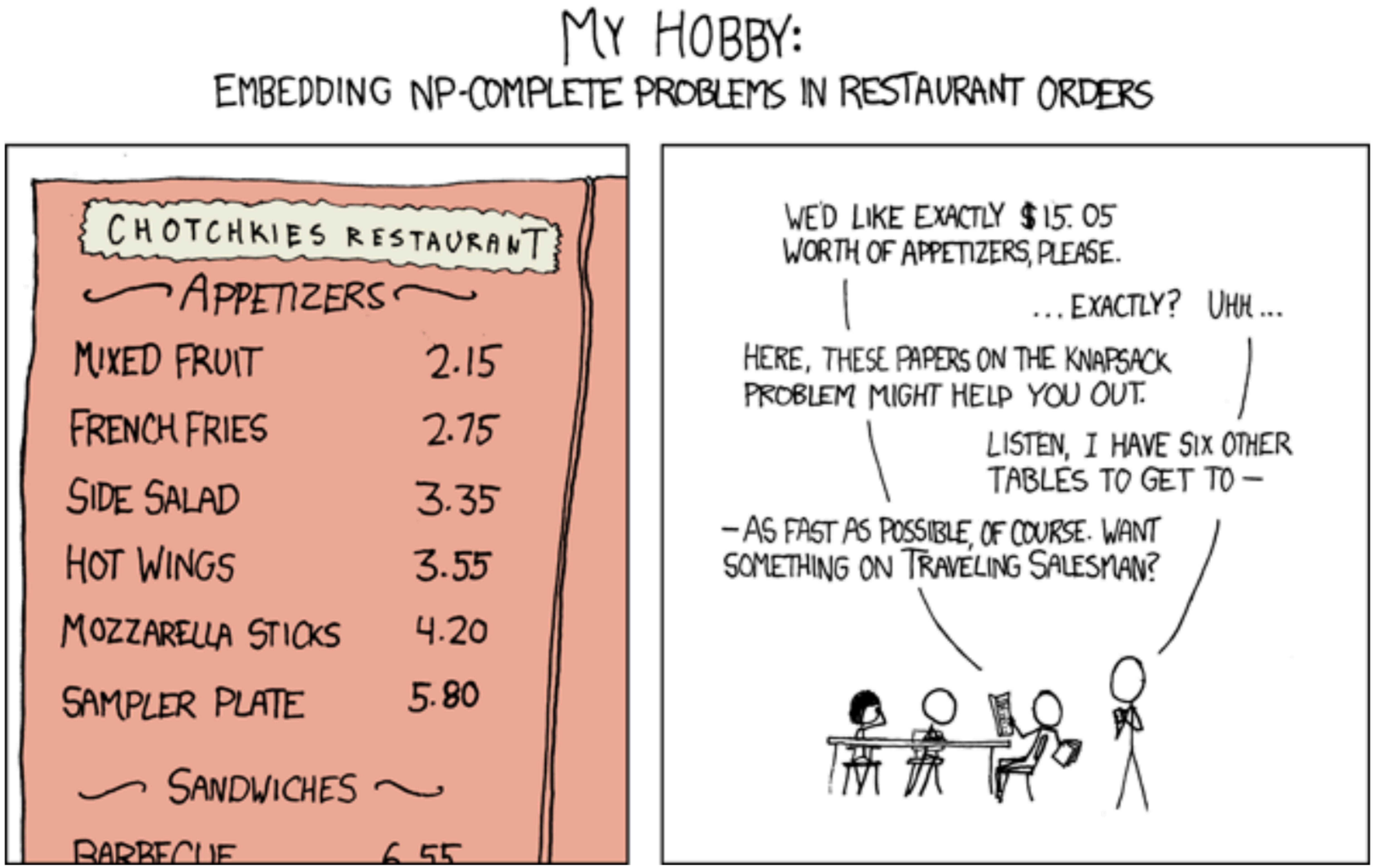


Enumerationsprinzip



- Exponentiell viele Fälle!
- Wie geht das systematisch?
- Wo kann man Arbeit sparen?

Alltagsanwendung: xkcd #287



Vielen Dank!

s.fekete@tu-bs.de